

## An Optimization Model for Reverse Logistics Network under Stochastic Environment Using Genetic Algorithm

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### Abstract

*Recovery of used products has become increasingly important recently due to economic reasons and growing environmental or legislative concern. Product recovery, which comprises reuse, remanufacturing and materials recycling, requires an efficient reverse logistic network. One of the main characteristics of reverse logistics network problem is uncertainty that further amplifies the complexity of the problem. The degree of uncertainty in terms of the capacities, demands and quantity of products exists in reverse logistics parameters. With consideration of the factors noted above, this paper proposes a probabilistic mixed integer linear programming model for the design of a reverse logistics network. This probabilistic model is first converted into an equivalent deterministic model. In this paper we proposed multi-product, multi-stage reverse logistics network problem for the return products to determines not only the subsets of disassembly centers and processing centers to be opened, but also the transportation strategy that will satisfies demand imposed by manufacturing centers and recycling centers with minimum fixed opening cost and total shipping cost. Then, we propose priority based genetic algorithm to find reverse logistics network to satisfy the demand imposed by manufacturing centers and recycling centers with minimum total cost under uncertainty condition. Finally, we apply the proposed model to a numerical example.*

**Keywords:** Reverse logistics network, Genetic algorithm (GA), Priority-based encoding, Stochastic programming

### 1. Introduction and literature review

#### 1.1. Reverse logistics

Increasing interest in reuse of products and materials is one of the consequences of growing environmental concern throughout the past decades. Waste reduction has become a prime concern in industrialized countries [1]. For a variety of economic, environmental or legislative reasons, companies have become more accountable for final products, after they sell those products. Reverse logistics is the process of moving goods from their typical final destination to another point, for the purpose of capturing value otherwise unavailable, or for the proper disposal of the products [2]. According to the American Reverse Logistics Executive Council, Reverse Logistics is defined as: “The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal.” A reverse logistics system comprises a series of activities, which form a continuous process to treat return-products until they are properly recovered or disposed of. These activities include collection, cleaning, disassembly, test and sorting, storage, transport, and recovery operations. The latter can also be represented as one or a combination of several main recovery options, like reuse, repair, refurbishing, remanufacturing, cannibalization and recycling [3]. Reverse logistics is practiced in many industries, including those producing steel, aircraft, computers, automobiles, chemicals, appliances and medical items. The effective use of the reverse logistics can help a company to compete in its industry.

Reverse logistics has become increasingly important as a profitable and sustainable business strategy. There are a number of situations for products to be placed in a reverse flow. Normally, return flows are classified into commercial returns, warranty returns, end-of-use returns, reusable container returns and others [2]. Implementation of reverse logistics especially in product returns would allow not only for savings in inventory carrying cost, transportation cost, and waste disposal cost due to returned products, but also for the improvement of customer loyalty and futures sales [4]. Reverse logistic systems are more complex than forward logistic systems. This complexity stems from a high degree of uncertainty due to the quantity and quality of the products [5]. Reverse logistics is receiving much attention recently due to growing environmental or legislative concern and economic opportunities for cost savings or revenues from returned products. Barros et al. [6] proposed a mixed integer linear programming (MILP) model based on a multi-level capacitated warehouse location problem for the sand and consider its optimization using heuristic procedures. The model determined the optimal number, capacities, and locations of the depots and cleaning facilities. Kirkke et al. [7] presented an MILP model based on a multi-level uncapacitated warehouse location model. They described a case study, dealing with a reverse logistics network for the returns, processing, and recovery of discarded copiers.

The model was used to determine the locations and capacities of the recovery facilities as well as the transportation links connecting various locations. Jayaraman et al. [8] proposed an MILP model to determine the optimal number and locations of distribution/remanufacturing facilities for electronic equipment. Jayaraman et al. [9] developed a mixed integer programming model and solution procedure for a reverse distribution problem focused on the strategic level. The model determines whether each remanufacturing facility is open considering the product return flow. Min et al. [10] proposed a Lagrangian relaxation heuristics to design the multi-commodity, multi-echelon reverse logistics network. Kim et al. [11] proposed a general framework for remanufacturing environment and a mathematical model to maximize the total cost saving. The model determines the quantity of products/parts processed in the remanufacturing facilities/subcontractors and the amount of parts purchased from the external suppliers while maximizing the total remanufacturing cost saving. Min et al. [12] proposed a nonlinear mixed integer programming model and a genetic algorithm that can solve the reverse logistics problem involving product returns.

Their study proposes a mathematical model and GA which aim to provide a minimum-cost solution for the reverse logistics network design problem involving product returns. Ko and Evans [4] presented a mixed integer nonlinear programming model for the design of a dynamic integrated distribution network to account for the integrated aspect of optimizing the forward and return network simultaneously. They also proposed a genetic algorithm-based heuristic for solving this problem. Lee et al. [13] proposed a multi-stage, multi-product, MILP model for minimizing the total of costs to reverse logistics shipping cost and fixed opening cost of facilities. They also proposed a hybrid genetic algorithm for solving this problem.

## **1.2. Stochastic programming**

In most of the real life problems in mathematical programming, the parameters are considered as random variables. The branch of mathematical programming which deals with the theory and methods for the solution of conditional extremum problems under incomplete information about the random parameters is called “stochastic programming”. Most of the problems in applied mathematics may be considered as belonging to any one of the following classes [14]:

1. Descriptive Problems, in which, with the help of mathematical methods, information is processed about the investigated event, some laws of the event being induced by others.

2. Optimization Problems in which from a set of feasible solutions, an optimal solution is chosen.

Besides the above division of applied mathematics problems, they may be further classified as deterministic and stochastic problems. In the process of the solution of the stochastic problem, several mathematical methods have been developed. However, probabilistic methods were for a long time applied exclusively to the solution of the descriptive type of problems. Research on the theoretical development of stochastic programming is going on for the last four decades. To the several real life problems in management science, it has been applied successfully [15]. The chance constrained programming was first developed by Charnes and Cooper [16]. Subsequently, some researchers like Sengupta [17], Contini [18], Sullivan and Fitzsimmons [19], Leclercq [20], Teghem et al. [21] and many others have established some theoretical results in the field of stochastic programming.

Stancu-Minasian and Wets [15] have presented a review paper on stochastic programming with a single objective function. Listes and Dekker [22] proposed a multi product stochastic mixed integer programming for recycling of the sand in reverse logistics network. Liu [23] introduced the stochastic programming methodology to characterize the stochastic traffic for a multi-commodity network model.

### **1.3. Genetic algorithm**

GA's are stochastic search techniques based on the mechanism of natural selection and natural genetics [24]. As one of the Evolutionary Computation (EC) techniques, the GA has been receiving great attention and successfully applied for combinatorial optimization problems [25]. GA is very useful when a large search space with little knowledge of how to solve the problem is presented. It belongs to the class of heuristic optimization techniques, which include simulated annealing (SA), Tabu search, and evolutionary strategies. It has been with great success in providing optimal or near optimal solution for many diverse and difficult problems [26].

Representation is one of the important issues that affect the performance of GAs. Usually different problems have different data structures or genetic representations. Tree-based representation is known to be one way for representing network problems. There are three ways of encoding tree: (1) edge-based encoding, (2) vertex-based encoding and (3) edge-and-vertex encoding [27].

Michalewicz et al. [28] used matrix-based representation GA which belongs to edge-based encoding for solving linear and non-linear transportation/distribution problems. When  $m$  and  $n$  are the number of sources and depots, respectively, the dimension of matrix will be  $m \times n$ . Although representation is very simple, this approach needs special crossover and mutation operators for obtaining feasible solutions.

Gen and Cheng [27] introduced spanning tree GA (st-GA) for solving network problems. They used Prüfer number representation for solving transportation problems and developed feasibility criteria for Prüfer number to be decoded into a spanning tree. Syarif et al. [29] proposed spanning tree-based genetic algorithm by using Prüfer number representation for solving a single product, three stage supply chain network (SCN) problem. Xu et al. [30] applied spanning tree-based genetic algorithm (st-GA) by the Prüfer number representation to find the SCN to satisfy the demand imposed by customers with minimum total cost and maximum customer services for multi objective SCN design problem. Although Prüfer number developed to encode of spanning trees, had been successfully applied to transportation problems, it needs some repair mechanisms to obtain feasible solutions after classical genetic operators.

In this study, to escape from these repair mechanisms in the search process of GA, we adopt at here the priority-based encoding method. Gen et al. [25] used priority-based encoding for a single-product, two-stage transportation problem. Altıparmak et al. [31] applied priority-based representation to a single-product, single-source, and three-stage SCN problem, Altıparmak et al. [32] proposed this encoding to a single-source, multi-product, multi-stage SCN problem. Lee et al. [13] proposed a hybrid genetic algorithm with priority-based encoding method.

One of the main characteristics of reverse logistics network problem is uncertainty that further amplifies the complexity of the problem. The degree of uncertainty in terms of the capacities, demands and quantity of products exists in reverse logistics parameters. An important issue, when manufacturing centers demand and recycling centers demand are random variables in reverse logistics network design problem, is to find the network strategy that can achieve the objective of minimization of total shipping cost and fixed opening costs of the disassembly centers and the processing centers. With consideration of the factors noted above, this paper proposes a probabilistic mixed integer linear programming model for the design of a reverse logistics network. This probabilistic model is first converted into an equivalent deterministic model. In this paper we propose multi-product, multi-stage reverse logistics network problem which consider the minimizing of total shipping cost and fixed opening costs of the disassembly centers and the processing centers in reverse logistics. In fact, this type of network design problem belongs to the class of NP-hard problems, so that priority based genetic algorithm will be presented in order to solve large size problem. Finally, we apply the proposed model to an example problem and show the numerical results.

This paper is organized as follows: in section 2, the stochastic constraint is explained and we present an approach to convert it into a deterministic for special case (normal distribution). The mathematical model of the reverse logistics network is introduced in section 3.

In section 4, the priority-based GA approach is explained in order to solve this problem. An illustrative numerical example is given in section 5. Finally, concluding remarks are outlined in section 6.

**2. Stochastic constraint**

If  $X \sim n(\mu, \sigma^2)$ , its density function is  $f(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ ;  $-\infty < x < +\infty$ .

$Z = \frac{X-\mu}{\sigma}$  transforms  $X$  to  $Z$  that has following properties:

$Z \sim n(0,1)$   $f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ ;  $-\infty < z < +\infty$

$$P(Z > z_\alpha) = \alpha, P(Z < z_\alpha) = 1 - \alpha$$

For example if  $\sum_{j=1}^n X_j \leq K$  is a constraint in a mathematical programming problem and  $K \sim n(\mu, \sigma^2)$  then  $P(\sum_{j=1}^n X_j \leq K) \geq 1 - \alpha$  is equivalent to:

$P\left(\frac{K-\mu}{\sigma} > \frac{\sum_{j=1}^n X_j - \mu}{\sigma}\right) \geq 1 - \alpha$  or  $P\left(Z > \frac{\sum_{j=1}^n X_j - \mu}{\sigma}\right) \geq 1 - \alpha$  that resulted in  $\frac{\sum_{j=1}^n X_j - \mu}{\sigma} \leq Z_{1-\alpha}$  or  $\sum_{j=1}^n X_j \leq \sigma \cdot Z_{1-\alpha} + \mu$ .

Therefore:

$$P\left(\sum_{j=1}^n X_j \leq K\right) \geq 1 - \alpha \approx \sum_{j=1}^n X_j \leq \sigma \cdot Z_{1-\alpha} + \mu \tag{1}$$

**3. Mathematical formulation**

In this section, we present a reverse logistics network problem for the return products to determines not only the subsets of disassembly centers and processing centers to be opened, but also the transportation strategy that will satisfies demand imposed by manufacturing centers and recycling centers with minimum fixed opening cost and total shipping cost. However, in reverse logistics network design problem, it is hard to describe these problem parameters as known variables because there are not sufficient enough data to analyze. The degree of uncertainty in terms of the capacities, demands and quantity of products exists in reverse logistics parameters. With consideration of the factors noted above, in this section, we present a probabilistic mixed integer linear programming model for the design of a reverse logistics network.

In the remanufacturing process, after dismantling products to parts, reusable parts are sent from disassembly centers to processing centers according to their types for inspecting, cleaning and preparing. These parts become new products by combined with another parts of processed or new in manufacturing centers.

In the recycling process, after dismantling products to parts, parts which are not reusable but are recyclable are sent directly from disassembly centers to recycling centers according to their types. Some products that do not need to disassemble send directly from returning centers to the processing centers, according to the product type (see Fig 1).

**Model assumptions**

- (1) The demand of manufacturing centers and recycling centers are regarded as random variables.
  - (2) The number of returning centers and manufacturing centers and recycling centers are known and constant.
  - (3) The number of potential processing centers and disassembly centers and their maximum capacities are known.
  - (4) Some products that do not need to disassemble should send from returning centers to processing centers directly, not through disassembly centers.
  - (5) Some parts should send form disassembly centers to recycling centers directly, not through processing centers.
- The notations used for the considered problem are listed below:

**Indices**

- $i$  is an index for returning center ( $i = 1,2,3, \dots, I$ ).
- $j$  is an index for disassembly center ( $j = 1,2,3, \dots, J$ ).
- $k$  is an index for processing center ( $k = 1,2,3, \dots, K$ ).
- $f$  is an index for manufacturing center ( $f = 1,2,3, \dots, F$ ).
- $r$  is an index for recycling center ( $r = 1,2,3, \dots, R$ ).
- $p$  is an index for product ( $p = 1,2,3, \dots, P$ ).
- $m$  is an index for part ( $m = 1,2,3, \dots, M$ ).

**Model variables**

$x_{ijp}$  is amount shipped from returning center  $i$  to disassembly center  $j$  for product  $p$ .  
 $x_{ikp}$  is amount shipped directly from returning center  $i$  to processing center  $k$  for product  $p$ .  
 $x_{jkm}$  is amount shipped from disassembly center  $j$  to processing center  $k$  for part  $m$ .  
 $x_{jrm}$  is amount shipped directly from disassembly center  $j$  to recycling center  $r$  for part  $m$ .  
 $x_{kfm}$  is amount shipped from processing center  $k$  to manufacturing center  $f$  for part  $m$ .  
 $x_{krm}$  is amount shipped from processing center  $k$  to recycling center  $r$  for part  $m$ .  
 $Y_{jm} = \begin{cases} 1, & \text{if disassembly center } j \text{ is open, } \forall j, m \\ 0, & \text{otherwise} \end{cases}$   
 $Q_{km} = \begin{cases} 1, & \text{if processing center } k \text{ is open, } \forall k, m \\ 0 & \text{otherwise} \end{cases}$

**Model parameters**

$I$  is the number of returning centers.  
 $J$  is the number of disassembly centers.  
 $K$  is the number of processing centers.  
 $F$  is the number of manufacturing centers.  
 $R$  is the number of recycling centers.  
 $P$  is the number of products.  
 $M$  is the number of parts  
 $a_{ip}$  is the capacity of returning center  $i$  for product  $p$ .  
 $b_{jm}$  is the capacity of disassembly center  $j$  for part  $m$ .  
 $u_{km}$  is the capacity of processing center  $k$  for part  $m$ .  
 $d_{fm}$  is the demand of part  $m$  in manufacturing center  $f$  (random variable).  
 $d_{rp}$  is the demand of product  $p$  in recycling center  $r$  (random variable).  
 $d_{rm}$  is the demand of part  $m$  in recycling center  $r$  (random variable).  
 $n_{mp}$  is the number of part for the part type  $m$  from disassembling one unit of product  $p$ .  
 $c_{ijp}$  is the unit cost of shipping from returning center  $i$  to disassembly center  $j$  for product  $p$ .  
 $c_{ikp}$  is the unit cost of shipping from returning center  $i$  to processing center  $k$  for product  $p$ .  
 $c_{jkm}$  is the unit cost of shipping from disassembly center  $j$  to processing center  $k$  for part  $m$ .  
 $c_{jrm}$  is the unit cost of shipping from disassembly center  $j$  to recycling center  $r$  for part  $m$ .  
 $c_{kfm}$  is the unit cost of shipping from processing center  $k$  to manufacturing center  $f$  for part  $m$ .  
 $c_{krm}$  is the unit cost of shipping from processing center  $k$  to recycling center  $r$  for part  $m$ .  
 $c_{jm}^{oc}$  is the fixed opening cost for disassembly center  $j$  for part  $m$ .  
 $c_{km}^{oc}$  is the fixed opening cost for processing center  $k$  for part  $m$ .  
 $1 - \alpha$  is the confidence level.

The problem can be formulated as follow:

$$\begin{aligned} \text{Min } Z = & \sum_{j=1}^J \sum_{p=1}^P c_{jp}^{oc} Y_{jm} + \sum_{k=1}^K \sum_{m=1}^M c_{km}^{oc} Q_{km} + \sum_{i=1}^I \sum_{j=1}^J \sum_{p=1}^P c_{ijp} x_{ijp} + \sum_{i=1}^I \sum_{k=1}^K \sum_{p=1}^P c_{ikp} x_{ikp} \\ & + \sum_{j=1}^J \sum_{k=1}^K \sum_{m=1}^M c_{jkm} x_{jkm} + \sum_{j=1}^J \sum_{r=1}^R \sum_{m=1}^M c_{jrm} x_{jrm} + \sum_{k=1}^K \sum_{f=1}^F \sum_{m=1}^M c_{kfm} x_{kfm} \\ & + \sum_{k=1}^K \sum_{r=1}^R \sum_{m=1}^M c_{krm} x_{krm} \end{aligned}$$

s. t

$$\sum_{j=1}^J x_{ijp} \leq a_{ip}, \forall i, p \tag{2}$$

$$\sum_{k=1}^K x_{ikp} \leq a_{ip}, \forall i, p \tag{3}$$

$$\sum_{k=1}^K x_{jkm} \leq b_{jm} Y_{jm}, \forall j, p, m \tag{4}$$

$$\sum_{r=1}^R x_{jrm} \leq b_{jm} Y_{jm}, \forall j, p, m \tag{5}$$

$$\sum_{f=1}^F x_{kfm} \leq u_{km} Q_{km}, \forall k, m \tag{6}$$

$$\sum_{r=1}^R x_{krm} \leq u_{km} Q_{km}, \forall k, m \tag{7}$$

$$\sum_{j=1}^J \sum_{k=1}^K x_{jkm} \leq n_{mp} \left( \sum_{i=1}^I \sum_{j=1}^J x_{ijp} \right) \forall m, p \tag{8}$$

$$\sum_{j=1}^J \sum_{r=1}^R x_{jrm} \leq n_{mp} \left( \sum_{i=1}^I \sum_{j=1}^J x_{ijp} \right) \forall m, p \tag{9}$$

$$P(\sum_{j=1}^J \sum_{k=1}^K x_{jkm} \geq \sum_{f=1}^F d_{fm}) \geq 1 - \alpha_{fm}, \forall f, m \tag{10}$$

$$P(\sum_{i=1}^I \sum_{k=1}^K x_{ikp} \geq \sum_{r=1}^R d_{rp}) \geq 1 - \alpha_{rp}, \forall r, p \tag{11}$$

$$P\left(\sum_{k=1}^K x_{kfm} \geq d_{fm}\right) \geq 1 - \alpha_{fm}, \forall f, m \tag{12}$$

$$P\left(\sum_{k=1}^K x_{krp} \geq d_{rp}\right) \geq 1 - \alpha_{rp}, \forall r, p \tag{13}$$

$$P \left( \sum_{j=1}^J x_{jrm} \geq d_{rm} \right) \geq 1 - \alpha_{rm}, \forall r, m \tag{14}$$

$$\sum_{j=1}^J Y_{jm} J, \forall m \tag{15}$$

$$\sum_{k=1}^K Q_{km} \leq K, \forall m \tag{16}$$

$$x_{ijp}, x_{ikp}, x_{jkm}, x_{jrm}, x_{kfm}, x_{krm}, x_{jdm}, \geq 0, \forall i, j, k, r, f, p, m \tag{17}$$

$$Y_{jm} = \{0,1\} \forall j, m$$

$$Q_{km} = \{0,1\} \forall k, m \tag{19}$$

The objective function minimizes the total cost of the reverse logistics. It consists of the reverse logistics shipping cost and fixed opening cost of the disassembly centers and processing centers.

Constraints (2),(3) explain about capacity of each returning center, (4),(5) explain about capacity of each disassembly center, (6),(7) explain about capacity of each processing center, (8),(9) are balance of parts produced by disassemble of products, and  $n_{mp}$  is a conversion coefficient, (10),(11) show that parts and product shipped from disassembly centers and returning centers to processing centers are pulled by demand of manufacturing centers and recycling centers respectively (12) is the constraint that demands of parts of manufacturing center, (13) is the constraint that demands of product of recycling ,(14) shows the constraint that demands of parts of recycling,(15),(16) limit the number of disassembly centers and processing centers that can be opened,(17) imposes the non-negativity restriction on the decision variables,(18),(19) impose the integrality restriction on the decision variables  $Y_{jp}, Q_{km}$ . In order to solve this probabilistic model, we have to transform it into deterministic model by Eq (1) in sec. 2.

#### 4. Solution approach

We proposed Priority-based genetic algorithm for solving the probabilistic reverse logistics network design problem in this paper.

##### 4. 1. Priority-based genetic algorithm

As it is known, in priority-based encoding, a gene in a chromosome is characterized by two factors: locus, the position of the gene within the structure of chromosome, and allele, the value the gene takes. In priority- based encoding, the position of a gene is used to represent a node (source/depot in transportation network), and the value is used to represent the priority of corresponding node for constructing a tree among candidates [27].

For a transportation problem, a chromosome consists of priorities of sources and depots to obtain transportation tree and its length is equal to total number of sources  $|K|$  and depots  $|J|$ , i.e.  $|K| + |J|$ . The transportation tree corresponding with a given chromosome is generated by sequential arc appending between sources and depots. At each step, only one arc is added to tree selecting a source (depot) with the highest priority and connecting it to a depot (source) considering minimum cost [31]. The decoding algorithm of the priority- based encoding is presented below.

##### Algorithm 1: Priority-based decoding:

**Inputs:**

- K: set of sources
- J: set of depots
- $b_j$ : demand on depot  $j, \forall j \in J$  for product  $p$  or part  $m, \forall p, m$

$a_k$ : capacity of source  $k$ ,  $\forall k \in K$  for product  $p$  or part  $m$ ,  $\forall p, m$   
 $c_{kj}$ : transportation cost of one unit of product  $p$  or part  $m$  from source  $k$  to depot  $j$ ,  $\forall k \in K, \forall j \in J$   
 $v(k + j)$ : chromosome,  $\forall k \in K, \forall j \in J$

**Outputs:**

$x_{kj}$  : the amount of product  $p$  or part  $m$  shipped from source  $k$  to depot  $j$

**While**  $\sum_{j=1}^J b_j \geq 0$

**Step 1:**  $x_{kj} = 0, \forall k \in K, \forall j \in J$

**Step 2:** select a node based on  $l = \arg \max \{v(t), t \in |k| + |j|\}, \forall k \in K, \forall j \in J$

**Step 3:** **if**  $l \in K$  **then** a source is selected  $k^* = l$ ,  
 $j^* = \arg \min \{c_{kj} \mid v(j) \neq 0, j \in J\}$  Select a depot with minimum cost  
**else**  $j^* = l$  a depot is selected  
 $k^* = \arg \min \{c_{kj} \mid v(j) \neq 0, k \in K\}$  Select a source with minimum cost

**Step 4:**  $x_{k^*j^*} = \min\{a_{k^*}, b_{j^*}\}$   
 Update demands and capacities  
 $a_{k^*} = a_{k^*} - x_{k^*j^*}$        $b_{j^*} = b_{j^*} - x_{k^*j^*}$

**Step 5:** **if**  $a_{k^*} = 0$  **then**  $v(k^*) = 0$   
**if**  $b_{j^*} = 0$  **then**  $v(j^*) = 0$

**Step 6:** **if**  $v(k + j) = 0, \forall j \in J$  **then** output  $x_{kj}$  and calculate transportation cost,  
**else return** step 1

**End**

Fig. 2 represents a transportation tree with 5 sources and 4 depots, its cost matrix and priority based encoding. Table 1 gives trace table of the decoding procedure to obtain transportation tree in Fig. 2

For the problem in this paper, we use a chromosome consists of six segments, in which each segment is related to one echelon of the reverse logistics network. We use segment I-J to represent the transportation pattern from returning centers to disassembly centers, segment I-K to represent the transportation pattern from returning centers to processing centers, segment J-K to represent the transportation pattern from disassembly centers to processing centers, segment J-R to represent the transportation pattern from disassembly centers to recycling centers, segment K-R to represent the transportation pattern from processing centers to recycling centers and segment K-F to represent the transportation pattern from processing centers to manufacturing centers (see Fig.3). The chromosome of reverse logistics network is decoded on the backward direction. Transportation tree between processing centers and manufacturing centers, processing and recycling centers, disassembly centers and recycling centers, disassembly centers and opened processing centers, returning centers and opened processing centers, returning centers and opened disassembly centers are obtained with decoding of the last, fifth, fourth, third, second and first segment of chromosome, respectively.

The decoding algorithm of a reverse logistics network chromosome is presented in Algorithm 2.

**Algorithm 2: Reverse logistics network decoding algorithm**

**Inputs:**  $a_{ip}, b_{jm}, u_{km}, d_{fm}, d_{rp}, d_{rm}, n_{mp}, c_{ijp}, c_{ikp}, c_{jkm}, c_{jrm}, c_{kfm}, c_{krm}, c_{jm}^{OC}, c_{km}^{OC}$

**Outputs:**  $x_{ijp}, x_{ikp}, x_{jkm}, x_{jrm}, x_{kfm}, x_{krm}, Y_{jm}, Q_{km}$

**Step 1:** calculate  $x_{kfm}, Q_{km} \quad \forall k \in K, f \in F, m \in M$  using Algorithm 1

**Step 2:** calculate  $x_{krm}, Q_{km} \quad \forall k \in K, r \in R, m \in M$  using Algorithm 1

**Step 3:** calculate  $x_{jrm}, Y_{jm} \quad \forall j \in J, r \in R, m \in M$  using Algorithm 1

**Step 4:** calculate  $x_{jkm}, Y_{jm} \quad \forall j \in J, k \in K, m \in M$  using Algorithm 1

**Step 5:** calculate  $x_{ikp} \quad \forall i \in I, k \in K, p \in P$  using Algorithm 1

**Step 6:** calculate  $x_{ijp} \quad \forall i \in I, j \in J, p \in P$  using Algorithm 1



## 4.2. Genetic operators

The proposed GA solution procedure used four genetic operators described below.

### 4.2.1. Parent selection operator

The parent selection operator is an important process that directs a GA search toward promising regions in a search space. Two parents are selected from the solutions of a particular generation by selection methods that assign reproduction opportunities to each individual parent in the population. There are a number of different selection methods, such as roulette wheel selection, tournament selection, rank selection, elitism selection, and random selection [27]. In this study, we used tournament selection method that two teams of individuals are chosen from the population randomly that each team consists of two chromosomes. The two best chromosomes that are taken from one of the two teams are chosen for crossover operations.

### 4.2.2. Crossover operator

The crossover operator generates new offspring by combining information contained in the chromosomes of the parents so that new chromosomes will have the best parts of the parent's chromosomes. The crossover is done to explore new solution space and crossover operator corresponds to exchanging parts of strings between selected parents. Several crossover operators have been proposed for permutation representation, such as partially mapping crossover (PMX), order crossover (OX), position-based crossover (PX), cycle crossover (CX), weight mapping crossover (WMX), Heuristic crossover, and so on [13]. In this paper, we used weight mapping crossover (WMX) operator that is one-cut point crossover for permutation representation. As one point crossover, two chromosomes (parents) would be chose a random cut-point and generate the offspring by using segment of own parent to the left of the cut-point, then remapping the right segment that base on the weight of other parent of right segment (Fig. 4). After recombination, some children undergo mutation. Similar to crossover, mutation is done to prevent the premature convergence and explores new solution space. However, unlike crossover, mutation is usually done by modifying gene within a chromosome. In this study, we used insert mutation that a digit is randomly selected and it is inserted a randomly selected new position in chromosome. Fig.5 represents insert mutation.

### 4.2.4. Evaluation operator

The evaluation aims to associate each individual with a fitness value so that it can reflect the goodness of fit for an individual. The evaluation process is intended to compare one individual with other individuals in the population. The choice of fitness function is also very critical because it has to accurately measure the desirability of the features described by the chromosome. The function should be computationally efficient since it is used many times to evaluate each solution [27]. In this study, the objective function has been taken as fitness function.

## 5. Numerical example

The proposed model is applied to the hypothetical problem. We take into account a three product (that each product is disassembled to parts:  $\square$  is disassembled to one A, two C's and one D,  $\Delta$  is disassembled to one B and one E,  $\odot$  send directly from returning centers to processing centers because it does not need to disassemble. A and B are reusable parts, C and  $\odot$  are recyclable and D and E are unusable parts that must dispose of.). Three stages reverse logistics network having two manufacturing centers, two recycling centers, four processing centers, four disassembly centers and three returning centers that upper limits of opened disassembly centers and processing centers are taken as three for each part or product.

Table 2 gives information about the capacities of returning centers for each product. Table 3 and 4, show the capacities of product and part and fixed cost for disassembly centers and processing centers, respectively. Table 5 gives information about the stochastic demand of manufacturing centers and recycling centers. Shipping cost from returning center to disassembly center and processing center is presented by table 6. Table 7 shows shipping cost from disassembly centers to processing centers and recycling centers. Table 8 gives information about the shipping cost from processing center to manufacturing centers and recycling centers. By considering  $1 - \alpha = 0.95$  as confidence level of constraints satisfaction for all  $\alpha$ 's and according to Eq (1) in section 2 by, the probabilistic mixed integer linear programming model of this problem has been converted into deterministic model. We used the priority-based genetic algorithm to solve this problem, when GA parameters such as crossover probability:  $P_c=0.8$ , mutation probability:  $P_m =0.15$  and population size: pop size = 50.

The transportation strategy of this example is shown by table 9 to 17.

### 4.2.3. Mutation operator

From the results of above, it can be seen that the best solution is reached by opening three disassembly centers and two processing centers for part A, two disassembly centers and three processing centers for part B, one disassembly centers for part C, and three processing centers for product ☼ though it is allowed to open three disassembly centers and three processing centers for each part or product.

The solution approach illustrated in this paper can easily be applied to other reverse logistics network problems. Therefore, this approach is the appropriate tool to solve other reverse logistics network problems in realistic environments with some stochastic parameters.

## 6. Conclusion

In this paper, we considered a probabilistic mixed integer linear programming model for the design of a reverse logistics network. The demand of manufacturing centers and recycling centers are regarded as random variables. This probabilistic model is first converted into an equivalent deterministic model. In This paper we proposed multi-product, multi-stage reverse logistics network problem which consider the minimizing of total shipping cost and fixed opening costs of the disassembly centers and the processing centers in reverse logistics. In fact, this type of network design problem belongs to the class of NP-hard problems, so that we utilized the priority-based genetic algorithm that is known to be an efficient and robust method to represent various logistics network problems. The proposed model was applied to the hypothetical problem. And then, computing results show that we can obtain solutions for reverse logistics network design problem with some stochastic parameters.

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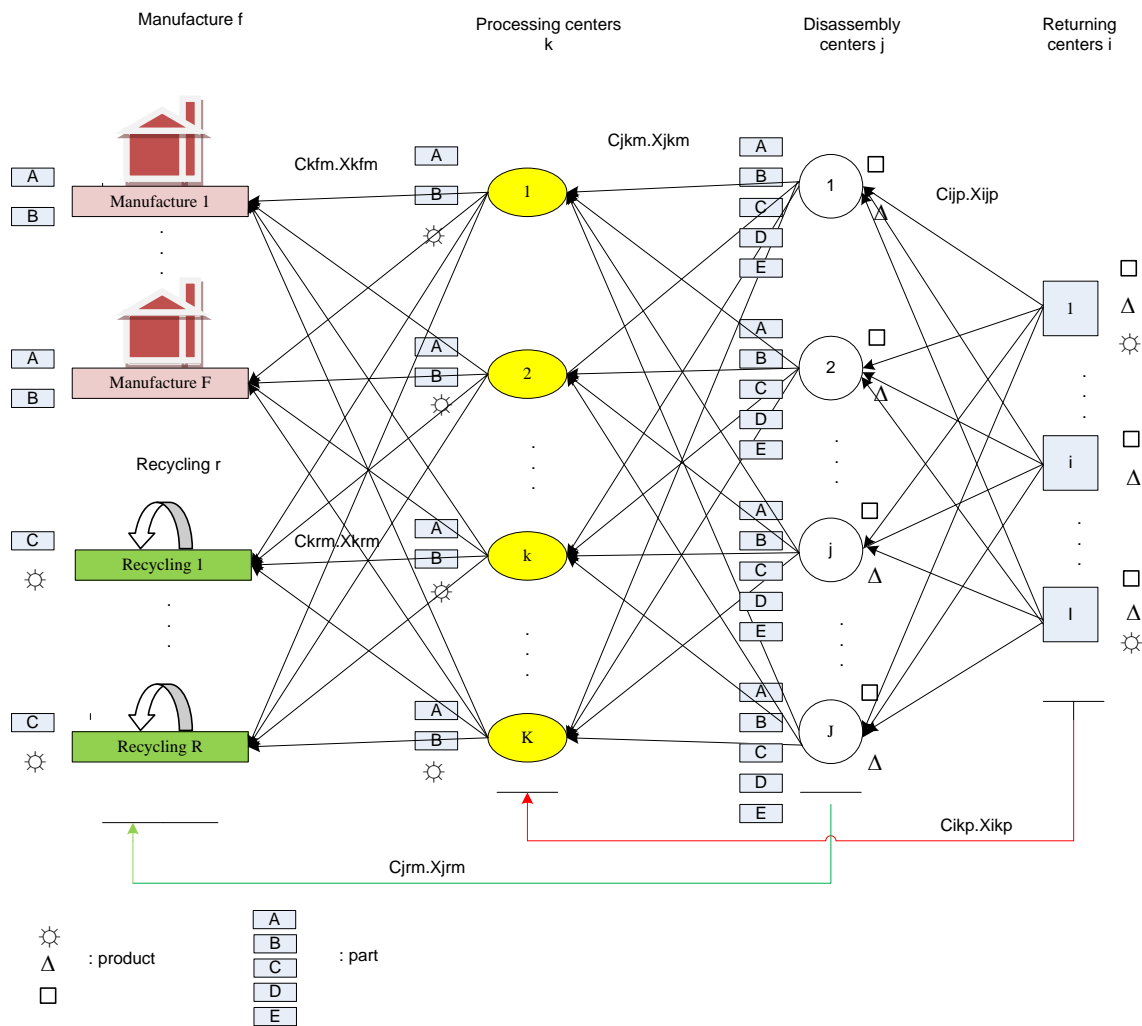


Fig.1. A simple network of multi-product, multi-stage reverse logistics network.

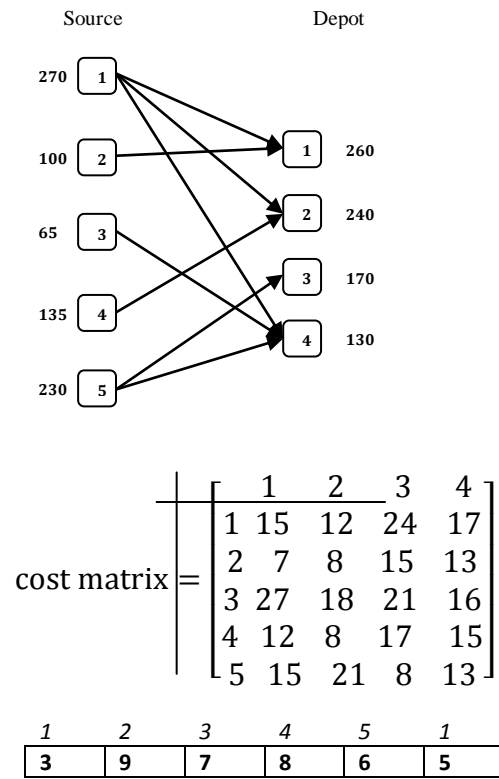
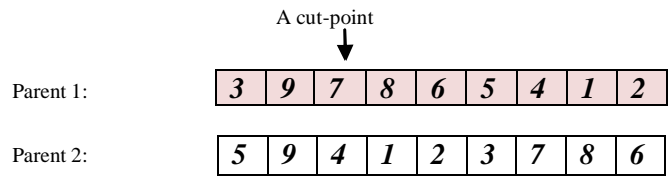


Fig.2. A sample of transportation tree and its encoding.

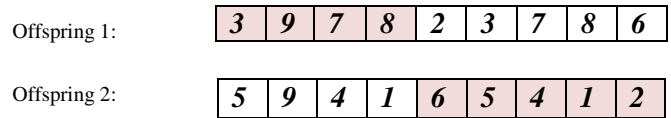
	First segment I-J				Second segment I-K				Third segment J-K				Forth segment J-R				Fifth segment K-R				Sixth segment K-F																			
node	1	2	3	1	2	3	4	1	2	3	1	2	3	4	1	2	3	4	1	2	1	2	3	4	1	2	1	2	3	4	1	2								
priority	2	1	4	7	5	3	6	6	7	3	5	1	4	2	6	8	3	1	5	2	7	4	5	4	1	6	2	3	6	3	1	5	2	4	3	1	4	5	2	6

Fig.3. An illustration of reverse logistics network model chromosome.

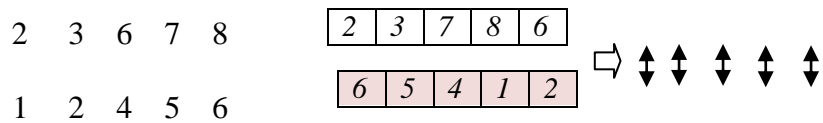
Step 1: select a cut-point



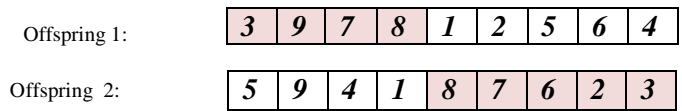
Step 2: exchange substrings between parents



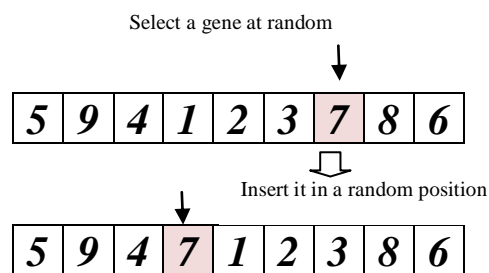
Step 3: mapping the weight of the right segment



Step 4: generate offspring with mapping relationship



**Fig .4.** Illustration of the WMX



**Fig .5.** Illustration of the insert mutation.

**Table 1.** Trace table of decoding procedure.

Iteration	v(k+j)	a	b	k	j	x <sub>kj</sub>
0	[3 9 7 8 6   5 4 1 2]	(270,100,65,135,230)	(260,240,170,130)		2 1	100
1	[3 0 7 8 6   5 4 1 2]	( 270, 0, 65,135,230)	(160,240,170,130)		4 2	135
2	[3 0 7 0 6   5 4 1 2]	( 270, 0,65,0,230)	(160,105,170,130)		3 4	65
3	[3 0 0 0 6   5 4 1 2]	(270,0,0,0,230)	(160,105,170,65)		5 3	170
4	[3 0 0 0 0   5 4 1 2]	(270,0,0, 0,60)	(160,105,0,65 )		1 1	160
5	[3 0 0 0 0   0 4 1 2]	(110,0,0, 0,60)	( 0,105,0,65)		1 2	105
6	[3 0 0 0 0   0 0 1 2]	(5,0,0,0,60)	(0,0,0,65)		1 4	5
7	[0 0 0 0 0   0 0 1 2]	(0,0,0,0,60)	(0,0,0,60)		5 4	60
8	[0 0 0 0 0   0 0 1 0]	(0,0,0,0,0)	(0,0,0,0)			

**Table 2.** Capacities of returning centers for each product.

	Returning center		Product	
	Δ	□	□	☀
1	20		50	35
2	25		40	20
3	20		50	25

**Table 3.** Capacities of product and part and fixed cost for disassembly centers.

Disassembly center	product		part					fixed cost				
	□	☀	A	B	C	D	E	A	B	C	D	E
1	35	70	35	70	70	35	70	85	60	75	50	70
2	30	60	30	60	60	30	60	90	110	110	100	60
3	20	75	20	75	40	20	75	70	55	80	65	100
4	20	50	20	50	40	20	50	100	110	140	110	120

**Table 4.** Capacities of part and fixed cost for processing centers.

Processing center	part			fixed cost		
	A	B	☀	A	B	☀
1	20	35	15	100	90	110
2	40	65	20	70	85	95
3	30	50	10	120	70	100
4	35	70	20	60	100	70

**Table 5.** Demand of manufacturing centers and recycling centers.

Plant	Part demand		recycling	Part demand	
	A~(μ <sub>A</sub> , σ <sub>A</sub> <sup>2</sup> )	B~(μ <sub>B</sub> , σ <sub>B</sub> <sup>2</sup> )		C~(μ <sub>C</sub> , σ <sub>C</sub> <sup>2</sup> )	☀~(μ <sub>☀</sub> , σ <sub>☀</sub> <sup>2</sup> )
1	N(40,16)	N(50,25)	1	N(20,4)	N(20,4)
2	N(30,9)	N(60,36)	2	N(10,1)	N(30,9)

**Table 6.** Shipping cost from returning center to disassembly center and processing center.

returning center	Disassembly center				Processing center			
	1	2	3	4	1	2	3	4
1	6	2	5	1	5	2	4	3
2	4	3	6	5	4	6	3	5
3	3	2	6	2	3	6	5	2

**Table 7.** Shipping cost from disassembly centers to processing centers and recycling centers.

Disassembly center	Processing center				Recycling center	
	1	2	3	4	1	2
1	3	5	2	4	4	2
2	6	2	5	1	5	6
3	4	3	6	5	3	5
4	2	4	3	2	2	3

**Table 8.** Shipping cost from processing center to manufacturing centers and recycling centers.

Processing center	Manufacturing center		Recycling center	
	1	2	1	2
1	4	3	5	3
2	3	5	4	5
3	1	6	3	6
4	2	4	6	4

**Table 9.** Transportation strategy from returning centers to disassembly centers for product □.

returning center	Disassembly center			
	1	2	3	4
1	-	30	5	-
2	-	-	15	-
3	25	-	-	-

**Table 10.** Transportation strategy from disassembly centers to processing centers for part A.

Disassembly center	Processing center			
	1	2	3	4
1	-	-	-	25
2	-	20	-	10
3	-	20	-	-
4	-	-	-	-

**Table 11.** Transportation strategy from processing centers to manufacturing centers for part A.

Processing center	Manufacturing center	
	1	2
1	-	-
2	8	32
3	-	-
4	35	-

**Table 12.** Transportation strategy from disassembly centers to recycling centers for part C.

Disassembly center	Recycling center	
	1	2
1	22	11
2	-	-
3	-	-
4	-	-

**Table 13.** Transportation strategy from returning centers to processing centers for product ☼.

returning center	Processing center			
	1	2	3	4
1	-	20	-	-
2	14	-	-	-
3	1	-	-	19

**Table 14.** Transportation strategy from Processing centers to recycling centers for product ☼.

Processing center	Recycling center	
	1	2
1	-	15
2	20	-
3	-	-
4	2	17

**Table 15.** Transportation strategy from returning centers to disassembly centers for product Δ.

returning center	Disassembly center			
	1	2	3	4
1	-	-	-	49
2	20	-	-	-
3	50	-	-	-

**Table 16.** Transportation strategy from disassembly centers to processing centers for part B.

Disassembly center	Processing center			
	1	2	3	4
1	20	-	50	-
2	-	-	-	-
3	-	-	-	-
4	15	-	-	34

**Table 17.** Transportation strategy from processing centers to manufacturing centers for part B.

Processing center	Manufacturing center	
	1	2
1	-	35
2	-	-
3	50	-
4	4	30