MODELING STOCK MARKET VOLATILITY USING GARCH MODELS EVIDENCE FROM SUDAN

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Abstract

This paper uses the Generalized Autoregressive Conditional Heteroscedastic models to estimate volatility (conditional variance) in the daily returns of the principal stock exchange of Sudan namely, Khartoum Stock Exchange (KSE) over the period from January 2006 to November 2010. The models include both symmetric and asymmetric models that capture the most common stylized facts about index returns such as volatility clustering and leverage effect. The empirical results show that the conditional variance process is highly persistent (explosive process), and provide evidence on the existence of risk premium for the KSE index return series which support the positive correlation hypothesis between volatility and the expected stock returns. Our findings also show that the asymmetric models provide better fit than the symmetric models, which confirms the presence of leverage effect. These results, in general, explain that high volatility of index return series is present in Sudanese stock market over the sample period.

KEY WORDS: Khartoum index; volatility; GARCH models; leverage Effect.

1. INTRODUCTION

Over the last few years, modelling and forecasting volatility of a financial time series has become a fertile area for research, this is simply because volatility is considered as an important concept for many economic and financial applications, like portfolio optimization, risk management and asset pricing. In simple words, volatility means "the conditional variance of the underlying asset return" [1]. A special feature of this volatility is that it is not directly observable, so that financial analysts are especially keen to obtain a precise estimate of this conditional variance process, and consequently, a number of models have been developed that are especially suited to estimate the conditional volatility of financial instruments, of which the most well-known and frequently applied model for this volatility are the conditional heteroscedastic models. The main objective of building these models is to make a good forecast of future volatility which will therefore, be helpful in obtaining a more efficient portfolio allocation, having a better risk management and more accurate derivative prices of a certain financial instrument.

Among these models, the Autoregressive Conditional Heteroskedasticity (ARCH) model proposed by Engle 1982 [2] and its extension; Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model by Bollerslev 1986 [3], and Taylor 1986 [4] were found to be the first models introduced into the literature and have become very popular in that they enable the analysts to estimate the variance of a series at a particular point in time Enders 2004 [5].[1] Since then, there have been a great number of empirical applications of modelling the conditional variance of a financial time series (See for example, Nelson 1991 [6], Bollerslev et al. 1992 [7], Engle and Patton 2001 [8], Shin 2005 [9], Alberg et al. 2008 [10], Shamiri and Isa 2009 [11] and Kalu 2010 [12]. These types of models were designed to explicitly model and forecast the time-varying conditional second order moment (variance) of a series by using past unpredictable changes in the returns of that series, and have been applied successfully in economics and finance, but more predominantly in financial market research.

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A lot of empirical studies have been done on modelling and forecasting stock market volatility by applying of ARCH – GARCH specifications and their large extensions,[2] most of these studies focus on developed markets, and to the best of our knowledge, there are no such empirical studies for the Sudanese stock market, so the current paper attempts to fill this gap. The main objective of this paper is to model stock return volatility for Khartoum Stock Exchange (KSE), by applying different univariate specifications of GARCH type models for daily observations of the KSE index series for the period 2nd January 2006 to 30th November 2010. As well as describing special features of the market in terms of trading activity and index components and calculations. The volatility models applied in this paper include the GARCH (1,1), GARCH-M (1,1), EGARCH(1,1), TGARCH(1,1), and Power GARCH(1,1). The remainder of this paper is organized as follows: Following this introduction, Section 2 provides a general overview of Khartoum Stock Exchange. Section 3 describes the data and provides the summary statistics. In the fourth section the GARCH methodology is presented, while the results of the estimation are discussed in section 5, and finally, section 6 concludes the paper.

2. OVERVIEW OF KHARTOUM STOCK EXCHANGE

The Khartoum Stock Exchange is the principal stock exchange of Sudan. It is located in Khartoum and its name is abbreviated to KSE. The principal stock index of the KSE is the Khartoum Index. KSE officially started operating in January 1995 with the assistance of the Common Market for Eastern and Southern Africa (CoMESA)[3], with the objective of regulating and controlling the issuance of securities, and mobilizing private savings for investment in securities. Securities traded in the KSE are ordinary shares and investment units. In addition to that a good number of mutual funds and Government Investment Certificates are also traded, (KSE Annual report, 2009). Orders are handled through brokers during trading hours and share prices are quoted in Sudanese Pound (SDG).[4] The trading is processed manually by continuous auction from Sunday to Thursday for one hour from 10.00 am to 11.00 am, with buy and sell orders being relayed to floor-based representatives of registered brokers for execution, and trading in securities is taking place in two markets, the primary and the secondary markets.[5]

As with most of the Sudan financial system, the KSE operates on the basis of Islamic *Shariaa* and is supervised and regulated by the Central Bank of Sudan. The key feature of Islamic *Shariaa* practices in Khartoum Stock Exchange is that it aimed at the formation of investment portfolios from common stocks of listed companies. These ideally satisfy three basic criteria: legitimate field of economic activity, interest-free dealings in both assets and liabilities, and the dominance of real assets. Thus a company must not be engaged in the production of illegitimate goods like alcoholic drinks; it must not deal with interest rate financing as a means to leverage its capital structure through fixed debt liabilities, or generate interest income from investment securities. Since a company's shares represent equity rights in its assets, the latter should be real assets, not liquid money or receivable debt as they cannot be sold freely at a profit, like real goods, real estate and machinery [13]

One of the most popular financial instruments introduced by Islamic Shariaa practices in the KSE activities is the existence of Government *Musharakah* Certificates (GMCs), which represents an Islamic equivalent of the conventional bonds (also known as *Shahama* bonds).[6] Through *Shahama* bonds the state borrows money in the domestic market instead of printing more banknotes. After one year, holders of GMCs can either cash or extend them.[7] These bonds are backed by the stocks and shares portfolio of various companies owned by the Ministry of Finance and therefore are asset-backed. The profitability of GMCs can reach 33 per cent per annum and depends on the financial results of the companies involved. Hence, the profit of a GMC is variable rather than fixed. The government issues these bonds on a quarterly basis and their placement is done very quickly- in just six days.
KSE is relatively small market as compared to the stock markets of the developed countries or even to some countries in the Arab region; the number of listed companies is few and most stocks are infrequently traded, market capitalization and traded value are very low (See Table 1).

Banks, Communications and Certificates sectors dominate the trading activity of the market in terms of volume of trading and number of shares (see Table 2). KSE currently lists 53 companies with a total market capitalization of SDG 6961.9 million (KSE Annual report, 2009).[8] The amount of capitalisation is very small (See Figure 1), but increased from SDG1.929 million in 2003 to SDG10.121 million in 2007, then declined to SDG8.541 million and SDG 6961.9 million in 2008 and 2009 respectively (see Table 3). The overall performance of the Khartoum stock market is measured by the KSE index, which is a market capitalization-weighted index. In September 2003, the KSE index was established and listed in the Arab Monetary Fund database. At the end of the first month the index closed at 961.74 points. At December 2005, the index closed at the highest level of 3259.17 points. In November 2010 it is 2365.66.

3. THE DATA AND BASIC STATISTICS:

3.1. Data for Analysis

The time series data used for modelling volatility in this paper is the daily closing prices of Khartoum Stock Exchange (KSE) index over the period from 2nd January 2006 to 30th November 2010, resulting in total observations of 1326 excluding public holidays. These closing prices have been taken from KSE website. In this study, daily returns (r_t) were calculated as the continuously compounded returns which are the first difference in logarithm of closing prices of KSE-Index of successive days:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where P_t and P_{t-1} are the closing market index of KSE at the current day and previous day, respectively.

Since October 18, 2009, the index on the Khartoum Stock Market has been declining. In only 16 trading days, the stock market index fell from 3077.12 October 18, 2009 to 2363.30 on November 10, 2009. Since that time, the KSE index was reporting to fluctuate around an average value of 2363.23. In order to see the impact of this sharp fall on the volatility modeling, the full data set is divided into two sub-periods: the first sub-period covers Jan. 2, 2006 to Oct. 18, 2009 with 1042 total observations, while the second sub-period ranges from Nov. 10, 2009 to Nov. 30 2010 resulting in 269 observations. So, the results will be presented separately in a three periods; for the period before the sharp fall, the period after that fall and period of the whole data set.

3.2. Basic statistics of KSE returns series

3.2.1. Summary statistics

To specify the distributional properties of the daily KSE return series (r_t) during the period of this study, various descriptive statistics were calculated and reported in Table 4. As it can be seen from Table 4, the average return for the first sub-period is higher than the average of the second sub-period and the full sample period. Skewness and excess kurtosis are clearly observed for the daily returns of KSE index for the three periods, which represent the nature of departure from normality.[9] Likewise, the Jarque-Bera (J-B) statistic, which is a test for normality, also confirms that the null hypothesis of normality for the daily KSE returns should be rejected at the 1% significant level. In summary, the KSE return series do not conform to normal distribution but display positive skewness (the distribution has a long right tail) for the three periods, in addition to that, a highly leptokurtic distribution is observed for all periods.The daily prices and returns for KSE index for the period under review are presented in Figure 2 and Figure 3

3.2.2 A Quantile-Quantile (Q-Q) Plot

The Q-Q graphical examination is also employed to check whether the KSE index return series is normally distributed. Q-Q plot is a scatter plot of the empirical quantiles (vertical axis) against the theoretical quantiles (horizontal axis) of a given distribution [14]. If the sample observations follow approximately a normal distribution with mean equal to the empirical mean μ and standard deviation equal to the empirical standard deviation σ, then the resulting plot should be roughly scattered around the 45-degree line with a positive slope, the greater the departure from this line, the greater the evidence for the conclusion that the series is not normally distributed. The results of this graphical examination are provided in Figure 4. The Q-Q plot against the normal distribution in Figure 3 shows that the KSE return data is not dispersed along the line for all specified periods, which means that the return series is not normally distributed confirming the results in Table 4.

3.2.3 Testing for Stationarity

To investigate whether the daily price index and its returns are stationary series, the Augmented Dickey –Fuller (ADF) test, Dickey and Fuller 1981 [15] has been applied for both series. The results of the tests are reported in Table 5. The ADF test for the KSE price index in level form indicates that it is stationary series for two periods (the first and the second sub-periods), and it was found to be a non-stationary times series for the full sample period.[10] But when applying the same test for return series, the results strongly reject the null hypothesis of a unit root for all periods. Therefore, we conclude that the return series is stationary at level in all three periods.

3.2.4 Testing for Heteroscedasticity

One of the most important issues before applying the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) methodology is to first examine the residuals for evidence of heteroscedasticity. To test for the presence of heteroscedasticity in residuals of KSE index return series, the Lagrange Multiplier (LM) test for ARCH effects proposed by Engle (1982) is applied. In summary, the test procedure is performed by first obtaining the residuals e_t from the ordinary least squares regression of the conditional mean equation which might be an autoregressive (AR) process, moving average (MA) process or a combination of AR and MA processes; (ARMA) process. For example, in ARMA (1,1) process the conditional mean equation will be as:

$$r_t = \phi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \tag{2}$$

after obtaining the residuals e_t, the next step is regressing the squared residuals on a constant and q lags as in the following equation:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \ldots + \alpha_q e_{t-q}^2 + \nu_t \tag{3}$$

The null hypothesis that there is no ARCH effect up to order q can be formulated as:

$$H_0 : \alpha_1 = \alpha_2 .. = \alpha_q = 0 \tag{4}$$

against the alternative:

$$H_1 : \alpha_i > 0 \tag{5}$$

for at least one i = 1, 2, …, q

The test statistic for the joint significance of the q-lagged squared residuals is the number of observations times the R-squared (TR^2) from the regression. TR^2 is evaluated against $\chi^2(q)$ distribution. This is asymptotically locally most powerful test [16].

In our case, we first employ an autoregressive moving average ARMA (1,1) model for the conditional mean in the return series as an initial regression, then, test the null hypothesis that there are no ARCH effects in the residual series from lag 1 up to lag 5. The results of this examination are summarized in Table 6. The ARCH-LM test results in Table 6 provide strong evidence for rejecting the null hypothesis for all lags included. Rejecting H_0 is an indication of the existence of ARCH effects in the residuals series and therefore the variance of the return series of KSE index is non-constant for all periods specified.

4. METHODOLOGY

Autoregressive conditional heteroscedasticity (ARCH) and its generalization (GARCH) models represent the main methodologies that have been applied in modelling and forecasting stock market volatility.[11] In this paper different univariate GARCH specifications are employed to model daily stock return volatility in Khartoum Stock Exchange and these models are GARCH (1,1), GARCH-M (1,1), EGARCH(1,1), TGARCH(1,1) and PGARCH (1,1). In presenting these different models, there are two distinct equations or specifications, the first for the conditional mean and the second for the conditional variance, this section briefly reviews this methodology.

4.1. Volatility Definition and Measurement

It is useful, before starting the description of volatility models to give a brief explanation of the term volatility, at least for the purpose of clarifying the scope of this paper. Volatility refers to the spread of all likely outcomes of an uncertain variable. Typically, in financial markets, we are often concerned with the spread of asset returns. Statistically, volatility is often measured as the sample standard deviation:

$$\hat{\sigma} = \sqrt{\tfrac{1}{T-1}\sum_{t=1}^{T}(r_t - \mu)^2} \tag{6}$$

where r_t is the return on day t and μ is the average return over the T-day period. Sometimes, variance, σ^2, is used also as a volatility measure. Volatility is related to, but not exactly the same as, risk.

Risk is associated with undesirable outcome, whereas volatility as a measure strictly for uncertainty could be due to a positive outcome [17]. In this paper, we use the variance as a measured of volatility.

4.2. Volatility Modeling Techniques

The existing models of volatility can be divided into two main categories, symmetric and asymmetric models. In the symmetric models, the conditional variance only depends on the magnitude, and not the sign, of the underlying asset, while in the asymmetric models the shocks of the same magnitude, positive or negative, have different effect on future volatility.

4.2.1. Symmetric GARCH Models

4.2.1.1. The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

In this model, the conditional variance is represented as a linear function of its own lags. The simplest model specification is the GARCH (1,1) model

Mean equation $$r_t = \mu + \varepsilon_t \qquad (7)$$

Variance equation $$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (8)$$

where $\omega > 0$ and $\alpha_1 \geq 0$ and $\beta_1 \geq 0$, and.

r_t = return of the asset at time t.

μ = average return.

ε_t = residual returns, defined as:

$$\varepsilon_t = \sigma_t z_t \qquad (9)$$

where z_t is standardized residual returns (i.e. *iid* random variable with zero mean and variance 1), and σ_t^2 is conditional variance.For GARCH(1,1), the constraints $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ are needed to ensure σ_t^2 is strictly positive [17].

In this model, the mean equation is written as a function of constant with an error term. Since σ_t^2 is the one – period ahead forecast variance based on past information, it is called the conditional variance. The conditional variance equation specified as a function of three terms:

- A constant term : ω
- News about volatility from the previous period, measured as the lag of the squared residula from the mean equation: ε_{t-1}^2 (the ARCH term)
- Last period forecast varince: σ_{t-1}^2 (the GARCH term).

The conditional variance equation models the time varying nature of volatility of the residuals generated from the mean equation. This specification is often interpreted in a financial context, where an agent or trader predicts this period's variance by forming a weighted average of a long term average (the constant), the forecast variance from last period (the GARCH term), and information about volatility observed in the previous period (the ARCH term). If the asset return was unexpectedly large in either the upward or the downward direction, then the trader will increase the estimate of the variance for the next period.
The general specification of GARCH is, GARCH (p, q) is as:

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-1}^2 \qquad (10)$$

where, p is the number of lagged σ^2 terms and q is the number of lagged ε^2 terms.

4.2.1.2 The GARCH-in-Mean (GARCH-M) Model

In finance, the return of a security may depend on its volatility. To model such a phenomenon one may consider the GARCH-M Model of Engle, Lilien, and Robins 1987 [18], where "M" stands for GARCH in the mean [1].

This model is an extension of the basic GARCH framework which allows the conditional mean of a sequence to depend on its conditional variance or standard deviation. A simple GARCH-M (1,1) model can be written as :

Mean equation $$r_t = \mu + \lambda\sigma_t^2 + \varepsilon_t \quad (11)$$

Variance equation $$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (12)$$

where μ and λ are constants. The parameter λ is called the risk premium parameter. A positive λ indicates that the return is positively related to its volatility. In other words, a rise in mean return is caused by an increase in conditional variance as a proxy of increased risk.

Engle, Lilien, and Robins assume that the risk premium is an increasing function of the conditional variance of ε_t; in other words, the greater the conditional variance of returns, the greater the compensation necessary to induce the agent to hold the long – term asset [5]. Other specifications of risk premium have also been used in the literature, including:

$$r_t = \mu + \lambda\sigma_t + \varepsilon_t \quad (13)$$

and

$$r_t = \mu + \lambda \ln\sigma_t^2 + \varepsilon_t \quad (14)$$

4.2.2. Asymmetric GARCH Models

An interesting feature of asset price is that (bad) news seems to have a more pronounced effect on volatility than does (good) news. For many stocks, there is strong negative correlation between the current return and the future volatility. The tendency for volatility to decline when returns rise and to rise when returns fall is often called the leverage effect [5]. The main drawback of symmetric GARCH models is that the conditional variance is unable to respond asymmetrically to rises and falls in ε_t and such effects are believed to be important in the behaviour of stock returns. In the linear GARCH (p,q) model the conditional variance is a function of past conditional variances and squared innovations; therefore, sign of returns cannot affect the volatilities [19].The symmetric GARCH models described above cannot account for the leverage effects observed in stock returns, consequently, a number of models have been introduced to deal with this phenomena. These models are called asymmetric models. This paper uses EGAECH, TGARCH and PGARCH for capturing the asymmetric phenomena.

4.2.2.1 The Exponential GARCH (EGARCH) Model

This model captures asymmetric responses of the time-varying variance to shocks and, at the same time, ensures that the variance is always positive. It was developed by Nelson (1991) with the following specification

$$Ln(\sigma_t^2) = \omega + \beta_1 Ln(\sigma_{t-1}^2) + \alpha_1\left\{\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - \sqrt{\frac{2}{\pi}}\right\} - \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (15)$$

where γ is the asymmetric response parameter or leverage parameter. The sign of γ is expected to be positive in most empirical cases so that a negative shock increases future volatility or uncertainty while a positive shock eases the effect on future uncertainty[12]. In macroeconomic analysis, financial markets and corporate finance, a negative shock usually implies bad news, leading to a more uncertain future. Consequently, for example, shareholders would require a higher expected return to compensate for bearing increased risk in their investment [20]. Equation (15) is an EGARCH (1,1) model. Higher order EGARCH models can be specified in a similar way; EGARCH (p, q) is as follows:

$$Ln(\sigma_t^2) = \omega + \sum_{j=1}^{p}\beta_j Ln(\sigma_{t-j}^2) + \sum_{i=1}^{q}\alpha_i\left\{\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| - \sqrt{\frac{2}{\pi}}\right\} - \gamma_i\frac{\varepsilon_{t-i}}{\sigma_{t-i}} \quad (16)$$

4.2.2.2. The Threshold GARCH (TGARCH) Model

Another volatility model commonly used to handle leverage effects is the threshold GARCH (or TGARCH) model; see Glosten, Jagannathan, and Runkle 1993 [21] and Zakoian 1994 [22]. In the TGARCH (1,1) version of the model[13], the specification of the conditional variance is:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \gamma d_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \quad (17)$$

where d_{t-1} is a dummy variable, that is:

$$d_{t-1} = \begin{cases} 1 \quad if \quad \varepsilon_{t-1} \prec 0, & bad \quad news \\ 0 \quad if \quad \varepsilon_{t-1} \geq 0, & good \quad news \end{cases} \tag{18}$$

the coefficient γ is known as the asymmetry or leverage term. When $\gamma = 0$, the model collapses to the standard GARCH forms. Otherwise, when the shock is positive (i.e., good news) the effect on volatility is α_1, but when the news is negative (i.e., bad news) the effect on volatility is $\alpha_1 + \gamma$. Hence, if γ is significant and positive, negative shocks have a larger effect on σ_t^2 than positive shocks [23].

In the general specification of this model, TGARCH (p,q), the conditional variance equation is specified as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i + \gamma_i d_{t-i}) \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{19}$$

α_i, γ_i and β_j are non-negative parameters satisfying conditions similar to those of GARCH models.

4.2.2.3. The Power GARCH (PGARCH) Model

Ding, Granger and Engle 1993 [24] also introduced the Power GARCH (PGARCH) specification to deal with asymmetry. Unlike other GARCH models, in this model, the standard deviation is rather modelled as against modelling of variance in most of the GARCH-family of models. In Power GARCH an optional parameter γ can be added to account for asymmetry in modelling up to order r. The model also affords one the opportunity to estimate the power parameter δ instead of imposing it on the model Ocran and Biekets 2007 [25].

The general asymmetric Power GARCH model specifies σ_t as of the following form:

$$\sigma_t^\delta = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^\delta + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-1} \right)^\delta \tag{20}$$

where α_i and β_j are the standard ARCH and GARCH parameters, γ_i are the leverage parameters and δ is the parameter for the power term, and:

$\delta > 0$, $|\gamma_i| \leq 1$ for $i = 1,2,\ldots,r$, $\gamma_i = 0$ for all $i < r$ and $r \leq p$.

The symmetric model sets $\gamma_i = 0$ for all i .

when $\delta = 2$, the above equation becomes a classic GARCH model that allows for leverage effects and when $\delta = 1$ the conditional standard deviation will be estimated. In addition, we can increase the flexibility of the PGARCH model by considering δ as another coefficient that must also be estimated [26].

5. EMPIRICAL RESULTS

As it was shown in the data description part when the residuals were examined for heteroscedasticity, ARCH-LM test provides strong evidence of ARCH effects in the residual series, which indicates that we can now proceed with the modelling of the index return volatility by using GARCH methodology. The results of estimating the different GARCH models for the KSE index returns are presented in this section for the full sample, first sub-period, and second sub-period, the models are estimated using maximum likelihood method under the assumption of Gaussian normal distribution. The log likelihood function is maximized using Marquardt numerical iterative algorithm to search for optimal parameters. To account for the sharp decline of the KSE index in the last part of October 2009, a dummy variable (DUM) will be introduced into the mean equation of the full sample period, which is set equal to 0 for the period before the sharp decline and 1 thereafter. Thus, for the full sample, the mean equation is adjusted as:

Mean equation $$r_t = \mu + DUM + \varepsilon_t \tag{19}$$

For the first and second sub-periods, the mean equation will still be used as specified before. Beside the estimation output of different GARCH models, diagnostics test results of these models are also provided to see whether there are still ARCH effects left in the estimated model. 14

Tables 7 to 11 in the Appendix show the parameter estimates of different GARCH models for the returns of KSE index for the three specified periods. GARCH (1,1) estimation The results of estimating the GARCH (1,1) model are reported in Table 7.

Table 7 about here

In the variance equation From Table 7, the first three coefficients ω (constant), ARCH term (α) and GARCH term (β) for GARCH (1,1) are highly significant and with expected sign for all periods. The significance of α and β indicates that lagged conditional variance and squared disturbance has an impact on the conditional variance, in other words this means that news about volatility from the previous periods has an explanatory power on current volatility. The sum of the two estimated ARCH and GARCH coefficients $\alpha + \beta$ (persistence coefficients) in the estimation process of the second sub-period is less than one, which is required to have a mean reverting variance process. In contrast, the sum up of these parameters for the first sub-period and full data sample is larger than one, suggesting that shocks to the conditional variance are highly persistent, i.e. the conditional variance process is explosive. This implies that large changes in returns tend to be followed by large changes and small changes tend to be followed by small changes, which will therefore, confirm that volatility clustering is observed in KSE index returns series. The ARCH-LM test statistics for all periods did not exhibit additional ARCH effect. This shows that the variance equations are well specified.

GARCH-M (1,1) estimation

The GARCH-M model is estimated by allowing the mean equation of the return series to depend on function of the conditional variance σ^2. Table 3 presents the estimation results for the mean and variance equations.

Table 8 about here

The estimated coefficient (risk premium) of σ^2 in the mean equation is positive for all periods, which indicates that the mean of return sequence considerably depends on past innovation and past conditional variance. In other words, conditional variance used as proxy for risk of return is positively related to the level of return. This result show that as volatility increases, the returns correspondingly increase by a factor of 0.282, 0.128, and 0.099 for first, second and full sample periods respectively. These results are consistent with the theory of a positive risk premium on stock indices which states that the higher returns are expected for asset with higher level of risk. The ARCH and GARCH coefficients are significant in all periods. The null hypothesis that there is no ARCH effect is accepted.

EGARCH (1,1) estimation

To investigate the existence of leverage effect in returns of the Khartoum stock market index during the sample period, the asymmetrical EGARCH (1,1) models were estimated. Results are provided in Table 9 in the Appendix.

Table 9 about here

The EGARCH(1,1) model estimated for the returns of KSE index in Table 9 indicates that all the estimated coefficients for all periods are statistically significant at 1% confidence level. The asymmetric (leverage) effect captured by the parameter estimate γ is also statistically significant with negative sign for all periods, indicate that negative shocks imply a higher next period conditional variance than positive shocks of the same sign, which imply that the existence of leverage effect is observed in returns of the Khartoum stock market index during the sample period. The null hypothesis of no heteroscedasticity in the residuals is accepted for first and second sub-periods, but not for the full sample period.

TGARCH (1,1) estimation

The second model used to test for the asymmetry in the volatility of KSE returns is the TGARCH (1,1) model. Results of the estimation are in Table 5.

Table 10 about here

In the estimated TGARCH(1,1) model, the coefficient of leverage effect is significant and positive for all periods which means asymmetry effect is accepted for this period, the significance of this coefficient indicates that negative shocks (bad news) is larger effect on the conditional variance (volatility) , than positive shocks (good news) of same magnitude. The null hypothesis that there is no ARCH effect is accepted.

Power GARCH (1,1) estimation

Unlike other GARCH models, in this model, the standard deviation is rather modelled as against modelling of variance in most of the GARCH-family of models. Results of PGARCH (1,1) are presented in Table 11.

Table 11 about here

From the results of PGARCH (1,1) in Table 6, the estimated coefficient γ is significant and positive for the first and second sub periods , indicating that positive shocks are associated with higher volatility than negative shocks. In contract, the estimated coefficient in the full sample period is negative and insignificant. The ARCH-LM test statistics did not exhibit additional ARCH effect. This shows that the variance equations are well specified.

6. CONCLUSIONS

Modelling and forecasting the volatility of returns in stock markets has become a fertile field of empirical research in financial markets. This is simply because volatility is considered as an important concept in many economic and financial applications like asset pricing, risk management and portfolio allocation This paper attempts to explore the comparative ability of different statistical and econometric volatility forecasting models in the context of Sudanese stock market namely; Khartoum Stock Exchange (KSE). A total of five different models were considered in this study. The volatility of the KSE index returns have been modelled by using a univariate Generalized Autoregressive Conditional Heteroscedastic (GARCH) models including both symmetric and asymmetric models that captures most common stylized facts about index returns such as volatility clustering and leverage effect, these models are GARCH(1,1), GARCH-M(1,1), exponential GARCH(1,1), threshold GARCH(1,1) and power GARCH(1,1).The first two models are used for capturing the symmetry effect whereas the second group of models is for capturing the asymmetric effect. The study used a stock market index from Sudan (KSE index), a country not previously considered in the volatility literature, for the period 2nd January 2006 to 30th November 2010. Based on the empirical results presented, the following can be concluded:

The paper finds strong evidence that daily returns could be characterised by the above mentioned models. KSE data showed a significant departure from normality and existence of conditional heteroscedasticity in the residuals series. For all periods specified, the empirical analysis was supportive to the symmetric volatility hypothesis, which means returns are volatile and that positive and negative shocks (good and bad news) of the same magnitude have the same impact and effect on the future volatility level. The parameter estimates of the GARCH (1,1) models (α and β) indicates a high degree of persistent in the conditional volatility of stock returns on the Khartoum Stock Exchange which means an explosive volatility. The parameter describing the conditional variance in the mean equation, measuring the risk premium effect for GARCH-M (1,1), is statistically significant in all periods, and the sign of the risk premium parameter is positive. The implication is that increase in volatility would increase returns, which is an expected result. To summarize, the results from all GARCH specifications applied in this paper for three periods explain that explosive volatility process is present in KSE index returns over the sample period.

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NOTES

1. A time series is said to be hetroscedastic if its variance changes over time, otherwise it is called homoscedastic.
2. GARCH-M models (Engle, Lilien, and Robins 1987), IGARCH model (Engle and Bollerslev 1986) [27], Exponential GARCH model (Nelson, 1991),Threshold GARCH model Zakoian (1994) and (Glosten et al., 1993) and Power ARCH model (Ding et al., 1993),
3. Member states are: Burundi, Comoros, Democratic Republic of Congo, Djibouti, Egypt, Eritrea, Ethiopia, Kenya, Libya, Madagascar, Malawi, Mauritius, Rwanda, Seychelles, Sudan, Swaziland, Uganda, Zambia and Zimbabwe.
4. SDG is the Sudanese *Gunaih* (Sudanese new pound), which replaced the old Sudanese Dinar (SDD) in 2007, (1$=2.4986 SDG as in 22 December 2010).
5. The Primary Market deals with the trading of new securities. When a company issues securities for the first time (i.e. IPO) , they are traded in the Primary Market through the help of issuing houses, Dealing /Brokerage Firms, Investment Bankers and or Underwriters. The acronym IPO stands for Initial Public Offering, which means the first time a company is offering securities to the general public for subscription. Once the securities (shares) of a company are in the hands the general public, they can be traded in the Secondary Market to enhance liquidity amongst holders of such financial securities. Thus, the Secondary Market facilitates the buying and selling of securities that are already in the hands of the general public (investors).
6. For a detailed discussion of the Islamic *Shariaa* principles and its practices on stock exchange see for example: El-Gamal 2006 [28] and Ayub 2007 [29].
7. "*Musharakah*" is a word of Arabic origin which literally means sharing. In the context of business and trade it means a joint enterprise in which all the partners share the profit or loss of the joint venture. It is an ideal alternative for the interest-based financing with far reaching effects on both production and distribution Usmani 1998 [30]
8. Out of this listed number there are 20 banks, 8 Insurance companies, 6 commercial companies, 2 in the industrial sector and 3 companies in the agricultural sector, 4 in the communication sector, 5 in services sector and 5 other companies with various activities.

9. In a normally distributed series, skewness is 0 and kurtosis is 3. Positive or negative skewness indicate asymmetry in the series and less than or greater than 3 kurtosis coefficient suggest flatness and peakedness, respectively, in the returns data.)
10. It is very important to point out that, there might be some bias towards accepting the null hypothesis of a unit root for the index series in level for the full sample period, this simply because of the clear existence of the break points in the series at the end of October 2009 (see Figure 2). ADF test fails in case of structural break and it has low power, one way to account for these structural breaks, Perron 1989 [31] introduced a dummy variable to the ADF test. For a detailed discussion of the structural breaks in unit root test see for example Mills and Markellos (2008)
11. The main feature of ARCH model is to describe the conditional variance as an autoregression process. However, most empirical time series require using long-lag length ARCH models and a large number of parameters must be estimated. The solution of the problem was GARCH models which gave better results (see Engle and Bollerslev 1986; Nelson 1991).
12. This is in contrast to the standard GARCH model where shocks of the same magnitude, positive or negative, have the same effect on future volatility.
13. The model uses zero as its threshold to separate the impacts of past shocks. Other threshold can also be used; see (Tsay 2010) for the general concepts of threshold models.

14- If the variance equation of GARCH model is correctly specified, there should be no ARCH effect left in the residuals

APPENDIX

Table 1. Trading Activity in some Arab Stock markets (2009)

Market	Value Traded (U.S.$)	Shares Traded (Million)	Market Capitalization (U.S.$)	No. of Transactions	No. of Listed Companies
Abu Dhabi	18,698.35	36,870.11	72,967.81	730742	67
Amman	13,615.91	5,991.44	31,985.19	2954322	272
Bahrain	460.01	831.36	16,141.33	29784	49
Beirut	934.60	50.92	18,297.99	35505	11
Casablanca	8,598.67	233.90	60,694.46	286269	73
Khartoum	949.21	172	2,979.25	8069	53
Doha	24,234.02	3,903.37	87,931.99	1630407	44
Dubai	46,659.87	109,646.92	58,829.91	1965131	67
Egypt	50,812.70	28,234.25	86,267.22	13300653	306
Kuwait	74,161.61	104,540.98	104,226.22	19063774	205
Saudi	322,432.10	54,443.71	318,784.68	12197799	135
Tunis	1,206.26	169.89	9,399.05	349700	52

Source: Compiled by authors based on data from Arab Monetary Fund website and KSE annual report.

Table 2. Volume of Trading in Khartoum Stock Exchange by Sectors (2003 – 2009)

Sector	Volume of trading (SDG millions)						
	2003	2004	2005	2006	2007	2008	2009
Banks	39.7	7.8	11.1	91.4	139.7	135.8	81.5
Insurance	0.1	0.01	0.0	0.0	1.9	1.8	0.1
Commercial	1.9	39.3	18.3	22.3	22.0	6.2	15.1
Industry	0.01	39	48.2	57.0	4.0	0.8	25.4
Agriculture	0.0	0.0	0.0	0.0	0.1	0. 1	0.0
Communication	-	-	-	-	432.2	320.1	122.5
Services	-	-	-	-	0.8	7.6	0.9
Funds	7.4	2.8	47.1	120.2	130.3	123.5	164.8
Certificates	62.7	113.7	194.4	799.9	1068.5	1283.2	1836.3
Others	132.3	245.1	897.7	977.3	0.1	0.04	0.0
Total	244.1	447.7	1216.8	2068.1	1799.6	1879.04	2246.6

Source: Compiled by authors based on data from Central Bank of Sudan annual reports.

Table 3. Summary of Trading Activity in Khartoum Stock Exchange Secondary market

Year	No. Of Listed companies *	No. Of traded shares (In thousands)	Market Capitalization (SDG millions)	Volume of trading (SDG millions)	No. of transactions	Certificates traded (In thousands)
2003	47	9.745.457	1.929.85	244.104	3109	
2004	48	2.186.127	3.689.88	447.723	3534	102.108
2005	49	1.731.670	7.473.27	1.216.833	3673	308.1
2006	52	7.567.782	9.312.42	2.068.054	5842	1,472.4
2007	53	9.411.559	10.121.6	1.799.600	7195	2,016.5
2008	53	289.008	8.541.5	1.879.410	8569	2,421.1
2009	53	172.359	6.961.9	2.246.600	8069	3,417.7

* The number of listed companies does not include Funds & Certificates.
Source: Compiled by authors based on data from KSE and Central Bank of Sudan annual reports.

Table 4. Descriptive Statistics of the KSE returns series

STATISTICS	*First Sub-period*	*Second Sub-period*	*Full Sample Period*
Mean %	0.01%	0.00%	-0.02%
Median %	0%	0%	0%
Maximum %	21%	1%	21%
Minimum %	-11%	-1%	-12%
Standard deviation	1.47%	0.71%	1.37%
Skewness	2.57	3.52	1.76
Kurtosis	65.31	82.71	73.72
Jarque- Bera	169550.4	71496.33	276573.2
Prob. of Jarque-Bera	0.000	0.000	0.000
No. of observations	1042	269	1326

Table 5. ADF Unit Root Test Output for the Price index and return Series

Period	KSE index Series				KSE Return Series			
	ADF statistic	Critical Values			ADF statistic	Critical Values		
		1%	5%	10%		1%	5%	10%
Full Period	-2.390(6)	-3.438	-2.864	-2.568	-18.584(5)**	-3.965	-3.413	-3.129
First sub-period	-2.671(5)*	-3.436	-2.864	-2.568	-29.419(1)**	-3.463	-2.864	-2.568
Second sub-period	-7.469(0)**	-3.455	-2.872	-2.572	-20.352(2)**	-3.455	-2.872	-2.572

Notes:
- Figures in parentheses denote the optimal lag lengths, which were automatically selected based on Schwarz Information Criterion (SIC).
- Critical values for unit root test are referred to MacKinnon (1996) one-sided p-value.
- * and ** indicate significant at 5% and 1% respectively.
- ADF test includes a constant term without trend.

Table 6. ARCH-LM Test for residuals of returns on KSE and CASE markets

	First Sub-period	*Second Sub-period*	*Full Sample Period*
ARCH-LM test statistic (TR^2)	13.59	43.97	59.02
Prob. Chi-square (5)	0.0184	0.0000	0.0000

Table 7. Estimation results of GARCH (1,1) model

Parameters		*Period*		
		First Sub-period	*Second Sub-period*	*Full sample period*
Mean equation	ϕ (Constant)	0.000192	-1.18E-05	0.000248
	DUM			-0.000293
Variance equation	ω (Constant)	3.52E-05*	6.00E-08*	3.19E-05*
	α (ARCH effect)	0.739287*	0.124267*	0.695345*
	β (GARCH effect)	0.406875*	0.650981*	0.428239*
	$\alpha+\beta$	1.146162	0.775248	1.123584
Log likelihood		3303.647	1627.045	4398.309
ARCH-LM Test for heteroscedasticity				
ARCH-LM test statistic($N*R^2$)		0.171356	3.341550	0.129301
Prob. Chi-Square(5)		0.9994	0.6475	0.9997

* Indicates significance at 1% level

Table 8. Estimation results of GARCH-M (1,1) model

Parameters		*Period*		
		First Sub-period	*Second Sub-period*	*Full sample period*
Mean equation	ϕ (Constant)	-0.002410*	-7.09E-05*	-0.000752*
	DUM			
	λ (risk premium)	0.282394*	0.127631*	0.099496*
Variance equation	ω (Constant)	3.70E-05*	6.05E-08*	2.32E-05*
	α (ARCH effect)	0.714649*	0.128282*	0.888347*
	β (GARCH effect)	0.377463*	0.646581*	0.420817*
	$\alpha+\beta$	1.092112	0.774863	1.309164
Log likelihood		3309.044	1627.207	4419.219
ARCH-LM Test for heteroscedasticity				
ARCH-LM test statistic($N*R^2$)		0.120292	2.982367	0.115306
Prob. Chi-Square		0.9997	0.7027	0.9998

* Indicates significance at 1% level

Table 9. Estimation results of EGARCH (1,1) model

Parameters		Period		
		First Sub-period	Second Sub-period	Full sample period
Mean equation	ϕ (Constant)	0.000572*	-1.90E-05	
	DUM			9.27E-05*
Variance equation	ω (Constant)	-3.952504*	-1.581487*	-5.523160*
	α (ARCH effect)	0.862935*	0.171235*	0.183655*
	β (GARCH effect)	0.605742*	0.900153*	0.417255*
	γ (Leverage effect)	-0.127306*	-0.123552*	-0.017766*
	$\alpha+\beta$	1.468677	1.071388	0.60091
Log likelihood		3309.860	1608.297	4059.788
ARCH-LM Test for heteroscedasticity				
ARCH-LM test statistic($N*R^2$)		0.205917	0.901964	27.03379
Prob. Chi-Square		0.9990	0.9701	0.0001*

* Indicates significance at 1% level

Table 10. Estimation results of TGARCH (1,1) model

Parameters		Period		
		First Sub-period	Second Sub-period	Full sample period
Mean equation	ϕ (Constant)	7.72E-05	-2.18E-05	8.91E-05
	DUM			-8.25E-05
Variance equation	ω (Constant)	3.55E-05**	5.82E-08**	2.90E-05**
	α (ARCH effect)	0.487554**	0.083005**	0.656613**
	β (GARCH effect)	0.399900**	0.657500**	0.426983**
	γ (Leverage effect)	0.524922**	0.086663*	0.239591**
	$\alpha+\beta$	0.887454	0.740505	1.083596
Log likelihood		3307.803	1627.809	4409.251
ARCH-LM Test for heteroscedasticity				
ARCH-LM test statistic($N*R^2$)		0.151712	1.741300	0.095918
Prob. Chi-Square		0.9995	0.8837	0.9999

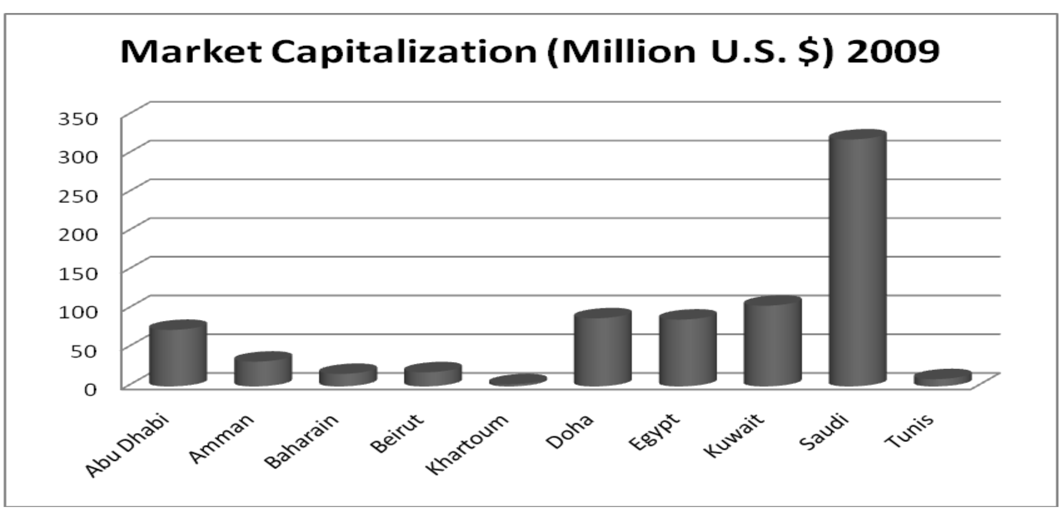
* and ** indicate significant at 5% and 1% respectively.

Table 11. Estimation results of PowerGARCH (1,1) model

Parameters		Period		
		First Sub-period	Second Sub-period	Full sample period
Mean equation	ϕ (Constant)	0.000128	-1.31E-05	0.000536
	DUM			-0.000475
Variance equation	ω (Constant)	8.64E-07	4.01E-08	0.000183*
	α (ARCH effect)	0.865**	0.122**	0.417**
	β (GARCH effect)	0.327**	0.640**	0.520**
	γ (Leverage effect)	0.190**	0.058**	-0.0386
	δ (Power Parameter)	2.794**	2.057**	1.666**
	$\alpha+\beta$	1.192	0.762	0.937
Log likelihood		3307.977	1636.926	4333.934
ARCH-LM Test for heteroscedasticity				
ARCH-LM test statistic($N*R^2$)		0.161	2.633	0.288
Prob. Chi-Square		0.999	0.756	0.999

* , ** indicate significant at 5% and 1% respectively.

Figure 1. Stock market capitalization for some Arab countries during 2009



Source: Compiled by author based on data from Arab Monetary Fund wesite and KSE annual report.

Figure 2. The trend graph of daily prices for the KSE index Jan.2006 – Nov.2010

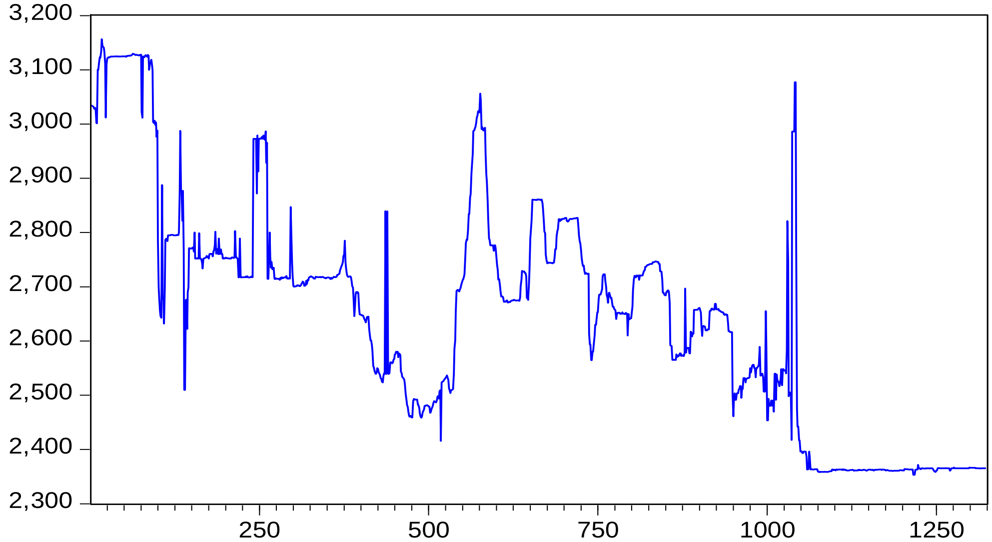


Figure 3. The trend graph of daily returns for the KSE index Jan.2006 – Nov.2010

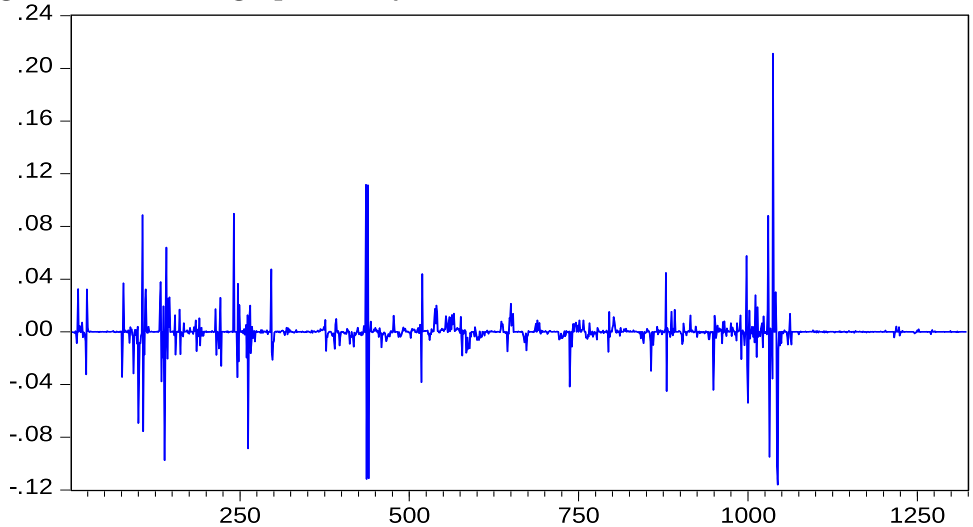


Figure 4. Normal Quantile-Quantile Plots for the Daily Stock Return 2006 – 2010: