Forecasting Exchange Rates: a Comparative Analysis

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Abstract

This research aims to analyze and to compare the ability of different mathematical models, such as artificial neural networks (ANN) and ARCH and GARCH models, to forecast the daily exchange rates Euro/U.S. dollar (USD), identifying which, among all the models applied, produces more accurate forecasts. By empirically comparing the different mathematical models developed in this research, the traditional indicators for assessing the relevance of the models show that the ARCH and GARCH models, especially in their static formulations, are better than the ANN for analyzing and forecasting the dynamics of the exchange rates.

JEL Classification: C3, C5, G14, G20.

Keywords: Exchange Rate; Forecasting; Artificial Neural Networks; ARCH and GARCH models.

1. Introduction

The recent international economic crisis has highlighted the need for banks to implement effective systems for estimating the market risks. In particular, the international activity of the largest banks and the increasing volatility of exchange rates emphasize the importance of exchange rate risk, whose active management by the banks require the use of effective forecasting models.

The study of the topic of forecasting in financial markets is based on the research hypotheses that:

- (h_1) the process of pricing in financial markets is not random;
- (h_2) the degree of information efficiency at Fama of the financial markets is not strong or semi-strong.

If the two research hypotheses proposed were not considered valid, it would be highly redundant and useless to study the issue of forecasting in financial markets.

This research aims to analyze and to compare the ability of different mathematical models, such as artificial neural networks (ANN) and ARCH and GARCH models, to highlight non-random and therefore predictable behaviour in a highly liquid market and therefore characterized by high efficiency, such as the exchange rate Euro/US dollar. So a non-linear model of ANN and different ARCH and GARCH models were developed and empirically tested to forecast the daily exchange rates Euro/U.S. dollar (USD), identifying which, among all the models applied, produces more accurate forecasts. After developing and applying empirically the ANN and alternative formulations of the ARCH and GARCH models with different number of parameters (lags p and q), this research compares these formulations using the traditional indicators for assessing the relevance of the models, leading to interesting conclusions about which is the model characterized by better forecasting ability.

2. A Literature Review

The economic theory has not yet provided econometric models to produce efficient forecasts of exchange rates, although many studies have been devoted to the estimation of the equilibrium of exchange rates from the 20s to the recent years [Cassel (1923); Samuelson (1964); Mundell (1968); Dornbusch (1973 and 1979); Allen and Kenen (1980); Frankel and Mussa (1985); MacDonald (1999); Rogoff (1999); Alba e Papell (2007); Kim B.H., Kim H.K. and Oh (2009); Taylor (2009); Grossmann, Simpson e Brown (2009)]. In particular, Meese and Rogoff (1983) found that none of the forecasting models of the exchange rate established by economic theory has a better ability to forecast, over a period lower than 12 months, rather than the forward rate models or random walk, emphasizing the paradox that the variations of exchange rates are completely random.

In the wake of the study of Meese and Rogoff, some authors, including Hsieh (1989), Refenes, Azema-Barac, Chen, Karoussous (1993), Nabney, Dunis, Dallaway, Leong, Redshaw (1996), Brooks (1996 and 1997), Tenti (1996), Lawrence, Giles, Tsoi (1997), Gabbi (1999), Gencay (1999), Soofi, Cao (1999), Alvarez and Alvarez-Diaz (2003, 2005 and 2007) Alvarez-Diaz (2008), Reitz and Taylor (2008), Anastakis and Mort (2009), Majhi, Panda and Sahoo (2009), Bereau, Lopez and Villavicencio (2010), Bildirici, Alp and Ersen (2010), have studied the predictability of the dynamics of exchange rates of non-linear models such as artificial neural networks, genetic algorithms, expert systems or fuzzy models, leading however to conflicting results.

Mandelbrot (1963) and Fama (1965) have shown that the time series of exchange rates are generally characterized by conditional heteroskedasticity, leptocurtosis and volatility clustering. These features of the series of the exchange rates therefore imply the rejection of the hypothesis of normality, as these financial series show alternating periods characterized by large fluctuations around the average value with periods characterized by smaller variations. In this framework, numerous studies on econometric models were carried out, such as on ARCH and GARCH models, which are able to analyze and perceive the time variability of the phenomenon of volatility, and are therefore useful tools to capture the non-linearity of the changes in exchange rates (Krager and Kugler, 1993; Rossi, 1995; Brooks, 1996 and 1997; Bali and Guirguis, 2007; Wang, Chen, Jin and Zhou, 2010).

The pioneers of the ARCH (*Autoregressive Conditional Heteroschedasticity*) models were Engle (1982) and Bollerslev (1986), who generalized the model of Engle opening the way for a new generation of models able to capture the dynamics of time series, the GARCH (*Generalized Autoregressive Conditional Heteroschedasticity*) models. Over the years other contributions have extended the GARCH models in to two directions: univariate and multivariate models. The first category includes the E-GARCH model (*Exponential* GARCH) of Nelson (1991), the T-GARCH model (*Threshold* GARCH) of Glosten, Jagannathan and Runkle (1993), the Q-GARCH model (*Quadratic* GARCH) of Sentana (1995). The second category includes the VECH model of Bollerslev, Engle and Wooldridge (1988), the BEKK model formalized by Engle and Kroner (1995), the O-GARCH model (*Orthogonal* GARCH) of Alexander and Chibumba (1996) and the GO-GARCH (*Generalized Orthogonal* GARCH) of Van der Weide (2002).

3. The Methodology

The prediction of the financial time series, as the exchange rates, requires the prior identification of a specific portfolio of variables (input data for forecasting models) which are explanatory of the phenomenon to be foreseen and therefore significantly influence the pricing (output for forecasting models). The forecasting models, in fact, will learn the characteristics of the phenomenon to be foreseen by the variables of input selected and by the historical data that represent the phenomenon analyzed. The models predicting exchange rates, developed by the economic theory over the years, can be classified into two main categories:

- structural prediction models or linear ones, such as econometric models as Autoregressive Conditional Heteroschedasticity (ARCH), Generalized Autoregressive Conditional Heteroschedasticity (GARCH), State Space, which are based on the general view that every action of traders can be explained by a model of behaviour and thus by a definite, explicit function that can bind variables determinants of the phenomenon to be foreseen;
- *black box* forecasting models or non-linear ones, such as artificial neural networks, genetic algorithms, expert systems or fuzzy models, which, through the learning of the problem analyzed, attempt to identify and predict the non random and non-linear dynamics of prices, but without explicit ties and logical functions that bind the variables analyzed.

This paper aims to analyze and to compare the ability of different mathematical models belonging from the two categories, such as artificial neural networks (ANN) and ARCH and GARCH models, to forecast the exchange rate Eur/ Usd.

3.1. The Methodology for the Development of the Artificial Neural Network Model (ANN_m)

The objective of the ANN developed is to predict the trend of the exchange rate Euro / USD up to three days ahead of last data available. The variable of output of the ANN designed is then the daily exchange rate Euro/Dollar and the frequency of data collection of variables of input and the output is daily.

The construction of the data base used to train the artificial neural network (ANN) developed was divided into the following three phases:

- data collection;
- data analysis;
- variable selection.

The phase of data collection must achieve the following objectives:

- regularity in the frequency of the data collection by the markets;
- homogeneity between the information provided to the ANN and that available for the market operators.

In the phase of the data collection, both macro-economic variables (*fundamental data*) and market data were, therefore, initially considered as variables of input, from which it was assumed that the behaviour of the exchange rate euro-dollar was conditional. The data were collected from the 1st January, 1999 to December 31, 2009[1].
Once collected all the data, there was the stage of their analysis, which aims to select the data, that will be used to train ANN, among those initially collected. This phase is crucial, because the learning capacity of the ANN depends on the quality of information provided, which is the capacity of this information to provide a true representation of the phenomenon without producing ambiguous, distorting or amplifying effects in the phases of training networks.

In this phase, the observation of the correlation or similarity coefficients allow to evaluate the nature of relations between the variables of input considered, suggesting the elimination of the variables highly correlated with each other and therefore capable to product amplifying or distorting effects during the training phases (Pacelli, Bevilacqua, Azzollini, 2011).

Following the analysis of the correlation coefficients, there was the stage of selection of variables and the variables with the following characteristics were eliminated:

- variables characterized by a Pearson correlation coefficient with at least one other variable considered above the threshold level of acceptance equal to 0,80;
- monthly variables, because, having developed a neural network with a daily frequency of data collection of variables of input and output, they were considered potentially able to produce ambiguous or redundant signals during the training of ANN.

As a result of the selection of variables conducted according to the criteria outlined above, the following seven variables of input of the ANN were selected:

- Nasdaq Index;
- Daily Exchange Rate Eur/Usd New Zeland;
- Gold Spot Price Usa;
- Average returns of Government Bonds - 5 years in the Usa zone;
- Average returns of Government Bonds - 5 years in the Eurozone;
- Crude Oil Price – CLA (Crude oil);
- Exchange rate Euro / US dollar of the previous day compared to the day of the output.

In establishing the final data set with data of the seven input variables, exceptional values, as the *outliers*, were also removed related to special historical events such as the terrorist attacks of September 11, 2001.
For each of these variables of input, historical memory was calculated, which is the number of daily observations in which it is very high the possibility that the daily value of the variables is self-correlated with the values of n days[2].

The historical memory was calculated by a polynomial interpolation with coefficient R^2 equal to 0,98 for 90% of cases. The historical memories calculated for each variable are:

- Nasdaq index: eight surveys;
- Daily exchange rate Euro / NZ Dollar: five surveys;

[1] Source of data are Bloomberg and Borsa Italiana.
[2] The construction of the data set of the ANN is based on the concept of historical memory as the objective of the ANN is to predict the trend of the exchange rate Euro / Dollar.

- Spot price of gold expressed in dollars per ounce: six surveys;
- Average returns of government bonds - 5 years in the USA: eight surveys;
- Average returns of government bonds - 5 years in the Eurozone: seven surveys;
- The price of crude oil (CLA): eight surveys;
- Exchange rate Euro / USD: seven surveys more output[3];

In order to predict the trend of historical memories of individual variables by determining the angular coefficients (m), it was used by the software MatLab the function *Polyfit*, whereas for the first experiments a degree of the polynomial approximation of 1.

Since the ANN uses values between -1 and 1 where it is used the activation function *Tansig*[4], it was necessary to normalize data through the interpolation performed with MatLab, assigning values between -1 and 1 to vary of the value of the angular coefficient (m) produced by the *Polyfit*, according to the following summary:

IF	0<=m<=0.1	Then	value =0.2
IF	0.1<m<=1.1	Then	value =0.4
IF	1.1<m<=3.1	Then	value =0.6
IF	3.1<m<=7.1	Then	value =0.8
IF	m>7.1	Then	value =1
IF	-0.1<=m<0	Then	value =-0.2
IF	-1.1<=m<-0.1	Then	value =-0.4
IF	-3.1<=m<-0.1	Then	value =-0.6
IF	-7.1<=m<3.1	Then	value =-0.8
IF	m<-7.1	Then	value =-1

As shown by the previous scheme, the change of the angular coefficient determines the change in trend growth or reduction of the exchange rate Euro / USD.

The inputs of the network were reduced by 49 (i.e. 7 input with their historical memories) to 7, while the records are 547.

An innovative genetic algorithm multi-objective was used to solve the problem of finding the optimal topology of a Multi Layer Perceptron (MLP) neural network as a trade-off between the performance in terms of precision and the performance in terms of generalization, avoiding the problems of overfitting during the training phase (Pacelli, Bevilacqua, Azzollini, 2011). In this paper each MLP neural topology developed for this research was trained on data sets described in this paragraph by monitoring two parameters of precision and generalization. Generalization and accuracy were calculated as mean square error over all 120 training examples and all 40 examples of validation considered. In particular, for the purposes of this research, the optimal MLP neural network topology has been designed and tested by means the specific genetic algorithm multi-objective Pareto-Based designed from Bevilacqua *et al.* (2006).

3.2. The Methodology: the ARCH and GARCH Models

The ARCH (*Auto Regressive Conditionally Heteroskedasticity*) model, introduced by Engle in 1982, is one of the main methods used to analyze financial time series.

In a simplified version of the model proposed by Engle, the ARCH process is expressed by the following relation:

$$Y_t = \sum_{j=1}^{k} \phi_j x_{t,j} + e_t \quad \text{(a)}$$

[3] To train the ANN, it is considered as current moment *t-2* for each variable, as to obtain two readings back in order to predict a trend output rate Eur / U.S. dollar equal to three days.

[4] Hyperbolic tangent sigmoid activation function: Tansig (n) = 2 / (1 + exp (-2 * n)) -1, where *n* is the matrix of inputs. The results of a function Tansig can vary between -1 and 1.

(b) $$h_t = \alpha_0 + \alpha_1 e_{t-1}^2$$

(c) $$e_t \sim N(0, h_t)$$

The first expression (a) represents the equation that can interpret the different events of the phenomenon observed through the linear combination of *k* explanatory variables of the phenomenon. In other words, the phenomenon depends on the different events $x_{t,j}$ and on the variable e_t, which represents the prediction error of the past.

The second expression (b) means the equation of conditional variance (h_t) with not negative parameters α_0 and α_1 to estimate. The residual term e_t is expressed by a normal distribution, with mean of 0 and variance (h_t). Generalizing the expressions described, the ARCH model of order *p* can be represented as follows:

(d) $$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \alpha_3 e_{t-3}^2 + \cdots + \alpha_p e_{t-p}^2$$

(e) with $\alpha_0 > 0; \alpha_1, \alpha_2, \alpha_3 \ldots . \alpha_p \geq 0$

The equation (d) considers the variance as a function of the forecast errors made in the past *p* periods. This model is based, therefore, on a moving average of past forecast errors squared. In other words, if there were an unexpected shock of the financial variable considered, this would lead to a prediction error, which in turn would produce an immediate rise in volatility prediction on the future period, if its alpha coefficient is positive.

The condition (e), called the "condition of regularity", guarantees the positivity of the variance.
The generalization introduced by Bollerslev (1986) has precisely the aim of making the model more flexible and able to achieve the same degree of accuracy, using a smaller number of lags[5]. The analytical formulation is:

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \cdots + \alpha_p e_{t-p}^2 + \beta_1 h_{t-1} + \beta_2 h_{t-2} + \cdots + \beta_q h_{t-q}$$

Compared to ARCH, GARCH conditional variance is modeled by inserting, in addition to *p* lags related to forecast errors, *q* lags related to the past values of the same variance, hence the name GARCH (p, q) with q> 0, which identifies the order of the process, $\alpha_j > 0$ with j = 0,1,2, ..., p and $\beta_i > 0$ with i = 0,1,2, ..., q.

The GARCH model is based, therefore, on a strategy of forecasting quite intuitive: the variance expected at a certain time is a combination of long-run variance and the variance expected in the previous period corrected for the shock of the last period. The basis of this assumption is in fact the belief that volatility, as well as changes over time, is characterized by a predictable component, which inevitably produces forecast errors.

The GARCH models therefore recognize explicitly the existence of a phenomenon of serial correlation, expressed through an autoregressive model, and give adequate importance to the new information incorporated in market shocks.

4. The Empirical Results by the ANN

To optimize the performance of the ANN, it was reduced the data set to avoid signals of distortion or enhancement of some information, using 160 examples of maximum variance, of which 75% (120 examples) for training set and 25% (40 samples) for the validation set.

In table 1, there are summarized the characteristics and performance of the three best ANN models designed for the purpose of this research which have provided, at the same performance of the training set of 100%, the best results for validation sets, respectively of 70%, 60% and 80%.

[5] In this work, the lags refer to the number of days in which it was analyzed the variable.

First ANN Topology with Technology Building designed trial and error				
N° inputs	First Layer	Second Layer	N° output	Performance
7	11	8	1	120/120 28/40
Activation Function	Tansig	Tansig	Tansig	

Second ANN Topology with Technology Building designed trial and error				
N° inputs	First Layer	Second Layer	N° outputs	Performance
7	12	9	1	120/120 24/40
Activation Function	Tansig	Tansig	Tansig	

Third and optimal ANN Topology designed with optimized construction technique				
N° inputs	First Layer	Second Layer	N° outputs	Performance
7	12	9	1	120/120 32/40
Activation Function	Tansig	Tansig	Tansig	

Table 1: Characteristics and performance of the three best topologies of ANN models

The first two ANN models are designed with the construction technique trial and error while the third ANN model with the optimized construction technique mentioned in paragraph 3.1.

The third topology of neural network designed with an optimized construction technique gives the best performance since it classifies correctly 120 examples of 120 in the training phase (performance of 100%) and 32 examples of 40 during validation (performance of 80%) using as classification decreasing range [-0,2; -0,04] and as a growing range of classification [0,04; 0,2]. The bandwidth of the network indecision is then amplitude namely 0,08 [-0,04; + 0,04].

Table 2 shows some indicators of statistical error that can provide useful information on the quite good predictive power of the third neural network topology designed with the optimized construction technique mentioned in paragraph 3.1.

Coefficient	**Result**
Coefficient of determination R^2	0,946
MAE (Mean Absolute Error)	0,0835
MSE (Mean Square Error)	0,0316
MSEP (Mean Square Percentage Error)	0,7911
RMSE (Root Mean Square Error)	0,1779
RMSEP (Root Mean Square Percentage Error)	0,8895

Table 2: Performance's statistical indicators of the best ANN topology

5. *The Empirical Results by the ARCH and GARCH Models*

An objective of this research is to apply empirically the ARCH and GARCH models described in the paragraph 3.2 for predicting the daily exchange rate Euro/US dollar up to twenty-three days forward from the last survey available, identifying, among all models used, which generates the most accurate predictions and comparing the performances of the ARCH and GARCH models with the ANN's ones.

After developing and applying empirically the alternative formulations of the ARCH and GARCH models with different number of parameters (lags p and q), this research compares these formulations using the traditional indicators for assessing the relevance of the models listed below:

- Akaike info criterion (AIC);
- Schwartz criterion (SIC);
- R-squared;

- Adjusted R-squared;
- Standard deviation.

The empirical analysis was conducted on a series of daily data of the exchange rate Euro/U.S. dollar related to the period from December 31, 2008 until December 31, 2009.

Firstly, the existence of the degree of autocorrelation and the partial autocorrelation between the data considered and the results of the *Ljung-Box Q test* performed on the squared residuals were verified. Because of the *p-value*, all zero, the hypothesis of zero correlation between the data series was rejected, also demonstrated by the autocorrelation values that are different from zero.

Analyzing the data in Table 3, we can see that the values of the autocorrelations decrease but never nullify. Excluding the moving average model, a first estimate of the AR (1)[6] was carried out, as it is clear from the partial autocorrelations that only the first term is relevant (Table 4).

	AC	PAC	Q-stat	P-value
1	0,981	0,981	276,46	0,000
2	0,963	-0,016	543,41	0,000
3	0,944	-0,019	800,76	0,000
4	0,926	0,036	1049,60	0,000
5	0,908	-0,027	1289,60	0,000
6	0,891	0,028	1521,70	0,000
7	0,872	-0,074	1744,80	0,000
8	0,854	0,006	1959,20	0,000
9	0,837	0,050	2166,20	0,000
10	0,821	-0,003	2366,10	0,000
11	0,808	0,008	2560,40	0,000
12	0,797	0,042	2750,10	0,000
13	0,789	0,078	2936,70	0,000
14	0,779	-0,054	3119,10	0,000
15	0,769	-0,013	3297,40	0,000
16	0,758	-0,002	3471,60	0,000
17	0,748	-0,010	3971,60	0,000
18	0,739	0,037	3808,30	0,000
19	0,730	-0,010	3641,60	0,000
20	0,720	-0,021	4131,10	0,000
21	0,711	0,041	4287,20	0,000
22	0,704	0,051	4440,70	0,000
23	0,696	-0,003	4591,50	0,000
24	0,692	0,085	4741,10	0,000
25	0,689	0,003	4889,70	0,000
26	0,683	-0,078	5036,40	0,000
27	0,677	0,005	5181,10	0,000
28	0,671	0,001	5323,90	0,000
29	0,666	0,022	5465,10	0,000
30	0,657	-0,104	5603,20	0,000
31	0,648	-0,008	5738,20	0,000
32	0,639	0,010	5869,90	0,000
33	0,629	-0,015	5998,00	0,000
34	0,621	0,049	6123,20	0,000
35	0,612	-0,003	6245,50	0,000
36	0,603	-0,080	6364,60	0,000

Table 3: Autocorrelation, partial autocorrelation and Ljung-Box Q

[6] The name of the AR (Auto Regressive) comes from the fact that it looks very much like a regression model where the explanatory variables are past values of the dependent variable. An AR (m), with $m \geq 1$, is represented by the equation:

$$x_t = b_0 + b_1 x_{t-1} + \cdots + b_m x_{t-m} + u_t = b_0 + \sum_{l=1}^{m} b_l x_{t-l} + u_t$$

where $b_l, \forall\, l = 0, 1, \dots, m$, are real constant and $\{u_t\}_{t \in T} \sim \mathrm{w.n.}(0, \sigma^2)$; u_t can be interpreted as the error of a regression model, as the difference between x_t and its conditional mean, where the set of conditioning is composed of past values x_t .

Variable	Coefficient	Standard Error	T Statistics	P-value
C	1,4528	0,0713	20,3845	0,0000
AR(1)	0,9864	0,0099	99,5965	0,0000
F Statistics			9919,4560	
P-value			0,0000	
log-likelihood			795,6307	
Akaike info criterion (AIC)			-6,1049	
Schwarz criterion (SIC)			-6,0775	

Table 4: AR (1) model estimate

A preliminary analysis of the data of Table 4 shows that the AR (1) model is already representative of the series, but an objective of this work is to verify the presence of ARCH and GARCH patterns in the series of exchange rate analyzed.

Firstly, the presence of heteroskedasticity in the series must be tested, and for this purpose, tests were performed on the residuals of the models AR (q).

It should be noted that, fixed a level of relevance equal to 5%, the null hypothesis of homoskedasticity in the series has been verified. The parameters, calculated using the method of *Ordinary Lost Squared* (OLS) for all the models used, are all non-zero, confirming the presence of heteroskedasticity, which does not depend on the number of parameters considered in the model, as evidenced by the results reported below (Table 5).

	AR (1)	AR (2)	AR (3)	AR (4)	AR (5)
Akaike info criterion	-13,95251	-13,981970	-13,972410	-13,979900	-13,983790
Schwarz criterion	-13,92504	-13,940650	-13,917170	-13,910660	-13,900460
R-squared	0,029115	0,067171	0,068264	0,085842	0,091565
Adjusted R-squared	0,025338	0,059854	0,057215	0,071274	0,073323
Standard deviation	0,000228	0,000228	0,000229	0,000229	0,000228

Table 5: Indicators of the dynamics interpretation

We observe a substantial disregard of the results obtained compared to the different degree of complexity of the models used. In other words, the value of the indexes that express the model's ability of interpretation does not vary significantly with the complexity of the model in terms of number of coefficients. It is therefore evident that the indicators do not constitute signage elements to improve the process of interpreting the dynamics of the exchange rate.

As mentioned, the identification of the model on the conditional variance was conducted testing alternative formulations of the ARCH and GARCH models with different number of parameters, using an estimator of *Maximum Likelihood method*, and comparing these models with the main indicators used (Table 6).

	R-squared	Adj. R-squared	Sum squared resid	AIC	SIC	Log-likelihood
ARCH (1)	0,974463	0,974364	0,033706	-6,106958	-6,052178	797,9045
ARCH (2)	0,974641	0,974543	0,033470	-6,118199	-6,049725	800,3659
ARCH (3)	0,974624	0,974525	0,033494	-6,118020	-6,035850	801,3426
ARCH (4)	0,974621	0,954523	0,033497	-6,123144	-6,027280	803,0088
ARCH (5)	0,974584	0,974485	0,033546	-6,134698	-6,025139	805,5107
GARCH(1,1)	0,974607	0,974508	0,033516	-6,196940	-6,128465	810,6022
GARCH(1,2)	0,974607	0,974509	0,033515	-6,189273	-6,107103	810,6054

Table 6: Principal indicators for the conditional variance model

The analysis of the data shows that the increase in the number of parameters considered in different formulations does not significantly improve the model.

By comparing the alternative ARCH and GARCH models used, there is a slight preference for ARCH (q) models with respect to GARCH (p, q) ones.

The models described above can then be used for interesting operating applications in finance, as forecasting financial series of data or trading.

Assuming that at the basis of all forecasts there is the volatility, as the uncertainty observed in the markets, and that it is identified with the conditional variance on previous information and available at a given instant of time, in this research ARCH and GARCH models, constructed and described previously, are used to predict the exchange rate Euro/US Dollar.

Using a sample consisting of N observations and fixed H <N, the aim is to make predictions for the times H +1, H +2, ..., N.

We represent two prediction models:

- the *static* forecast (Table 7): the prediction uses only the number of observed data, increased by a factor at every step: in this case $f_{H+1}$with the series $x_1, x_2, \ldots, x_H$, f_{H+2} with $x_1, x_2, \ldots, x_H, x_{H+1}$,.. and f_N with $x_1, x_2, \ldots, x_H, x_{H+1}, \ldots, x_{N-1}$ are calculated;
- the *dynamic* forecast (Table 8): expected values are calculated using the series of observed data at the period before the prediction: after the first period, the observed data are replaced by the corresponding amounts previously provided for, as f_{H+1} is calculated using the series $x_1, x_2, \ldots, x_H$, the value of f_{H+2} is calculated with the series $x_1, x_2, \ldots, x_H, f_{H+1}$, and so on until it is estimated f_N with the series $x_1, x_2, \ldots, x_H, f_{H+1}, \ldots, f_{N-1}$.

All predictions were made with twenty-three days after the last survey. In order to test the forecasting ability of the models used, in addition to the theoretical values obtained by the empirical application of the models, we have calculated the actual historical values, that represent the terms of comparison in the process of performance evaluation of the forecasting models used.

So the values expected by the models, or the results obtained in-sample, are compared with the corresponding out-sample values tested, or the actual historical values. This comparison has highlighted that the average level of the absolute error (MAE) obtained on the sample is significantly lower.

	ARCH (1)	**ARCH (2)**	**ARCH (3)**	**ARCH (4)**	**ARCH (5)**	**GARCH(1,1)**	**GARCH(1,2)**
RMSE	0,008356	0,007916	0,008002	0,007989	0,008120	0,008131	0,008130
MAE	0,006037	0,005761	0,005813	0,005802	0,005880	0,005900	0,005900
MAPE	0,414198	0,395050	0,398663	0,397914	0,403329	0,404745	0,404724
TIC	0,002864	0,002713	0,002743	0,002738	0,002783	0,002787	0,002786
Bias	0,218378	0,145327	0,160516	0,157602	0,180503	0,185400	0,185279
Variance	0,011337	0,005672	0,006762	0,006843	0,008309	0,007137	0,007094
Covariance	0,770285	0,849001	0,832721	0,835555	0,811188	0,807463	0,807627

Table 7: The static forecast

	ARCH (1)	**ARCH (2)**	**ARCH (3)**	**ARCH (4)**	**ARCH (5)**	**GARCH(1,1)**	**GARCH(1,2)**
RMSE	0,063340	0,045915	0,049341	0,048896	0,054022	0,053601	0,053537
MAE	0,055724	0,040815	0,043773	0,043447	0,047792	0,047433	0,047378
MAPE	3,862409	2,827702	3,032976	3,010330	3,311878	3,286989	3,283163
TIC	0,021335	0,015547	0,016690	0,016564	0,018247	0,018108	0,018086
Bias	0,748332	0,728558	0,734804	0,734204	0,741081	0,740705	0,740631
Variance	0,098383	0,266803	0,263958	0,264209	0,205441	0,212329	0,213381
Covariance	0,153285	0,004639	0,001238	0,001587	0,534780	0,046966	0,045988

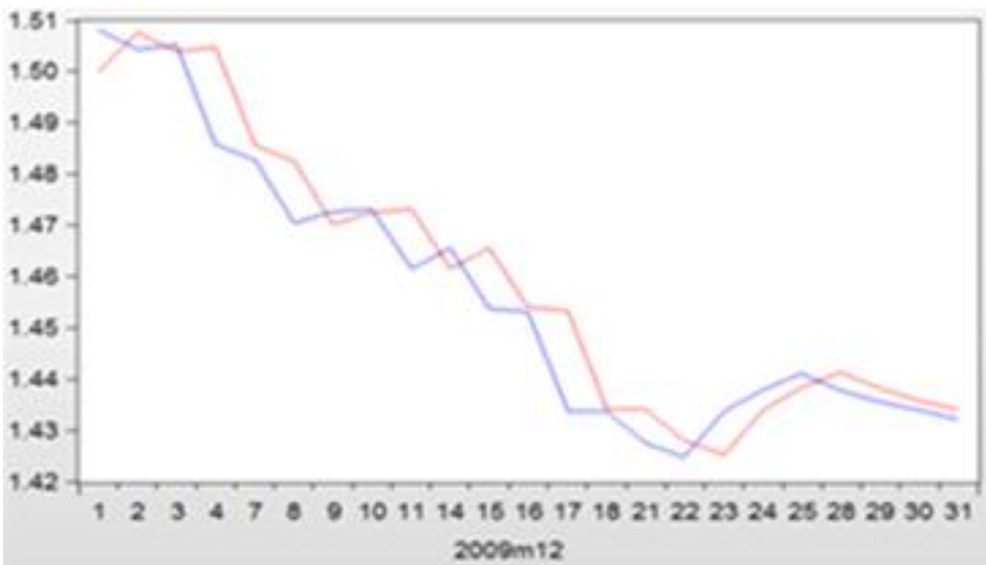
Table 8: The dynamic forecast

From the Tables 7 and 8, we can see that the indicators of static predictions are significantly more tending to zero than those of the dynamic ones, confirming the superiority of the static prediction than the dynamic one. In both static and dynamic methods, however, the TIC values are very low, thus demonstrating the predictive power of both methods. The model which showed the better predictive ability is the ARCH (2) model with a static approach. The greater relevance of the prediction model obtained through the ARCH (2) is demonstrated by the values of RMSE, MAE and MAPE, which are the lowest among all the models tested.

6. Concluding Remarks

By the analysis of the empirical results, it is possible to say, first of all, that the empirical research conducted largely support the two research hypothesis discussed in section 1, justifying the attempt to forecast the exchange rate Euro / USD performed in this research. The good forecasting performances of the different models developed show that the process of formation of the exchange rates is not completely governed by noise.

Furthermore by comparing the ability of the different mathematical models developed in this research, such as an artificial neural network (ANN) and different ARCH and GARCH models, the traditional indicators for assessing the relevance of the models show that the ARCH and GARCH models, especially in their static formulations, are better than the ANN for analyzing and forecasting the dynamics of the exchange rates (see Table 2, 7 and 8). In particular, the model which showed the best predictive ability is the ARCH (2) model with a static approach. In Graph 1, with reference to this ARCH (2) model, the comparison between the observed series and the expected ones of daily exchange rate Euro / US dollar is represented. The analysis of Graph 1 shows the very good ability of the model to predict the daily changes in the exchange rate up to 23 days forward from the last daily survey available.



Graph 1: Comparison of observed series (blue line) and expected ones (red line) by the model Arch (2)

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