A Research on Non-Stationary Stochastic Demand Inventory Systems

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Abstract

Non-stationary stochastic lot sizing problem appears in many industrial environments where demand is non-stationary and stochastic, and replenishments require fixed freight fees. This paper presents the cost performance assessment of three inventory control strategies proposed for the stochastic lot sizing problem, namely Hua et al. (2008)'s strategy (HYHX), Tarim and Kingsman (2006)'s model (TK) and rolling horizon integrated version of Tarim and Kingsman's model (Rolled TK), against the optimal policy, non-stationary (s, S). The computational analyses show that the rolling horizon method implementation to TK escalates the model's performance, and HYHX and Rolled TK perform quite close to each other in terms of expected cost.

Keywords: Inventory control, stochastic lot sizing problem, Non-stationary demand, rolling horizon framework

1. Introduction

This research considers the periodic review inventory system with non-stationary stochastic demands, fixed replenishment setup costs, and linear holding and penalty costs over a finite planning horizon. The problem is to decide upon when and how much to replenish inventory so as to minimize total expected costs. The practical relevance of the problem is evident, since it appears in many industrial environments where demand is non-stationary and stochastic (e.g. due to short product life cycles), and replenishments require fixed freight fees (e.g. due to distant offshore suppliers) (Tunc et al., 2011, 2014).

The aforementioned problem has been revived by Hua et al. (2008). They presented an approach which is essentially an application of the static-dynamic uncertainty strategy (see Bookbinder and Tan, 1988) in the context of a rolling horizon framework. They proposed a static-dynamic uncertainty model which divides the planning horizon into a series of disjoint sub-intervals, i.e. replenishment cycles, and sets an order-up-to level for each of those cycles. Their approach relies on the idea of solving this model in each and every period within the planning horizon while implementing the decisions regarding only the imminent period. That is, if the model calls for replenishment for the first period, then a replenishment order is placed so as to increase the inventory position up to the inventory position specified by the model, and if not, no order is placed. The planning horizon is then rolled forward by one period. Hua et al. (2008) examined the performance of their approach by means of a numerical study, and used the model by Tarim and Kingsman (2006) as a benchmark. Their results indicated that their strategy is superior to the Tarim and Kingsman's model (Hua et al., 2008, p1259).

We acknowledge the efficiency and applicability of Hua et al.'s strategy; however the comparison is not fair. They use rolling horizon framework in their strategy whereas Tarim and Kingsman's model is not implemented in the rolling horizon scheme. We argue and show that the evaluation of the respective optimal policy in Hua et al. (2008) is also not acceptable. Accordingly, our focus here is on three alternative inventory control strategies, namely Hua et al.'s strategy, Tarim and Kingsman's model and rolling horizon integrated version of Tarim and Kingsman's model. It has been considered necessary to accurately assess the cost performance of these inventory control strategies against the optimal one, non-stationary (s, S), under a fair comparison.

The rest of the paper is structured as follows. The next section presents a formal description of the studied problem. The subsequent section discusses the literature on alternative inventory control strategies. This section is followed by computational results. The last section presents conclusions.

2. Problem Definition

We start our analysis by providing a formal definition of the problem based on that of Hua et al. (2008). Consider a finite planning horizon comprising N periods. Demand in each period t, d_t is non-negative, independent and discrete random variables with known probability distribution functions, $g_t(d_t)$, and occurs instantaneously at the beginning of periods. The mean rate of demand may vary from period to period and there is a constant coefficient of variation, $cv = \sigma_t / \mu_t$ (Standard deviation/ Demand mean). A fixed holding cost h is incurred on any unit carried in inventory from one period to the next.

We assume that if the system is out of stock, back ordering is possible and the backlogged demand is filled as soon as an adequate supply arrives. A fixed shortage cost p is incurred for each unit of demand back ordered. A fixed procurement (ordering or set-up) cost K is incurred each time a replenishment order is placed and it is invariable against the size of the order. The initial inventory level I_0 and the unit procurement cost v are set to zero, and it is assumed that there is no replenishment lead-time. The aim of the problem is to decide upon when and how much to replenish inventory so as to minimize total expected costs.

3. Alternative Inventory Control Strategies

Different inventory control strategies can be employed to solve stochastic lot sizing problems. Bookbinder and Tan (1988) call these strategies as static uncertainty strategy, dynamic uncertainty strategy and static-dynamic uncertainty strategy respectively. The basic differences of these aforementioned strategies are on replenishment times and replenishment sizes as shown in Table 1. None of these strategies are totally superior to any of the others, since there is a there is a tradeoff between practical implementation and total cost. The properties of each of these strategies and the developed solution methods for solving them are discussed as follows:

	Static Uncertainty Strategy	Static-Dynamic Uncertainty Strategy	Dynamic Uncertainty Strategy
Replenishment periods (z_t)	Static	Static	Dynamic
Replenishment sizes (X_t)	Static	Dynamic	Dynamic

Table 1: Alternative inventory control strategies

3.1. Static Uncertainty Strategy

Static Uncertainty Strategy requires that both the timing and the size of the replenishments must be decided once and for all in the beginning of the planning horizon, before any of the demands become known. A general mathematical model for this problem can be written as follows (Sox, 1997; Tarim and Kingsman, 2006; Hua et al., 2008):

$$\min \quad \sum_{t=1}^{N} \left[K z_t + hE[I_t^+] + pE[I_t^-] \right]$$

$$\text{s.t.} \quad I_t = \sum_{n=1}^{t} X_n - \sum_{n=1}^{t} d_t \quad t = 1, 2, \ldots, N$$

$$z_t = \begin{cases} 1 & \text{if } X_t > 0 \\ 0 & \text{otherwise} \end{cases} \quad t = 1, 2, \ldots, N$$

$$z_t \in \{0, 1\} \text{ and } X_t \in \mathbb{R}_+ \quad t = 1, 2, \ldots, N$$

where I_t is the net inventory at the end of period t, including on hand inventory and backlogs; $E[I_t^+] = E[\max\{I_t, 0\}]$, $E[I_t^-] = E[\max\{-I_t, 0\}]$.

In this problem the goal is determining replenishment periods (z_t) and associated replenishment sizes (X_t) in the beginning of the planning horizon to minimize the expected total cost. In this respect it is the stochastic version of the Wagner-Whitin dynamic lot-size model and a study by Vargas (2009) presents an approach to solve this problem optimal. In terms of nervousness criterion static uncertainty strategy is the best strategy since the inventory plan which will be implemented during the whole periods is determined and fixed in the beginning of the planning horizon. However, it is also possible to employ another strategy to decrease expected total cost by compromising on system nervousness.

3.2. Static-Dynamic Uncertainty Strategy

Static-Dynamic Uncertainty Strategy requires that the timing of replenishments must be decided upon in advance whereas order sizes are dynamically determined in the beginning of each period. This strategy is a good option for building long term agreements with other supply chain members. It is noted in Heisig (2001) and Inderfurth (1994) that setup instability which is nervousness due to deviations in order setups is considered as the most serious in practice. In this context investigating models and solution procedures for the static-dynamic uncertainty strategy is potentially important from the practical application perspective. A general mathematical model for this problem can be written as follows:

$$
\begin{aligned}
\min \quad & \sum_{t=1} \left[K z_t + hE[I_t^+] + pE[I_t^-] \right] \\
\text{s.t.} \quad & I_t = S_t - d_t \quad t = 1, 2, \ldots, N \\
& S_t \geq I_{t-1} \quad t = 1, 2, \ldots, N \\
& z_t = \begin{cases} 1 & \text{if } S_t > I_{t-1} \\ 0 & \text{otherwise} \end{cases} \quad t = 1, 2, \ldots, N \\
& S_t \in \mathbb{R}_+ \quad t = 1, 2, \ldots, N
\end{aligned}
$$

Where S_t denotes order-up-to-level for this period, if there is replenishment in period t; otherwise it is opening stock level for this period.

In the preliminary work conducted by Bookbinder and Tan (1988) service level approach is integrated to the static-dynamic uncertainty model instead of shortage cost and for solving the aforementioned model a solution heuristic which has two stage is presented. In the first stage replenishment times are determined and fixed. Then, in the second stage adjustments on planned orders are determined according to the realized demand. In another major study, (Tarim and Kingsman, 2006), (R_n, S_n) model is developed for solving the static-dynamic uncertainty model. Since, the cost function of the model is in the non-linear form, Tarim and Kingsman propose a piecewise linear approximation. Then, a mixed integer linear programming model which gives the replenishment periods and associated order up levels instantaneously in the beginning of the planning horizon has been developed. Throughout this paper, we refer to (R_n, S_n) model as TK.

Hua et al. ignore negative replenishments, so that if the actual stock exceeds the order-up-to-level for that review, this excess stock is carried forward and not returned to the supply source (Hua et al., 2008, p1255). This assumption allows them to solve their model by implementing a Wagner-Whitin type of dynamic programming algorithm strategy, since allowing the non-negative replenishment means removing also the links between cycles. In this case, each of the cycle turns to a classic newsvendor problem and the whole problem can be solved by dynamic programming. It is known from the literature that under service level approach, this type of shortest route algorithms have been already employed in previous studies on inventory control such as Tarim (1996) and Rossi et al. (2008). Throughout this paper, we refer to Hua et al.'s static-dynamic uncertainty model as HYHX.

3.3. Dynamic Uncertainty Strategy

Dynamic Uncertainty Strategy requires that whether a replenishment order is placed or not as well as its size is decided upon in the beginning of each period. If the system nervousness is put aside, the best strategy is implementing this policy, since it is the optimal inventory policy for our problem. Scarf (1960) developed optimal strategy called (s, S) for dynamic uncertainty strategy. Under non-stationary demand, (s, S) policies are characterized by two critical numbers s_n and S_n for each period n, such that, the inventory is replenished up to a target level S_n whenever the inventory position at the beginning of the period is lower than a re-order level s_n. Scarf proved the optimality of (s, S) model by introducing K-convexity concept.

The dynamic program proposed by Scarf is given as follows (Scarf, 1960):

$$C_n(x) = min\{L_n(x) + E\{C_{n+1}(x - d_n)\}, \quad K + L_n(S_n) + E\{C_{n+1}(S_n - d_n)\}\}$$

Where x is the inventory position at the beginning of the time period; $C_n(x)$ is the expected cost of following the optimal policy from period n onwards and $L_n(x)$ is the expected period cost function if the opening inventory position is x. When the variable of x is continuous, finding the optimal (s_n, S_n) levels is extremely difficult. Bollapragada and Morton (1999) develop a simple heuristic to compute optimal (s_n, S_n) levels by discretizing x. In this paper, their approach is employed to determine the optimal non-stationary (s, S) policy.

4. Numerical Analyses

The conducted numerical experiment in Hua et al. (2008) to compare HYHX with TK is not conclusive. The experiment includes only two cases: Case 1 (cv = 0.1) and Case 2 (cv = 0.4). The other parameters are K = 250, h = 1, p = 10, I_0 = 0. In both cases, only 30 realizations are taken into account and the cost performances of the models are compared. It is claimed that HYHX is superior to TK (Hua et al., 2008, p1259). This is not a fair comparison, since HYHX is applied in a rolling horizon framework whereas not TK. However, TK can be used in a rolling horizon framework as well. In the numerical experiment of this study, we compare HYHX with the rolling horizon integrated version of TK. Throughout this paper; we refer to rolling horizon integrated version of TK as Rolled TK.

Hua et al. (2008) compare HYHX with stationary (s, S) model (see Scarf, 1960) under nonzero lead time and the different cost parameters than the previous analysis. They claim that HYHX is superior to the (s, S) policy for the inventory control problem with multiple-period replenishment lead time (Hua et al., 2008, p1259). However, it is not a proper action to employ stationary (s, S) model to the non-stationary stochastic inventory problem and compare its performance with their non-stationary HYHX model. We use the non-stationary (s, S) policy (see Bollapragada and Morton, 1999) as a frame of reference in terms of cost performance, since it is proven to be optimal both in stationary and non-stationary demand cases.

Here primarily we repeat the analysis of Hua et al. (2008). Nevertheless, we extend their parameter set and increase extremely the number of simulation run. We evaluate non-stationary (s, S), HYHX, TK and Rolled TK's performances in terms of total expected cost.

4.1. Experimental Design

To test the models, 36 different sample cases are generated. We use 3 different coefficient of variation cv = {0.1, 0.2, 0.3}, 3 different unit shortage cost p = {5, 10, 15} and 4 different setup cost K = {150, 250, 350, 450}. The planning horizon is set to 8 periods with zero initial inventory. The holding cost is set at 1, and unit cost and lead time is ignored. It is assumed that demands are discretized truncated normals and demand means are the same as those given in Hua et al. (2008). Mean demands of periods 1–8 are 200, 100, 70, 200, 300, 120, 50, and 100, respectively. Non-stationary (s, S), HYHX, TK and Rolled TK are implemented on each 36 generated sample case. Then, models' performances are observed over 1000 different instances for each case. Since non-stationary (s, S) is optimal in terms of the cost performance, we present the relative cost performances of HYHX, TK and Rolled TK by using their % differences from the optimal cost. We denote the % differences of HYHX, TK and

$$\Delta_{Method} = (C_{Method} - C_{s,S})/C_{s,S}$$

Rolled TK from Non-stationary (s, S) by Δ_{HYHX}, Δ_{TK} and $\Delta_{RolledTK}$ respectively. The formulation of % difference is calculated as follows:

where C_{Method} and $C_{s,S}$ denotes the average expected cost of chosen method for 1000 instances and the average expected cost of (s, S) for 1000 instances respectively.

4.2. Results

We illustrate the results of our numerical study in Table 2. In each row of the table, case number, varying parameters of associated case, and Δ_{HYHX}, Δ_{TK} and $\Delta_{RolledTK}$ are given respectively.

#	K	p	cv	Δ_{HYHX} (%)	Δ_{TK} (%)	$\Delta_{RolledTK}$ (%)
1	150	5	0.1	0.47	0.56	0.55
2			0.2	1.95	4.54	2.47
3			0.3	1.98	7.52	2.91
4		10	0.1	0.28	0.47	0.33
5			0.2	1.96	5.13	2.43
6			0.3	1.03	8.45	2.07
7		15	0.1	0.56	0.90	0.72
8			0.2	2.79	6.32	2.40
9			0.3	1.56	7.49	0.96
10	250	5	0.1	0.35	2.05	0.26
11			0.2	0.24	5.49	0.26
12			0.3	2.44	9.02	3.07
13		10	0.1	0.05	2.53	0.04
14			0.2	0.97	7.41	0.73
15			0.3	3.36	11.31	3.52
16		15	0.1	0.31	3.35	0.26
17			0.2	1.92	8.32	1.92
18			0.3	3.88	13.10	3.21
19	350	5	0.1	0.02	0.23	0.01
20			0.2	0.94	3.69	0.69
21			0.3	2.69	7.17	2.41
22		10	0.1	0.15	0.82	0.11
23			0.2	3.02	6.34	2.73
24			0.3	2.10	9.13	1.95
25		15	0.1	0.47	1.45	0.29
26			0.2	3.52	7.64	3.33
27			0.3	2.91	10.81	2.06
28	450	5	0.1	0.01	0.17	0.02
29			0.2	0.44	2.28	0.28
30			0.3	1.51	5.76	1.28
31		10	0.1	0.06	0.51	0.02
32			0.2	1.45	4.52	1.30
33			0.3	2.56	7.71	2.49
34		15	0.1	0.14	0.86	0.05
35			0.2	1.97	5.32	1.79
36			0.3	2.19	8.45	1.94
			Avg.	1.45	5.19	1.41

Table 2: Comparison between models

The results of our numerical analyses are quite revealing in several ways. First, non-stationary (s, S) model performs better than HYHX, TK and Rolled TK in 36 different cases each of which has 1000 instances. In any case this result is expected, since non-stationary (s, S) model is optimal policy for the periodic review inventory problem with non-stationary stochastic demands under the used assumptions. Second, the rolling horizon method implementation to TK escalates the model's performance. The difference between non- stationary (s, S) model and TK decreases from 5.19% to 1.41% on the average when the rolling horizon is integrated to the model. The reason is that observing realized demand at the end of each period and revising inventory plan in the light of this information provide advantage to the model. Third, Rolled TK performs better than HYHX on the average. However, the average difference between Rolled TK and HYHX is 0.04% which means that these two models perform quite close to each other in terms of expected cost. To conclude, the numerical analyses help to reveal the cost performance of these inventory control strategies against the optimal one, non-stationary (s, S), under a fair comparison.

5. Conclusion

This paper presents a comparison between non-stationary (s, S), HYHX, TK and Rolled TK models for the periodic review inventory problem with non-stationary stochastic demands. The results of this study support the literature that the non-stationary (s, S) model shows the best performance in terms of expected cost among the other models and the idea that the rolling horizon implementation to TK will increase the performance of the model.

Our computational experiments demonstrate that HYHX and Rolled TK models perform quite close to each other in terms of expected cost. Accordingly, we do believe that this study is worthwhile for the literature of the non-stationary stochastic inventory problems.

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