Estimating the Upper Value of an Assets Portfolio

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Abstract

The idea that the value of assets can be predicted is not reasonable. The typical approach to calculate asset value using dividends is placed under discussion and a formula which calculates asset values and portfolio values based solely on asset prices based on the idea that dividends or premiums paid are reflected upon the prices of the assets is discussed and its robustness proven statistically. It is recommended to diversify investment and to invest aimed at the medium to long term, not trying to make money in the short term (the latter called arbitrage).

Keywords: Finance, stocks, bonds, assets, portfolio, value.

Introduction

The literature is full with formulas which calculate the value of assets (corporate stocks or government bonds) based on their dividend or their premiums. However, it is the statement made in this paper that dividends paid are reflected upon the value or price of the stock, and that it is precisely these changes in price that bring about a given return over the investment, being that positive or negative.

The idea that asset prices follow a random walk is discussed in light of the relevant literature. Although some argue that the asset prices do not follow a random walk, most researchers agree on the random walk hypothesis. Thus, as the financial crisis from 2007 to 2009 shows, it is bordering on insanity to pretend it is possible to predict asset values, although in the medium to long term assets tend to increase in value. It is speculated that the random walk is not entirely random but rather with a slight increase in the value of the assets, although such claim is not proven. That is the reason why investing in the stocks or bonds markets means making money if the investment is diversified and aimed at the medium to long term.

Furthermore, the value of any given portfolio of assets is not calculated using the dividends probably paid in each case, but rather using a formula based on the price of each asset in order to calculate the value of the portfolio. These formula is experimentally tested using a statistical test of the equality of the two means obtained using alternative approaches to calculate the value of the portfolio: the first approach calculates the value of the portfolio for each day using the upper limit formula and then averages the results; the second approach averages the results of all the prices of each day for each asset and then applies the upper limit formula to these averages for all assets. The hypothesis that these two measures are statistically equivalent within a given marginal error is proven (or rather, not rejected), which shows the robustness of the upper limit approach calculation.

Valuing Assets

There is considerable research that has been done on the concept of money through time and how to value a business (Neugebauer, 1951; Parker, 1968; Stevin, 1582; Wellington, 1887; Pennell, 1914; Marshall, 1907; Bohm-Bawerk, 1903; Fisher, 1907, 1930; Boulding, 1935; Keynes, 1936; Samuelson, 1937). Damodaran (2009) identifies three approaches to valuation: a) discounted cash flow valuation (Dodd & Graham, 1934; Durand, 1957; Gordon, 1962; Fuller & Hsia, 1984; Michaud & Davis, 1981; Schiller, 1981; Poterba& Summers, 1988; Fama& French, 1988; Sorensen & Williamson, 1985; Baker &Wurgler, 2004; Ohlson, 1995); b) liquidation and accounting valuation (Daniels, 1934; Walter, 1966; Lundholm& O'Keefe, 2001; Barth, Beaver & Landsman, 2001); and c) relative valuation (Fernandez, 2001; Liu, Nissim& Thomas, 2002; Beaver & Morse, 1978).

As can be seen, the research on valuing assets (typically corporate stocks) is considerable. However, in all of these cases, the researchers often use the dividends paid for any given stock for valuing such stock on the idea that a stock is valuable only because of the dividends it pays.

However, it is the hypothesis here proposed that dividends any given corporate stock and the premiums any given government bond pay are reflected upon their respective prices (in short, assets, since that word includes both corporate stocks and government bonds). That is, the price of any given asset at any given point in time reflects the actual value of such asset, which includes the dividends or premiums paid. Since such asset can be bought or sold, that price is the only measure of value that should be considered. The latter is not only reasonable, but also simpler to apply in practice, because there is no need to consider dividends or premiums paid, nor the actual value in accounting books of the company, the equity or any other consideration.

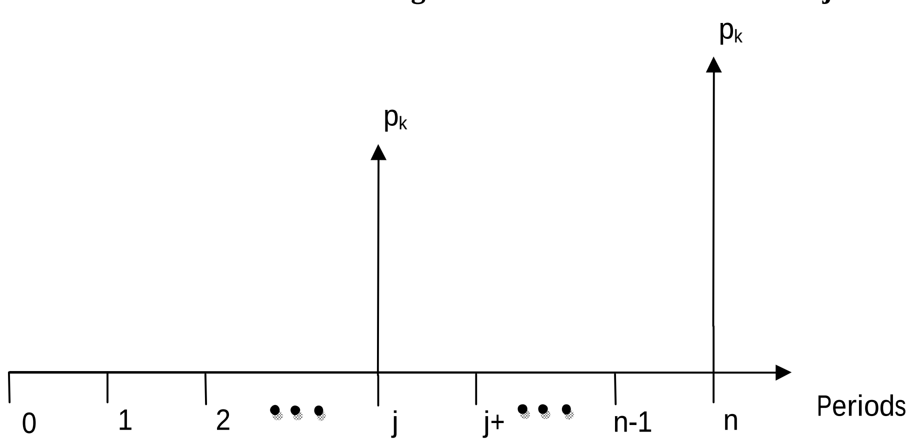
Although not everybody agrees (Lo & MacKinlay, 1988; Smith, 2007), a careful experimental analysis on any given sequence of asset prices would reveal that it is impossible to accurately predict the future behavior in the price of such asset, since the prices tend to behave following a random walk (Granger, 1968; Kassouf, 1968; Fama, 1965; Narayan & Smyth, 2006; Narayan & Smyth, 2005; Solnik, 1973). The reason why people make money by investing in the stock market or in government bonds is because these prices tend to increase in the long run due to the fact that dividends or premiums are paid, which are reflected in the assets prices. To pretend that it is possible and reasonable to accurately predict the value (price) of any given asset in the short term borders insanity. The financial crisis from 2007 to 2009 showed that stock markets sometimes plummet without prior warning (Auer & Schuster, 2011). Assets only bring about value in the medium to long term.

If price is the only measure of value to consider, how can such changes in pricebe analyzed? What information can actually be derived from historical asset prices? That question is a little more complicated to answer, and it first requires considering the concept of the value of money through time, but this time, using a different paradigm.

Let p_{kj} and p_{kn} be the prices of asset k at periods j and n, respectively, where $j \leq n$, and the periods can be days, months, quarters, and so on. Figure 1 illustrates the situation. The relationship between p_{kj} and p_{kn} is given by equation (1), assuming p_{kj} and p_{kn} are the daily prices of asset k at days j and n, respectively, and IRR_{kj} is the Internal Rate of Return of the investment between days j and n.

$$p_{k_n} = p_{k_j}\left(1 + IRRp_{k_j}\right)^{n-j} \qquad (1)$$

Figure 1: Asset k Prices at Periods j and n.



Solving for $1+IRR_{kj}$ yields equation (2), where s is the total number of assets or maximum portfolio size.

$$\left(\frac{p_{k_n}}{p_{k_j}}\right)^{\frac{1}{n-j}} = 1 + IRR_{k_j} \forall k = 1, \ldots, s and j = 1, \ldots, n; j < n \qquad (2)$$

The yearly IRR_{kj} for asset k at day j is given by equation (3).

$$IRR_{k_j} = \begin{cases} \left(\left(\frac{p_{k_n}}{p_{k_j}}\right)^{\frac{365}{n-j}} - 1\right) \times 100\% \; \forall k = 1, \dots, s where j is in a nonleap year \\ \left(\left(\frac{p_{k_n}}{p_{k_j}}\right)^{\frac{366}{n-j}} - 1\right) \times 100\% \; \forall k = 1, \dots, s where j is in a leap year \\ \left(\frac{p_{k_n}}{p_{k_j}} - 1\right) \times 100\% \; \forall k = 1, \dots, s where j is in the last year \end{cases} \tag{3}$$

The average IRR_k for asset k is calculated according to equation (4), where n is the total number of days in the block of prices and IRR data.

$$IRR_k = \frac{\sum_{j=1}^{n} IRR_{k_j}}{n} \forall k = 1, \dots, s \tag{4}$$

The average price for asset k (p_k) can be calculated in a similar way according to equation (5), where n is also the same value as in equation (4), that is, the total number of days being considered.

$$p_k = \frac{\sum_{j=1}^{n} p_{k_j}}{n} \forall k = 1, \dots, s \tag{5}$$

This average price (p_k) can be used as the price estimated for asset k. The expected return over the investment is given by IRR_k. It is possible to further expand the mathematics in order to calculate the risk associated to each investment, but that is beyond the scope of this paper.

Valuing Portfolios

Most research on valuing portfolios has been done on the expected return over the investment of the entire portfolio of assets, and that is based on the expected dividends (Stapleton, 1971; Lintner, 1965). It is the postulate here that only the price of each asset (p_k for asset k) is required. Such price is given according to equation (5), whereas the expected return over the investment of asset k is given by equation (4).

In the literature hardly anybody cares about calculating the value of the portfolio based on the prices of the assets it contains. If there are s assets in a portfolio and all the assets have the same weight, each asset contributes with (1/s)x100% of the average value of the portfolio. Thus, equation (6) should apply for the average value of the portfolio (p).

$$p = \sum_{k=1}^{s} \frac{1}{s} p_k \tag{6}$$

However, this calculation requires us to assume the relative importance of each asset. Such importance can be different. One possibility is to assign the weight of asset k (w_k) as a function of the relative value of the total investment made in such asset when compared to the total investment in the portfolio. Let q_k be the number of assets k purchased. Then, the relative importance of such asset (w_k) can be given according to equation (7). Thus, the value of the portfolio can be calculated as indicated by equation (8).

$$w_k = \frac{q_k p_k}{\sum_{j=1}^{s} q_j p_j} \tag{7}$$

$$p = \sum_{k=1}^{s} w_k p_k \tag{8}$$

Nevertheless, Copertari (2014) notes that a better measure of the portfolio's value is given according to his application of the Pythagorean formula to the s-dimensional case, indicated in equation (9), which provides the upper limit to the portfolio's value.

$$\lceil p \rceil = \sqrt{\sum_{j=1}^{s} p_j^2} \tag{9}$$

The advantage of equation (9) is that it only requires the price of each asset in the portfolio (p_k), regardless of the number of assets acquired (q_k). But, is the value provided by equation (9) really consistent when evaluating the value of the portfolio?

Experimental Testing

In order to test the validity of equation (9), a numerical test is proposed. A total 10,000 simulated asset prices for 12 simulated stocks are generated following random distributions. The first two assets follow uniform distributions with parameters randomly generated. Assets k=3 to k=8 follow normal distributions with mean and standard deviation also randomly generated in such way that there are no negative prices. Finally, assets k=9 to k=12 follow a chi-squared distribution with parameters randomly generated such that there is never a negative asset price.

Each asset price out of the 10,000 asset prices for each asset represents the asset price for a given day. Equation (9) is applied to all 12 assets for each day. Then, the average of the portfolio value for each of the 10,000 days is calculated. This is called $\overline{x}_1$. On the other hand, the average asset price for each of the 12 assets is calculated for all 10,000 asset prices for each asset k, where k = 1,…,12. Then, equation (9) is applied to these 12 average asset prices. This value is called $\overline{x}_2$.

The question is whether or not both values ($\overline{x}_1$and $\overline{x}_2$) are statistically the same (Devore, 2012). Considering a confidence level of 5%, a statistical test for these two (mean) values is carried out. The statistical difference (d_0) is 5% of the average value of $\overline{x}_1$and $\overline{x}_2$. The null hypothesis is H_0: $\mu_1-\mu_2 = d_0$. The alternative hypothesis is H_1: $\mu_1-\mu_2 \neq d_0$. (Notice that μ_1 is the population value for the estimate $\overline{x}_1$ and μ_2 is the population value for the estimate $\overline{x}_2$.) The standardized normal estimate is given according to equation (10), where σ_1 is the standard deviation for the $\overline{x}_1$ calculation and σ_2 is the standard deviation for the $\overline{x}_2$ calculation. The total sample for the first calculation is n_1 = 10,000 and for the second calculation is n_2 = 12.

$$z = \frac{(\overline{x}_1-\overline{x}_2)-d_0}{\sqrt{(\sigma_1^2/n_1)+(\sigma_2^2/n_2)}} \tag{10}$$

Notice that this is a two tails test, so that the reference value from the standardized normal table corresponds to 5%/2 = 2.5%, that is, $\alpha/2$, where α=5%. The value from normal tables to consider, $z_{\alpha/2}$ is ±1.96. A total of 10 runs with 10,000 asset price values for each of the 12 assets were carried out, and the hypothesis tested in each case.

Discussion and Conclusion

The results from the calculations performed are shown in Table 1.

As can be seen, in all 10 runs, the result was the same: the means do not statistically differ or, to say it more appropriately, there is no statistical evidence suggesting that the means are not equal within a margin of d_0, that is, that it is not valid to reject the null hypothesis.

Consequently, the approach to calculate the portfolio's value depicted in equation (9) is consistent and seems to be appropriate in order to obtain an upper limit to the value of the portfolio. This result is very important because it provides a proven approach to calculate the upper limit to the value of any given assets portfolio. It is now possible to expand the hypothesis to include risk and experiment with different portfolio assets allocations.

Table 1: Results for the 10 Runs of the Statistical Test of Two Means

Run	$\overline{x}_1$	$\overline{x}_2$	σ_1	σ_2	Z	d_0	Result
1	69.60	67.90	5.07	5.97	-1.01	3.44	Meansnotdifferent
2	72.13	70.52	5.50	6.80	-1.00	3.57	Meansnotdifferent
3	72.40	71.07	4.56	8.14	-0.96	3.59	Meansnotdifferent
4	70.29	68.96	4.67	8.65	-0.86	3.48	Meansnotdifferent
5	64.26	62.76	4.43	6.03	-0.96	3.18	Meansnotdifferent
6	74.12	72.44	5.65	7.52	-0.92	3.66	Meansnotdifferent
7	74.35	72.77	5.03	6.99	-1.04	3.68	Meansnotdifferent
8	76.59	75.09	5.17	6.41	-1.24	3.79	Meansnotdifferent
9	62.96	61.49	4.31	6.13	-0.93	3.11	Meansnotdifferent
10	70.28	68.56	5.14	5.82	-1.04	3.47	Meansnotdifferent

The recommendation to investors besides the known idea of diversification is to ensure that the investments are made in the medium to long term. Arbitrage, although being a profession for some, is very risky and in theory provides only marginal gains.

Further research should include risk in consideration as well as portfolio allocations of assets. There are strategic considerations that need to be taken into account.

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