Using Pitman Closeness to Compare Stock Return Models

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Abstract

This paper provides an alternate method of evaluating portfolio performance of stock pricing models. We apply Pitman Closeness Criterion to compare the accuracy of three popular pricing models. This comparison is used to assess which, if any, model outperforms the others. In assessing model performance over a long period of time, we find that the Fama-French three-factor model and the Carhart four-factor model each show better performance in prediction of stock returns than CAPM. When we limit the study to more recent data, the Carhart model shows better performance for more portfolios than the Fama-French and the CAPM models.

Keywords: Asset Pricing Models, Model Specification, Pitman Closeness

JEL:G12, G11, C13, C12

1 Introduction

A long standing challenge in financial markets is finding an efficient model for predicting the future prices of stocks. Inherent variation in price and the volatility in markets make the assessment a challenge over time. In this study, we focus on a different method of comparing the model performances. We use Pitman Closeness Criterion (Pitman, 1937) to look for insights into the conditions under which each model performs better than the others. We evaluate the performance of the Capital Asset Pricing Model (CAPM), the Fama-French three-factor model, and the Carhart four-factor model. Prior studies have evaluated the constructs of pricing models. For example, Pennywell, Chow, and Javine (in press) use a similar approach using Pitman Closeness to compare performances of industry returns, identifying a performance change predictive models in the Energy sector in the time period following the Enron bankruptcy.

2. The Models

The CAPM, defined by Sharpe (1964), Lintner (1965), and Mossin (1966), identifies the beta (β_i) of the security, that captures the non-diversifiable part of the securities risk as it related to the market as a whole. In this model, the lone relevant source of risk that explains the volatility in security returns is the market risk. Beta can then be considered as indexing the security's risk to the risk of the relevant benchmarked portfolio.

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + \varepsilon_{it} \quad (1)$$

Where,

R_{it}= realized return on security i at time t;
R_{mt}= realized return on the market at time t. This is obtained from the Kenneth French Website and it is described as the value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus the one-month Treasury bill rate.
R_{ft}= nominal risk-free rate of return at time t;
α_i= the intercept, constant term for security i;
β_i= slope coefficient for security i on the market risk factor; and
ε_i= the residual excess return on portfolio i during time t.

Fama and French (1993 and 1996) extends the CAPM into a model of three-factors. With the three-factor model, variation in security returns is dependent on three different factors; market risk, the difference in returns between small and large companies (SMB), and the difference in returns between firms with high book-to-market ratios and low book-to-market ratios (HML). These two added factors, SMB and HML, intend to capture the risk associated with firm size and growth, respectively.

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + S_i(SMB_t) + H_i(HML_t) + \varepsilon_{it} \quad (2)$$

Where,

S_i = slope coefficient for security i on SMB;
H_i = slope coefficient for security i on HML;
SMB_t = the difference in returns on small versus large firms during time t; and
HML_t = the difference in return on high versus low book-to-market ratios during time t.

The last model, Carhart (1997), expands the Fama-French model by adding a price momentum factor as a fourth source of risk to explain the variation in returns. This added factor is intended to account for the tendency for companies with positive (negative) past returns to produce positive (negative) returns in the future. The price momentum factor is the mean return of the best performing stocks over a prior period minus the mean return of the worst performing stocks over the prior period.

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + S_i(SMB_t) + H_i(HML_t) + M_i(MOM_t) + \varepsilon_{it} \quad (3)$$

Where,

M_i = slope coefficient for security i on MOM;
MOM_t = the difference in the average returns on positive versus negative performing firms during time t.

3. Data

The sample used in this study consists of the twenty-five portfolios provided in the Fama-French data formed based on size and book-to-market ratios as extracted from the Dartmouth Data Library.[1] Size is measured by the market value of equity. Book value of equity-to-market value of equity measures the investment style. There are five categories for size. The portfolios can be very-small, small, medium, mid/large or large. There are five categories for book to market; low, 2, 3 4, and high. Each size is paired with each book-to-market to get the 25 portfolio combinations. We concentrate our attentions for this analysis on these 25 portfolios because they are the standard portfolios used in much of the existing literature. Table 1 displays the summary statistics relative to the number of firms in each portfolio, with the mean number of firms in each of the portfolios ranging from 18.9 to 386.6.

Table 1: Summary Statistics for the Number of Firms

	Mean	Standard Deviation	Skewness	Kurtosis	Max	Median	Minimum
SL	293.9200	333.1800	0.7854	2.2186	1166.0000	105.0000	1.0000
S2	194.8889	191.6146	0.4070	1.6027	625.0000	136.0000	2.0000
S3	200.9834	191.7971	0.5154	1.8587	681.0000	113.0000	3.000
S4	249.6901	231.3969	0.7287	2.3961	893.0000	138.5000	17.0000
SH	387.5858	345.1792	0.5349	1.7747	1185.0000	204.0000	36.0000
S2L	97.7339	91.0553	0.6332	2.1338	348.0000	57.0000	4.0000
S22	78.4464	55.5437	0.4518	1.7183	209.0000	55.0000	10.0000
S23	81.2378	51.6474	0.6078	2.0036	195.0000	52.0000	15.0000
S24	75.6452	40.6568	0.6444	2.3619	176.0000	61.0000	17.0000
S2H	58.7983	26.5467	0.2506	1.9683	124.0000	54.0000	18.0000
M3L	79.1189	58.0918	0.8088	3.0422	285.0000	65.0000	15.0000
M32	65.0877	35.9786	0.4518	1.8975	157.0000	49.0000	17.0000
M33	61.7905	26.8451	0.3367	2.1453	134.0000	55.0000	12.0000
M34	52.7047	20.0282	0.1206	1.9243	101.0000	50.0000	18.0000
M3H	37.6686	14.0067	-0.1130	2.1622	67.0000	37.0000	11.0000
B4L	76.1803	37.8895	0.9141	4.4046	240.0000	65.0000	20.0000
B42	61.6365	22.9620	0.4726	2.4422	124.0000	59.0000	20.0000
B43	50.0916	17.9883	0.2775	2.2877	102.0000	49.0000	19.0000
B44	39.6696	17.2729	0.2261	1.6974	76.0000	40.0000	11.0000
B4H	27.1930	14.1981	0.3582	1.9693	57.0000	24.0000	4.0000
BL	89.8382	39.0704	1.6652	7.5661	277.0000	85.0000	30.0000
B2	55.9220	16.684	0.0860	2.8295	107.0000	58.0000	24.0000
B3	40.9016	14.9904	0.1484	1.8532	72.0000	40.0000	14.0000
B4	32.2193	16.1859	0.3022	1.7774	69.0000	28.0000	7.0000
BH	18.9483	12.3796	0.6454	2.5436	51.0000	15.0000	0.0000

This table shows the summary statistics for returns on the 5x5 portfolios on size and book-to-market. The first character denotes the size and the second character denotes the book-to-market group for the portfolios with two characters. For example, SL denotes small and low book-to-market, S2 denotes small and second lowest book-to-market group. For the portfolios with three characters as in M3L, the portfolio label indicates that is the middle (third) size portfolio and the lowest book-to-market portfolio.

4. Methodology

A number of differing methods have been used in past studies to determine how well a predictor of stock return pricing estimates the actual results. A different approach to this evaluation process is the utilization of a measure of determining which of two or more estimators is closer to the actual parameter being predicted. Pitman (1937) proposed a method of looking at the closeness of each predictor estimator to the actual value of the parameter under investigation. This method is based on the probability of the absolute difference between the estimator and the value of the true parameter. Using Pitman Closeness, if we are considering two estimators of θ, call them $\hat{\theta}_1$ and $\hat{\theta}_2$, the closer estimate would be the one with the probability greater than 0.5 of its absolute difference being smaller than the other's.

While it has been noted that using the Pitman Closeness method of comparison may provide a difference that is not relevant (Fountain, Keating, & Maynard, 1996), an important note is that Pitman criterion only takes into consideration the estimator which is closer to the true value of the parameter being estimated. Pitman (1937) pointed out that in identifying a closer or best estimator, from a practical sense, the application of the estimator being considered needs to be assessed to look at the consequence resulting in error. Several practical approaches to the use of the Pitman Closeness method have been reported in several differing areas.

Chow, Chow, Hannumath, & Wagner (2007) used Pitman Closeness in Quality Control applications, while Wenzel (2002) applied Pitman Closeness in the comparison of forecast components.

In this approach, we utilize Pitman Closeness to evaluate the performance of several portfolio models. We find the absolute difference between the value of an estimate in a given time period and the value of the actual portfolio value for the same time period. Over each time period, we compare the absolute differences and count how many times each estimator provides the smaller absolute difference. We divide the number of times each estimator provided the smaller absolute deviation by the total comparisons to determine the probability that the estimator is closer to the parameter in question. If the probability is greater than 0.5 we consider the estimator to be Pitman Closer than the other.

In comparing two estimators at a time, we find that one of the estimators will nearly always produce a probability greater than 0.5. Chow, Tressler, and Woodford (2013) noted that in performing comparisons, a true difference may only occur when the probabilities are rounded at a significant number of places (they found differences at 15 decimal places). This must be viewed in Pitman's concept of what is appropriate for the application. Other challenges can occur when comparing more than two estimators at once. A result of this could be that no one estimator provides a probability of being closer that is greater than 0.5. Keaton, Mason, and Sen (1993) point out that when an estimator is closer than the others, but has a probability of less than 0.5, the estimator should be considered Pitman Nearer. For these potential reasons, we consider head-to-head comparisons for all of the estimators in this study, finding which are Pitman Closer than the other and then drawing conclusions based on the Pitman Closeness findings.

The monthly value-weighted portfolio returns were predicted through calculations using each of the models for each month between January 1927 and December 2011. The summary statistics are presented in Table 2 for the returns over all of the years contained in the study period. The summary statistics for each portfolio return from January 2001 through December 2011 are provided in Table 3. The mean returns and the standard deviation for each portfolio are smaller during the more recent years than the average return and standard deviation for each portfolio during the entire sample period.

Table 2: Summary Statistics of Returns for the Entire Sample from January 1927 to December 2011

Firm Category	Mean	Standard Deviation	Skewness	Kurtosis	Max	Median	Minimum
SL	0.7296	12.2524	2.7071	30.8072	147.5000	0.5500	-49.3600
S2	1.1016	10.5924	4.4004	60.0082	139.2700	0.9650	-43.0900
S3	1.3105	9.2366	1.7637	18.3963	81.0400	1.2500	-36.5800
S4	1.4506	8.6630	2.7270	33.1779	105.0700	1.4650	-34.7800
SH	1.6691	9.5969	3.0664	33.0888	105.3100	1.5050	-34.8700
S2L	0.8676	8.0029	0.3447	7.8696	54.1300	1.1700	-32.8200
S22	1.2390	7.8974	1.8687	23.9229	84.4100	1.5000	-32.5000
S23	1.3212	7.3602	2.0566	24.8702	78.7900	1.5400	-30.5800
S24	1.3671	7.6306	1.6732	20.8613	72.5700	1.5050	-32.7700
S2H	1.4789	8.7759	1.7497	20.3454	87.3700	1.6450	-34.6400
M3L	0.9676	7.6622	1.0054	13.3326	60.7500	1.3950	-29.5700
M32	1.1571	6.6234	0.2698	9.4155	44.3200	1.3300	-29.1900
M33	1.2664	6.7718	1.0006	17.0907	64.2700	1.5650	-33.4900
M34	1.2687	6.8501	1.1574	15.8447	56.2100	1.4550	-31.5800
M3H	1.4221	8.6448	1.8813	22.3202	82.0600	1.3650	-37.2800
B4L	0.9640	6.2574	-0.2075	6.4166	34.4700	1.2300	-28.8800
B42	1.0273	6.3157	0.8201	14.9335	57.5600	1.3600	-28.8300
B43	1.1253	6.4264	0.9342	17.3202	64.9100	1.5350	-32.0300
B44	1.2239	7.0355	1.7834	23.1420	70.6700	1.5200	-34.4500
B4H	1.3241	8.9998	2.0118	24.6640	86.4300	1.5200	-40.0800
BL	0.8772	5.4882	-0.0236	8.2363	35.5200	1.0650	-28.2100
B2	0.8775	5.2500	-0.0874	8.0397	32.2400	1.0150	-25.1000
B3	0.9401	5.7676	0.8082	17.1524	48.4100	1.1500	-31.1200
B4	0.9702	6.9131	1.8396	26.3158	65.0400	1.0750	-36.4200
BH	1.2157	7.5777	0.6876	14.0862	56.8200	1.2900	-45.5600

This table shows the summary statistics for returns on the 5x5 portfolios on size and book-to-market from January 1927 to December 2011. The first character denotes the size and the second character denotes the book-to-market group for the portfolios with two characters. For example, SL denotes small and low book-to-market, S2 denotes small and second lowest book-to-market group. For the portfolios with three characters as in M3L, the portfolio label indicates that is the middle (third) size portfolio and the lowest book-to-market portfolio. Returns are reported as percentages.

For our study, we set out to find the predicting ability of the different models when Pitman Closeness is the method of comparison. In these comparisons, we take actual returns and compare those with the predictive values to find residuals for each portfolio. We use the residuals in the Pitman Closeness method to evaluate which of the estimators provided closer estimates to the actual portfolio returns. We use each of the three models for comparison of each of the 25 portfolios over the entire sample period of time. Next, we take a more recent time period, and run our assessment for the time period of January 2001 to December 2011. Our findings are presented in the results section.

Table 3: Summary Statistics of Returns from January 2001 to December 2011

	Mean	Standard Deviation	Skewness	Kurtosis	Max	Median	Minimum
SL	0.2124	8.0910	0.0501	3.5618	27.3600	0.6050	-23.3400
S2	0.8494	6.7932	-0.2316	3.2000	17.5800	0.7550	-19.7000
S3	0.9208	5.9662	-0.3264	3.3775	15.5300	1.0050	-17.8600
S4	1.0353	5.8893	-0.3785	3.2998	15.4100	1.1650	-15.5600
SH	1.2603	6.9989	-0.5910	3.8889	18.8100	0.8350	-24.1900
S2L	0.5970	7.1416	-0.3229	3.1843	17.1900	1.6600	-22.4400
S22	0.8606	6.2809	-0.3966	3.6929	16.8300	1.2950	-20.7900
S23	1.0478	5.9987	-0.3610	3.7801	17.2900	1.1600	-19.0100
S24	0.8564	6.1600	-0.7031	4.2773	15.8800	1.3400	-22.4700
S2H	0.9519	7.4382	-0.7571	4.0431	19.1400	1.5400	-22.7300
M3L	0.5481	6.5742	-0.4964	3.4526	14.5400	1.5250	-22.5600
M32	0.7723	5.8184	-0.2430	3.8280	18.0900	1.1250	-18.2000
M33	0.9390	5.5466	-0.3529	3.5500	16.1500	1.5000	-17.2500
M34	0.8274	5.8648	-0.3622	3.8704	16.1300	1.1800	-19.5400
M3H	1.1914	6.4175	-0.6030	3.9047	15.0100	1.2250	-19.5700
B4L	0.6195	5.9394	-0.4778	3.8574	16.2700	1.3200	-19.8700
B42	0.7340	5.5517	-0.6007	4.4440	15.9000	1.2050	-21.3700
B43	0.5825	5.9128	-0.7378	5.7356	18.7600	1.1500	-26.0300
B44	0.7445	5.7840	-0.6920	4.1007	12.9700	1.3500	-21.3200
B4H	0.5841	6.4361	-0.5820	4.3868	19.9600	1.2950	-19.9100
BL	0.1782	4.6242	-0.4913	3.8408	10.4200	0.1850	-15.8700
B2	0.4647	4.3477	-0.5620	3.6724	10.7700	0.9650	-14.0200
B3	0.3300	4.8306	-0.4273	3.8183	12.8200	0.9400	-15.2600
B4	0.2323	4.8391	-0.8537	5.0339	11.1500	0.4700	-19.3200
BH	0.2300	6.2093	-0.3394	3.5134	17.5700	0.8350	-19.1300

This table shows the summary statistics for returns on the 5x5 portfolios on size and book-to-market from January 2001 to December 2011. The first character denotes the size and the second character denotes the book-to-market group for the portfolios with two characters. For example, SL denotes small and low book-to-market, S2 denotes small and second lowest book-to-market group. For the portfolios with three characters as in M3L, the portfolio label indicates that is the middle (third) size portfolio and the lowest book-to-market portfolio. Returns are reported as percentages.

5. Results

We initially begin the evaluation by assessing the portfolios with the methods utilized in previously published studies.

Table 4 provides the factor regressions for the monthly excess returns on the 25 portfolios of size and book-to-market in the time period between January 1927 and December 2011 for each of the models being studied. Of the 25 portfolios being studied, 11 had pricing errors that were significantly different from zero at the 5% level when using the CAPM model. Additionally, eight of the 11 portfolios are in the smallest size or lowest book-to-market groups. Consistent with expectations, the Fama-French model seems to be better as the number of portfolios with pricing errors decreased. Eight of the 25 portfolios contain pricing errors significantly different from zero at the 5% level when using the Fama-French model and four of the eight are in the smallest size or lowest book-to-market groups. Consistent with prior literature, adding the momentum factor also reduces the number of portfolios with pricing errors. The Carhart Model had pricing errors significantly different from zero in four of the 25 portfolios. Moreover, three of the portfolios are in the smallest size group and one of the portfolios is in the lowest book-to-market group. In addition to these findings, the Gibbons, Ross, &Shanken (1989) F-test rejected the null hypothesis that all of the 25 portfolios are jointly equal to zero for each of the models tested. The results from the above tests are support the findings of previously published studies.

Table 4: Factor Regression for Monthly Excess Returns on 25 Size and Book-to-Market Portfolios, 1/1927-12/2011 (1020 Months)

	CAPM			Fama-French 3-Factor Model					Carhart 4-Factor Model					
Portfolio	Alpha	$R_M - R_{rf}$	Adj R^2	Alpha	$R_M - R_{rf}$	SMB	HML	Adj R^2	Alpha	$R_M - R_{rf}$	SMB	HML	MOM	Adj R^2
SL	-0.563 -2.12 (0.04)	1.627 33.65 (0.00)	0.526	-0.839 -3.65 (0.00)	1.308 29.01 (0.00)	1.290 17.76 (0.00)	0.398 6.09 (0.00)	0.649	-0.722 -3.06 (0.002)	1.283 27.59 (0.00)	1.282 17.67 (0.00)	0.346 4.97 (0.00)	- 0.116 -2.15 (0.03)	0.650
S2	-0.088 -0.40 (0.69)	1.459 36.48 (0.00)	0.566	-0.386 -2.62 (0.009)	1.084 37.56 (0.00)	1.609 34.60 (0.00)	0.335 8.00 (0.00)	0.807	-0.318 -2.10 (0.036)	1.070 35.93 (0.00)	1.605 34.51 (0.00)	0.305 6.85 (0.00)	- 0.066 -2.10 (0.04)	0.808
S3	0.168 1.00 (0.32)	1.382 45.25 (0.00)	0.668	-0.114 -1.05 (0.30)	1.075 50.36 (0.00)	1.180 34.33 (0.00)	0.464 14.99 (0.00)	0.862	0.017 0.15 (0.88)	1.047 48.08 (0.00)	1.172 34.47 (0.00)	0.407 12.47 (0.00)	- 0.128 -5.07 (0.00)	0.865
S4	0.360 2.29 (0.02)	1.297 45.39 (0.00)	0.669	0.037 0.50 (0.62)	0.964 65.57 (0.00)	1.225 51.72 (0.00)	0.586 27.47 (0.00)	0.926	0.070 0.91 (0.37)	0.957 63.10 (0.00)	1.222 51.63 (0.00)	0.572 25.16 (0.00)	- 0.032 -1.82 (0.07)	0.926
SH	0.522 2.82 (0.005)	1.390 41.29 (0.00)	0.626	0.090 1.15 (0.25)	0.985 64.06 (0.00)	1.346 54.37 (0.00)	0.906 40.62 (0.00)	0.934	0.156 1.95 (0.05)	0.971 61.49 (0.00)	1.342 54.45 (0.00)	0.877 37.08 (0.00)	- 0.065 -3.56 (0.00)	0.934
S2L	-0.188 -1.4 (0.16)	1.241 50.82 (0.00)	0.717	-0.242 -2.98 (0.003)	1.069 66.69 (0.00)	1.053 40.95 (0.00)	- 0.265 - 11.43 (0.00)	0.897	-0.222 -2.66 (0.008)	1.064 64.57 (0.00)	1.051 40.85 (0.00)	-0.274 -11.08 (0.00)	- 0.020 -1.04 (0.30)	0.897
S22	0.166 1.39 (0.17)	1.269 58.31 (0.00)	0.769	-0.012 -0.18 (0.86)	1.042 80.83 (0.00)	0.987 47.56 (0.00)	0.190 10.16 (0.00)	0.931	0.046 0.96 (0.49)	1.029 77.82 (0.00)	0.983 47.63 (0.00)	0.1648.29 (0.00)	- 0.057 -3.75 (0.00)	0.932
S23	0.300 2.71 (0.007)	1.185 58.84 (0.00)	0.773	0.089 1.54 (0.12)	0.958 84.65 (0.00)	0.863 47.36 (0.00)	0.357 21.72 (0.00)	0.939	0.080 1.35 (0.18)	0.960 82.10 (0.00)	0.864 47.32 (0.00)	0.361 20.59 (0.00)	0.009 0.65 (0.52)	0.939

The data for the 1-month Treasury bill rate (R_f), the Fama-French factors, and the 25 size and book-to-market portfolios are from Kenneth French's website. The table reports the alphas and factor coefficients for each factor in the three models. The p-value for each coefficient is in parentheses.

Table 4: Factor Regression for Monthly Excess Returns on 25 Size and Book-to-Market Portfolios, 1/1927-12/2011 (1020 Months) (Continued)

	CAPM			Fama-French 3-Factor Model					Carhart 4-Factor Model					
Portfolio	Alpha	$R_M - R_{rf}$	Adj R^2	Alpha	$R_M - R_{rf}$	SMB	HML	Adj R^2	Alpha	$R_M - R_{rf}$	SMB	HML	MOM	Adj R^2
B44	0.211 2.29 (0.02)	1.171 69.87 (0.00)	0.827	0.015 0.24 (0.81)	1.044 84.74 (0.00)	0.201 10.14 (0.00)	0.587 32.84 (0.00)	0.921	0.071 1.11 (0.27)	1.032 81.63 (0.00)	0.198 10.01 (0.00)	0.562 29.70 (0.00)	-0.055 -3.76 (0.00)	0.922
B4H	0.150 1.07 (0.28)	1.434 56.59 (0.00)	0.759	-0.174 -2.11 (0.04)	1.228 75.89 (0.00)	0.298 11.42 (0.00)	0.985 41.95 (0.00)	0.916	-0.082 -0.97 (0.33)	1.208 73.06 (0.00)	0.292 11.30 (0.00)	0.944 38.11 (0.00)	-0.091 -4.75 (0.00)	0.918
BL	-0.008 -0.17 (0.87)	0.964 107.56 (0.00)	0.919	0.084 2.20 (0.03)	1.031 138.50 (0.00)	-0.152 -12.63 (0.00)	-0.251 -23.21 (0.00)	0.952	0.106 2.73 (0.006)	1.027 133.81 (0.00)	-0.153 -12.77 (0.00)	-0.261 -22.69 (0.00)	-0.022 -2.49 (0.01)	0.953
B2	0.020 0.41 (0.68)	0.918 102.82 (0.00)	0.912	0.047 1.02 (0.31)	0.957 106.85 (0.00)	-0.188 -13.03 (0.00)	-0.010 -0.78 (0.44)	0.925	0.048 1.01 (0.31)	0.957 103.38 (0.00)	-0.188 -13.01 (0.00)	-0.011 -0.76 (0.45)	-0.001 -0.08 (0.93)	0.925
B3	0.046 0.67 (0.51)	0.978 78.70 (0.00)	0.859	-0.018 -0.33 (0.74)	0.976 89.35 (0.00)	-0.218 -12.41 (0.00)	0.313 19.77 (0.00)	0.907	0.006 0.11 (0.91)	0.971 86.14 (0.00)	-0.220 -12.5 (0.00)	0.302 17.92 (0.00)	-0.024 -1.85 (0.06)	0.907
B4	-0.014 -0.14 (0.89)	1.125 62.09 (0.00)	0.791	-0.200 -3.32 (0.001)	1.054 89.07 (0.00)	-0.173 -9.06 (0.00)	0.713 41.51 (0.00)	0.924	-0.116 -1.90 (0.06)	1.036 86.19 (0.00)	-0.178 -9.48 (0.00)	0.676 37.53 (0.00)	-0.083 -5.94 (0.00)	0.927
BH	0.170 1.25 (0.21)	1.157 46.26 (0.00)	0.680	-0.082 -0.81 (0.42)	1.045 51.98 (0.00)	-0.010 -0.32 (0.75)	0.828 28.61 (0.00)	0.823	-0.152 -1.46 (0.14)	1.058 51.47 (0.00)	-0.005 -0.17 (0.87)	0.858 28.02 (0.00)	0.070 2.91 (0.004)	0.825
GRS F-Test			3.378 (0.00)					3.094 (0.00)						2.494 (0.00)

The data for the 1-month Treasury bill rate (R_f), the Fama-French factors, and the 25 size and book-to-market portfolios are from Kenneth French's website. The table reports the alphas and factor coefficients for each factor in the three models. The p-value for each coefficient is in parentheses. The GRS F-test provides the Gibbons et al. (1989) F-statistics testing the intercepts of all 25 portfolios are jointly zero, and the p-value is in parentheses.

The next step is to use the Pitman Closeness Criterion to assess the predictive power of the three models. The results for the Pitman Criteria are provided in Tables 5 and 6. When looking over the entire sample period from January 1927 to December 2011, the Fama-French three-factor Model and the Carhart four-factor Model each outperformed the CAPM Model for all portfolios as determined by the Pitman Closeness Criterion.

Table 5: Pitman Closeness Criterion for Monthly Excess Returns on 25 Size and Book-to-Market Portfolios, 1/1927-12/2011 (1020 Months)

	CAPM (1) vs. Fama-French 3-Factor Model (3)			CAPM (1) vs. Carhart 4-Factor Model (4)			Fama-French 3-Factor Model (3) vs. Carhart 4-Factor Model (4)		
Portfolio	P(1>3)	P(3>1)	Pitman Closer Model	P(1>4)	P(4>1)	Pitman Closer Model	P(3>4)	P(4>3)	Pitman Closer Model
SL	.362	.638	3	.347	.653	4	.478	.532	4
S2	.362	.638	3	.369	.631	4	.518	.482	3
S3	.304	.696	3	.293	.707	4	.468	.532	4
S4	.293	.707	3	.302	.698	4	.502	.498	3
SH	.277	.723	3	.274	.726	4	.508	.492	3
S2L	.278	.722	3	.277	.723	4	.480	.520	4
S22	.338	.662	3	.332	.668	4	.464	.536	4
S23	.299	.701	3	.298	.702	4	.514	.486	3
S24	.289	.711	3	.569	.431	1	.507	.493	3
S2H	.259	.741	3	.258	.742	4	.485	.515	4
M3L	.338	.662	3	.333	.667	4	.474	.526	4
M32	.339	.661	3	.340	.660	4	.552	.448	3
M33	.352	.648	3	.356	.644	4	.548	.452	3

The table shows the probability of one model outperforming another model. When the probability is greater than 0.5, then the model is considered Pitman Closer.

Table 5: Pitman Closeness Criterion for Monthly Excess Returns on 25 Size and Book-to-Market Portfolios, 1/1927-12/2011 (1020 Months) (Continued)

	CAPM (1) vs.Fama-French 3-Factor Model (3)			CAPM (1) vs.Carhart 4-Factor Model (4)			Fama-French 3-Factor Model (3) vs.Carhart 4-Factor Model (4)		
Portfolio	P(1>3)	P(3>1)	Pitman Closer Model	P(1>4)	P(4>1)	Pitman Closer Model	P(3>4)	P(4>3)	Pitman Closer Model
M34	.330	.670	3	.333	.667	4	.520	.480	3
M3H	.306	.694	3	.311	.689	4	.488	.512	4
B4L	.382	.618	3	.381	.619	4	.488	.512	4
B42	.429	.571	3	.420	.580	4	.458	.545	4
B43	.395	.605	3	.389	.611	4	.477	.523	4
B44	.378	.622	3	.378	.622	4	.472	.528	4
B4H	.351	.649	3	.336	.664	4	.480	.520	4
BL	.339	.661	3	.339	.661	4	.512	.488	3
B2	.436	.564	3	.435	.565	4	.489	.511	4
B3	.449	.551	3	.443	.557	4	.489	.511	4
B4	.392	.608	3	.382	.618	4	.486	.514	4
BH	.370	.630	3	.374	.626	4	.517	.483	3

The table shows the probability of one model outperforming another model. When the probability is greater than 0.5, then the model is considered Pitman Closer.

However, there is no primary better model when comparing the Fama-French Model and the Carhart Model as the two models alternate outperforming the other for various portfolios. The Fama-French model outperforms the Carhart Model in 15 of the 25 portfolios and the Carhart Model outperforms the Fama-French in 10 of the 25 portfolios. The results for reduced sample are presented in Table 6.

Table 6: Pitman Closeness Criterion for Monthly Excess Returns on 25 Size and Book-to-Market Portfolios, 1/2001-12/2011 (120 Months)

	CAPM (1) vs. Fama-French 3-Factor Model (3)			CAPM (1) vs. Carhart 4-Factor Model (4)			Fama-French 3-Factor Model (3) vs. Carhart 4-Factor Model (4)		
Portfolio	P(1>3)	P(3>1)	Pitman Closer Model	P(1>4)	P(4>1)	Pitman Closer Model	P(3>4)	P(4>3)	Pitman Closer Model
SL	.362	.638	3	.347	.653	4	.478	.532	4
S2	.362	.638	3	.369	.631	4	.518	.482	3
S3	.304	.696	3	.293	.707	4	.468	.532	4
S4	.293	.707	3	.302	.698	4	.492	.508	4
SH	.277	.723	3	.274	.726	4	.433	.567	4
S2L	.278	.722	3	.277	.723	4	.480	.520	4
S22	.338	.662	3	.332	.668	4	.464	.536	4
S23	.299	.701	3	.298	.702	4	.442	.558	4
S24	.289	.711	3	.569	.431	1	.507	.493	3
S2H	.259	.741	3	.258	.742	4	.485	.515	4
M3L	.338	.662	3	.333	.667	4	.474	.526	4
M32	.339	.661	3	.340	.660	4	.552	.448	3
M33	.352	.648	3	.356	.644	4	.492	.508	4

The table shows the probability of one model outperforming another model. When the probability is greater than 0.5, then the model is considered Pitman Closer.

Table 6: Pitman Closeness Criterion for Monthly Excess Returns on 25 Size and B/M Portfolios, 1/2001-12/2011 (120 Months) (Continued)

	CAPM (1) vs. Fama-French 3-Factor Model (3)			Fama-French 3-Factor Model			Carhart 4-Factor Model		
Portfolio	P(1>3)	P(3>1)	Pitman Closer Model	P(1>4)	P(4>1)	Pitman Closer Model	P(3>4)	P(4>3)	Pitman Closer Model
M34	.330	.670	3	.333	.667	4	.425	.575	4
M3H	.306	.694	3	.311	.689	4	.488	.512	4
B4L	.382	.618	3	.381	.619	4	.488	.512	4
B42	.429	.571	3	.420	.580	4	.458	.545	4
B43	.395	.605	3	.389	.611	4	.477	.523	4
B44	.500	.500	TIE	.542	.458	1	.525	.475	3
B4H	.351	.649	3	.336	.664	4	.480	.520	4
BL	.339	.661	3	.339	.661	4	.467	.533	4
B2	.436	.564	3	.435	.565	4	.525	.475	3
B3	.449	.551	3	.443	.557	4	.489	.511	4
B4	.517	.483	1	.382	.618	4	.486	.514	4
BH	.370	.630	3	.374	.626	4	.517	.483	3

The table shows the probability of one model outperforming another model. When the probability is greater than 0.5, then the model is considered Pitman Closer.

When comparing the last 10 years, some changes occur, so that the Carhart Model outperforms the others in 19 of the 25 portfolios. There is no discernible pattern with respect to size or book-to-market. These findings are not inconsistent with those of Bello (2008), who compared the same models over a different time period to equity mutual fund data using a statistical goodness of fit method. The implications of the results from this study are that investors should consider using the Carhart Model, when estimating risk-adjusted returns for their portfolios because it is a better estimator of returns according to the Pitman Closeness Criterion.

6. Conclusion

In this study we take a different approach to evaluating the predictive ability of several popular stock pricing models. Applying Pitman Closeness Criterion, we determine that over the long sample period, the Carhart four-factor model and the Fama-French three-factor models performed better in predicting prices for all of the portfolios evaluated than that CAPM model over the same time period. We also conclude that the Carhart model provides a Pitman Closer estimate for more portfolios than does the Fama-French model. Our findings using the Pitman Closeness Criterion are similar and in line with those of other studies using traditional methods of model comparison.

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