Small Firms, their Growth and Product Differentiation

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Abstract

This paper investigates a model of endogenous product differentiation between large well established and small newly established firms. Small firms should have greater growth potential than large mature firms whose growth potential tapers off once they reach a certain size relative to the capacity of the market. The small firm with its newness has a product that is typically seen as being of a lower quality than the products of their larger counterparts. The model explores the interaction of product quality, firm size and the growth of small firms. This paper shows that firms choose size (large or small) and they will maintain their decision throughout any stage of the game. Large firms are more efficient at producing quality, therefore, despite a growth rate advantage, small firms remain small. Driving this result is the fact that the payoff from remaining small outweighs the payoff from its growth potential since becoming large is accompanied by heavy costs. This property ensures that the principle of maximal product differentiation is sustained.

JEL Classifications: LO, L2.

Key Words: Small Firms, Large Firms, Firm Growth, Product Differentiation.

1 Introduction and Literature Review

Models that investigate firm dynamics have predicted that small firms grow at a faster rate than large ones. Hall (1987) investigated the dynamics of firm growth in the US Manufacturing sector using econometric techniques. He suggested that possible reasons why small firms may grow faster than large ones are due to differences in the rate and direction of innovative activity, or simply because the economy is finite and it is expected that diminishing returns will eventually take effect. He observed large differences in the variance of growth rates across size classes of firms and that the smaller firms in the sample grow faster, but made no claim to be able to distinguish clearly the reasons for these differences. This paper investigates, firm size and their respective growth rates under variable marginal cost. Our analysis presents a two-stage non-cooperative game, in which the firm chooses its size in stage one and prices given its own type and that of the other firm in stage two. Shaked and Sutton (1983) showed that where two firms choose distinct qualities, they will both enjoy positive profit at equilibrium. The intuition behind their result is that where the product qualities converge, a price competition will result between both firms which will serve to erode their profits.

This equilibrium runs counter to the wisdom of Hotelling's (1929) model of the linear city in which he concluded that the equilibrium outcome is characterized by minimal differentiation. In applying the wisdom of Shaked and Sutton we consider two firms of distinctive types; small and large. It is believed that small newly established firms should have greater growth potential than large mature firms whose growth potential tapers off once they reach a certain size relative to the capacity of the market. However, it has been argued that due to the product differentiation advantage that the large well established firm has and the demand disadvantage that may face the small firm it might be difficult for the small firm with the lower quality to settle in the market. Schmalensee (1982) reported that, once consumers are convinced that the first in any product class performs satisfactorily, that brand is seen as the standard against which subsequent entrants are judged. As a result, it presents a greater challenge for later entrants to attract consumers to invest in learning about their qualities than that faced by the first brand.The idea that there is product differentiation advantage for established sellers has been well supported by empirical investigations.

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Schmalensee (1982), in his article reported that the standard view in marketing and various empirical research supports the notion that there are significant gains from being the first entrant in some markets. Schmalensee (1982) further noted that Joe Bain's empirical work on conditions of entry led him to conclude that product differentiation advantages of established sellers are the most important barrier to entry. Bresnahan (1987) argued that products that are more distant (than adjacent) in quality scale have zero cross-price elasticities of demand. Also, Maez and Waterson (2001) highlighted the importance for firms to differentiate their products. They stated that firms will cover a broad range of qualities in order to discriminate among consumers with heterogeneous preferences (as in the monopoly situation). In extending the analysis to consider the relationship between firm size, and growth, it cannot be ignored that under vertical production differentiation, price is not the only variable of interest. When product quality differs, products are not homogenous and consumer preferences lie on a continuum in terms of quality. Allowing marginal cost to vary with quality so that higher quality products are more expensive to manufacture allowed for the delineation of firms by cost. Coincidentally, the marginal cost of the higher quality products produced by larger firms should be higher than that of lower quality products produced by small firms. Thomas (2010) concluded that when marginal costs are allowed to vary with firm type (quality) then maximal product differentiation is not necessarily the outcome, but under the assumption that all firms face positive demand, maximal product differentiation can be supported.

2 The Model

Since Consumers differ in their preferences, firms typically wish to differentiate themselves from their competitors. This is the standard case of product differentiation. This model presents a two stage game where duopoly firms choose their size in the first stage and price in the second stage. Marginal cost is expressed as a function of the exogenous variable size, hence large type firms will have a larger marginal cost than small type firms. More formally, consider a two stage game where firm $i \in I = \{1,2\}$ makes the following choices.

Stage 1: Each firm i chooses size, δ_i such that $\delta_i \in (\left(\underline{\delta}, \overline{\delta}\right) \subset \Re_+$ and denote the other firm's size by δ_{-i} .
Stage2: Each firm i chooses its price $P_i \in \Re_+$ given its own size and that of the other firm.

In stage 1, the firm's face a strictly non-negative and increasing size dependent marginal cost
$w_\delta = w_i(\delta), \forall, \delta$; that is marginal cost does not vary with output but instead is a function of the firms' size.

In this duopoly model, one firm chooses large size (L) and the other chooses small size (S) where L is to the right of S on the line segment representing the consumers' preferences. The small firm has a product that is typically seen as being of a lower quality than the larger firm.

Consider that small firms have greater growth potential than larger firms. Formally small firms are characterized by a growth path that is convex, while large firms sit on a concave growth path. As firms become larger the growth trajectory shifts from being convex to being concave, hence small firms grow rapidly and large firms tend to experience sluggish growth on average. Further, denote the growth rate of each firm as t_L (large firm) and t_S (small firm) which are increasing, bounded and twice differentiable functions in their own size. The consumers receive a payoff from consumption in accordance with their preferences given by:[1]

$$\begin{cases} t_S\, US - p_s & \text{if } S \text{ is chosen} \\ t_L UL - p_L & \text{if } L \text{ is chosen} \end{cases}$$

U Is a preference parameter, $U \sim u[0,1]$

To find the preference parameter, we solve the consumers' problem by finding the consumer who is indifferent between the two firms. As such we set the payoff from going to either firm equal. That is:

$$t_L UL - p_L = t_S US - p_S$$

$$\Leftrightarrow U = \frac{p_L - p_S}{t_L L - t_S S} > 0 \qquad (1)$$

If the price that the large firm charges is greater than that of the small firm, as will be shown below, then:

$$U > 0 \Rightarrow t_L L > t_S S$$

[1] Assume that the utility from consuming the outside good is zero

From the consumers problem we obtain the following demand functions.

$$D_L(P_L, P_S) = \begin{cases} 0 & if\ U \geq 1 \\ 1-U & if\ 0 < U < 1 \\ 1 & if\ U \leq\ 0 \end{cases}$$

$$D_S(P_L, P_S) = \begin{cases} 1 & if\ U \geq 1 \\ U & if\ 0 < U < 1 \\ 0 & if\ U \leq 0 \end{cases}$$

We can now write each firm's profit as a function of prices, size and it's growth rate as well as that of the competitor.

$$\pi_L(P_L, P_S) = (P_L - w_L)[D_L(L, S, P_L, P_S, t_L, t_S)] \qquad (2)$$

$$\pi_S(P_L, P_S) = (P_S - w_S)[D_S(L, S, P_L, P_S, t_L, t_S)] \qquad (3)$$

From the profit functions, equations (2) and (3) above, it can be seen that where price and/or quanity equals zero (corner solutions) the firms will face negative or zero profits. We assume that each firm will charge a price that is greater than its marginal cost. To find the Nash equilibrium price given the firms choice of size in stage 1, we solve each firm's profit maximization problem by taking the first order condition (F.O.C.) of the profit functions of both firms with respect to their respective prices.

$$\pi_S = (P_S - w_S)U \Rightarrow (P_S - w_S)[\frac{P_L - P_S}{t_L L - t_S S}]$$

Therefore the first order condition is:

$$\frac{\partial \pi_S}{\partial P_S}: \frac{P_L - P_S}{t_L L - t_S S} - \frac{P_S - w_S}{t_L L - t_S S} = 0$$

$$\Leftrightarrow P_L + w_S - 2P_S = 0$$

$$\Leftrightarrow P_S = \frac{P_L + w_S}{2} \qquad (4)$$

And

$$\pi_L = (P_L - w_L)(1 - U) \Rightarrow (P_L - w_L) - (P_L - w_L)[\frac{P_L - P_S}{t_L L - t_S S}]$$

With first order conditions:

$$\frac{\partial \pi_S}{\partial P_S}: 1 - \frac{P_L - P_S}{t_L L - t_S S} - \frac{(P_L - w_L)}{t_L L - t_S S} = 0$$

$$\Leftrightarrow [t_L L - t_S S - 2P_L + P_S + w_L] = 0$$

$$\Leftrightarrow P_L = \frac{(t_L L - t_S S) + P_S + w_L}{2} \qquad (5)$$

Substituting (4) into (5) gives:

$$P_L = \frac{(t_L L - t_S S)}{2} + \frac{1}{2}\frac{P_L + w_S + 2w_L}{2}$$

$$= \frac{2(t_L L - t_S S) + 2w_L + w_S}{3}$$

And substituting (6) into (4) we obtain:

$$P_S = \frac{(t_L L - t_S S) + 2w_s + w_L}{3}$$

Equations (6) and (7) are the Nash equilibrium prices. We can tell from observation that equation (6) is greater than equation (7), suggesting that the larger type firm charges a higher price than the small firm. From observation, it can be seen that the price charged by each firm is a function of its own marginal cost, the marginal cost of its competitor, its own size choice as well as the size of its competitor. The price of output for both firms is positively related to its own marginal cost and that of the competitor and increases in the distance between their respective size choices, with the price of the large firm's output being strictly greater than the price of the small firm. From the findings, the following propositions are true[2]

Proposition 1

Price competition is less intense the more distinct the other firm type makes itself if $2\left[t_S + t^{'}(s)S\right] > w^{'}(s)$.

Proof:

$$\frac{\partial P_L}{\partial S} = \frac{w^{'}(S) - 2[t_S + t^{'}(S)S]}{3} \leq 0; \ \frac{\partial P_S}{\partial L} = \frac{w^{'}(L) + [t_L + t^{'}(L)L]}{3} \geq 0$$

Specifically, as shown here, the more differentiated one firm makes its size, the higher the price the other firm will be able to charge. This softens price competition such that both firms can make positive profits.

Proposition 2

Output prices increase in the ratio $\frac{2}{3}:\frac{1}{3}$ *in the firm's own and its competitor's marginal cost of output.*

Proof:

$$\frac{\partial P_\delta}{\partial w_\delta} = \frac{2}{3}, \qquad \frac{\partial P_\delta}{\partial w_{-\delta}} = \frac{1}{3}, \qquad \delta \in \{L, S\};$$

Proposition 3

Output prices are increasing in large firm's growth rate and decreasing in the small firm's growth rate.

Proof:

$$\frac{\boldsymbol{\partial P_\delta}}{\boldsymbol{\partial t_L}} > 0; \ \frac{\partial \mathrm{P}_\delta}{\partial \mathrm{t}_\mathrm{S}} < 0$$

This implies that for output prices of both firms to increase it is optimal for the large firm to grow and for the small firm not to act on its growth potential. This supports the theory of maximal production differentiation.

Proposition 4

The large firm who provides a higher quality service will charge a higher price for output.

Proof:

$$(P_L - P_S) = \frac{t_L L - t_S S) + (w_L - w_S)}{3} > 0 \qquad (8)$$

This outcome is reasonable and supports the assumption that both firms face positive demand. Any other outcome would be infeasible as it would make acquiring the service from the high type firm a strictly dominant strategy for consumers and the low type firm would disappear from the market.

The derivation of equations (6) and (7) concludes the second stage of the game. With this done, we can obtain the first stage profit functions exclusively as a function of firms' sizes and growth rates (equations 9 & 10 below)

For Large Firms the Profit Functions are:

$$\pi_L(L,S,t) = \left[\frac{2(t_L L - t_S S) + (w_S - w_L)}{3}\right]\left[\frac{2(t_L L - t_S S) + (w_S - w_L)}{3(t_L L - t_S S)}\right]$$
$$= \frac{1}{(t_L L - t_S S)}\left[\frac{2(t_L L - t_S S) + (w_S - w_L)}{3}\right]^2 \qquad (9)$$

For Small Firms the Profit Functions are:

$$\pi_S(L,S,t) = \left[\frac{(t_L L - t_S S) + (w_L - w_S)}{3}\right]\left[\frac{(t_L L - t_S S) + (w_L - w_S)}{3(t_L L - t_S S)}\right]$$

[2] All proofs for propositions stated here are derived from equations (6) and (7).

$$= \frac{1}{(t_L L - t_S S)} \left[\frac{(t_L L - t_S S) + (w_L - w_S)}{3}\right]^2 \qquad (10)$$

From equations (9) and (10), it can be seen that each firm operates profitably which implies that at least in the short run there will be more than 1 firm in the market. Additionally, we can also conclude from our results that firms' choice of size under conditions of variable marginal cost brings another dimension to the analysis of product differentiation developed by Shaked and Sutton (1983).

3 Comparative Statics

So far we have assumed a strictly increasing, size dependent marginal cost. Which stated explicitly denotes the marginal cost curve faced by the respective firms as a function of their choice of size. We can conceptualize this idea by reasoning that in order to produce higher quality the large firm must put more (or higher quality/more expensive) inputs into the production process. The argument can therefore be made that the marginal cost function of the small firm will be distinct from its large competitor and the cost of moving closer toward the higher quality firm may be considered prohibitive.

We have so far defined the consumer preference parameter

$$U = \frac{P_L - P_S}{t_L L - t_S S}$$

This inherently presents consumers' preference as a function of the exogenous variables size (L, S) which was determined in stage one of the model. From the following assumption, we can proceed with comparative statics with respect to changes in firms' size. This assumption also ensures that the principle of maximal product differentiation is sustained.

$$A1: \quad Max \left\{\frac{2w'(S)}{3[t_S + 2t'(S)S]}, \frac{3(t_L L - t_S S)w'(L)}{2t_L(t_L L - t_S S) + (w_S - w_L)}\right\}$$

$$< U < \frac{2[t_S + t'(S)S - w'(S)}{3[t_S + 2t'(S)S]} < \frac{2w'(L)}{3[t_L + 2t'(L)L]}$$

The fundamental idea behind A1 is that no firm will cover the market such that it negates the influence of the other firm. It therefore identifies a set of precise conditions that confirms the existence of both firms in equilibrium. In this environment maximal product differentiation is confirmed as the dominant strategy. It must be noted that nothing precludes the terms on the lower bound of A1 from being negative, except that U must be positive.To show that maximal product differentiation does exist, we show that there is a unique subgame Nash equilibrium. Moreover, the profit of each firm increases with the distance between types. This is demonstrated in propositions 5 and 6 which follow.

Proposition 5

(A1) ensures a unique solution where each firm chooses a different size from that of its competitor in equilibrium.

The proof of proposition 5 consists of the following derivatives and all the results presented leading up to and including proposition 7. To prove proposition 5, we differentiate each firm's profit function with respect to its respective size. We show that each firm maximizes profit the more differentiated they are in size.

Proof of Proposition 5:

Large Firms[3]:

$$\pi_L(P_L, P_S) = (P_L - w_L)[D_L(L, S, P_L, P_S, t_L, t_S)]$$

$$\frac{\partial \pi_L}{\partial L} = (P_L - w_L)\left[\frac{\partial D_L}{\partial L} + \frac{\partial D_L}{\partial P_S}\frac{\partial P_S}{\partial L} + \frac{\partial D_L}{\partial t_L}\frac{\partial t_L}{\partial L}\right] - \frac{\partial w_L}{\partial L} D_L$$

Aside:

$$\frac{\partial D_L}{\partial L} = \frac{(P_L - P_S)\left[t_L + t'(L)L\right]}{(t_L L - t_S S)^2} = \frac{U\left[t_L + t'(L)L\right]}{(t_L L - t_S S)};$$

[3] By the envelope theorem, we need not take derivatives with respect to P_L

$$\frac{\partial D_L}{\partial P_S} = \frac{1}{t_L L - t_S S}; \qquad \frac{\partial P_S}{\partial L} = \frac{t_L + t'(L)L + w'(L)}{3}$$

$$\frac{\partial D_L}{\partial P_S}\frac{\partial P_S}{\partial L} = \frac{t_L + t'(L)L + w'(L)}{3(t_L L - t_S S)};$$

$$\frac{\partial D_L}{\partial t_L} = \frac{L(P_L - P_S)}{(t_L L - t_S S)^2} = \frac{LU}{(t_L L - t_S S)}; \quad \frac{\partial t_L}{\partial L} = t'(L)$$

$$D'(t_L)t'(L) = \frac{Ut'(L)L}{t_L L - t_S S};$$

$$\frac{\partial w_L}{\partial L} = w'(L); \;\; D_L = 1 - U \quad \Rightarrow \;\; w'(L)D = w'(L)(1-U);$$

Therefore:

$$\frac{\partial \pi_L}{\partial L} = (P_L - w_L)\left[\frac{U\left[t_L + t'(L)L\right]}{(t_L L - t_S S)} + \frac{t_L + t'(L)L + w'(L)}{3(t_L L - t_S S)} + \frac{Ut'(L)L}{t_L L - t_S S}\right] - w'(L)(1-U)$$

$$= \frac{\left[(P_L - w_L)\;\; t_L + t'(L)L + w'(L) + 3U(t_L + 2t'(L)L)\right]}{3(t_L L - t_S S)} - w'(L)(1-U)$$

$$= \frac{(P_L - w_L)\;\; t_L + t'(L)L + w'(L) + w'(L)3U(t_L L - t_S S)}{3(t_L L - t_S S)} + \frac{(P_L - w_L)6Ut'(L)L}{3(t_L L - t_S S)} + \frac{(P_L - w_L)3Ut_L - w'(L)3(t_L L - t_S S)}{3(t_L L - t_S S)} \qquad (11)$$

It is required that (11) be greater than zero to achieve maximal product differentiation. The first and second terms are distinctly greater than zero and (A1) ensures that the third term is also greater than zero. It is also necessary to prove that the third term in equation (11) is greater than zero. This implies that $(P_L - w_L)3t_L U - w'(L)3(t_L L - t_s S) > 0$ since the denominator is positive[4].

This implies that $(P_L - w_L)3Ut_L - w'(L)3(t_L L - t_S S) > 0$ since the denominator is positive.

$$\Leftrightarrow \;\; (P_L - w_L)3Ut_L > w'(L)3(t_L L - t_S S)$$

$$\Leftrightarrow \qquad U > \frac{w'(L)(t_L L - t_S S)}{(P_L - w_L)t_L}$$

Substituting the denominator from (6) yields:

$$U > \frac{3w'(L)(t_L L - t_S S)}{t_L[(t_L L - t_S S) + (w_S - w_L)]} \qquad Q.E.D.$$

Small Firms[5]:

The profit of the small firm is given by:

$$\pi_S(P_L, P_S) = (P_S - w_S)[D_S(L, S, P_L, P_S, t_L, t_S)]$$

$$\frac{\partial \pi_S}{\partial S} = (P_S - w_S)\left[\frac{\partial D_S}{\partial S} + \frac{\partial D_S}{\partial P_L}\frac{\partial P_L}{\partial S} + \frac{\partial D_S}{\partial t_S}\frac{\partial t_S}{\partial S}\right] - \frac{\partial w_S}{\partial S}D_S$$

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[4] The general conclusion is that the sign is ambiguous even though (A1) is sufficient to generate the desired result.

[5] Note that by the Envelope Theorem we need not take derivatives with respect to P_s.

<u>Aside:</u>

$$\frac{\partial D_S}{\partial S}=\frac{(P_L-P_S)\left[t_S+t'(S)S\right]}{(t_L L-t_S S)^2}=\frac{U\left[t_S+t'(S)S\right]}{(t_L L-t_S S)};$$

$$\frac{\partial D_S}{\partial P_L}=\frac{1}{t_L L-t_S S}; \qquad \frac{\partial P_L}{\partial S}=\frac{w'(S)-2[t_S+t'(S)S]}{3}$$

$$\frac{\partial D_S}{\partial P_L}\frac{\partial P_L}{\partial S}= \frac{w'(S)-2[t_S+t'(S)S]}{3(t_L L-t_S S)};$$

$$\frac{\partial D_S}{\partial t_S}=\frac{S(P_L-P_S)}{(t_L L-t_S S)^2}=\frac{US}{(t_L L-t_S S)}; \qquad \frac{\partial t_S}{\partial S}=t'(S)$$

$$D'(t_S)t'(S)=\frac{Ut'(S)S}{t_L L-t_S S};$$

$$\frac{\partial w_S}{\partial S}=w'(S); \;\; D_S=U \quad \Rightarrow \;\; w'(S)D_S=w'(S)U;$$

Therefore:

$$\frac{\partial \pi_S}{\partial S}= (P_S-w_S)\left[\frac{U\left[t_S+t'(S)S\right]}{(t_L L-t_S S)}+\frac{w'(S)-2[t_S+t'(S)S]}{3(t_L L-t_S S)}+\frac{Ut'(S)S}{t_L L-t_S S}\right]-w'(S)U$$

$$= (P_S-w_S)\left[\frac{3U\left[t_S+2t'(S)S\right]+w'(S)-2\left[t_S+t'(S)S\right]}{3(t_L L-t_S S)}\right] - w'(S)U \qquad (12)$$

Again it is required that (12) be less than zero to achieve maximal product differentiation. The second term is distinctly less than zero. Hence to achieve the desired result it must be shown that the first term is also negative, guaranteed by (A1). To show this, set the numerator to be less than zero.

$$\Leftrightarrow 3U\left[t_S+2t'(S)S\right]+w'(S)-2\left[t_S+t'(S)S\right]<0$$

$$\Leftrightarrow \qquad U<\frac{2\left[t_S+t'(S)S\right]-w'(S)}{3[t_S+2t'(S)S]} \qquad Q.E.D.$$

Proposition 6

Firms' profitability is increasing in the distance between types under A(1).

To prove proposition 6 we differentiate each firms profit with respect to the other firm's size to show that each firm will encourage the other to differentiate its size.

Proof:

<u>Large Firms</u>:

$$\frac{\partial \pi_L}{\partial S}= (P_L-w_L)\left[\frac{\partial D_L}{\partial S}+\frac{\partial D_L}{\partial P_S}\frac{\partial P_S}{\partial S}+\frac{\partial D_L}{\partial t_S}\frac{\partial t_S}{\partial S}\right]$$

Note that

$$\frac{\partial D_L}{\partial S}=-\frac{(P_L-P_S)\left[t_S+t'(S)S\right]}{(t_L L-t_S S)^2}=-\frac{U\left[t_S+t'(S)S\right]}{(t_L L-t_S S)};$$

$$\frac{\partial D_L}{\partial P_S}=\frac{1}{t_L L-t_S S}; \qquad \frac{\partial P_S}{\partial S}=\frac{2w'(S)-[t_S+t'(S)S]}{3}$$

$$\frac{\partial D_L}{\partial P_S}\frac{\partial P_S}{\partial S}= \frac{2w'(S)-[t_S+t'(S)S]}{3(t_L L-t_S S)};$$

$$\frac{\partial D_L}{\partial t_S} = -\frac{S(P_L - P_S)}{(t_L L - t_S S)^2} = \frac{US}{(t_L L - t_S S)}; \quad \frac{\partial t_S}{\partial S} = t'(S)$$

$$\frac{\partial D_L}{\partial t_S}\frac{\partial t_s}{\partial S} = -\frac{Ut'(S)S}{(t_L L - t_S S)}$$

Combining:

$$\frac{\partial \pi_L}{\partial S} = -(P_L - w_L)\left[\frac{U\left[t_S + t'(S)S\right]}{(t_L L - t_S S)} + \frac{t_S + t'(S)S - 2w'(S)}{3(t_L L - t_S S)} + \frac{Ut'(S)S}{t_L L - t_S S}\right]$$

$$= -(P_L - w_L)\left[\frac{U\left[t_S + 2t'(S)S\right]}{(t_L L - t_S S)} + \frac{t_S + t'(S)S - 2w'(S)}{3(t_L L - t_S S)}\right]$$

$$= -(P_L - w_L)\left\{\left[\frac{3U\left[t_S + 2t'(S)S\right] - 2w'(S)}{3(t_L L - t_S S)}\right] + \frac{t_S + t'(S)}{3(t_L L - t_S S)}\right\}$$

For maximal product differentiation to obtain the entire expression in curly brackets must be positive thus we require the first term in square brackets to be greater than zero. This will be obtained by setting the numerator of the said term greater than zero.

$$\Leftrightarrow \qquad 3U\left[t_S + 2t'(S)S\right] - 2w'(S) > 0$$

$$\Leftrightarrow \qquad U > \frac{2w'(S)}{3[t_S + 2t'(S)S]} \qquad Q.E.D.$$

<u>Small Firms</u>:

$$\frac{\partial \pi_S}{\partial L} = (P_S - w_S)\left[\frac{\partial D_S}{\partial L} + \frac{\partial D_S}{\partial P_L}\frac{\partial P_L}{\partial L} + \frac{\partial D_S}{\partial t_L}\frac{\partial t_L}{\partial L}\right]$$

<u>Aside:</u>

$$\frac{\partial D_S}{\partial L} = -\frac{(P_L - P_S)\left[t_L + t'(L)L\right]}{(t_L L - t_S S)^2} = -\frac{U\left[t_L + t'(L)L\right]}{(t_L L - t_S S)};$$

$$\frac{\partial D_S}{\partial P_L} = \frac{1}{t_L L - t_S S}; \qquad \frac{\partial P_L}{\partial L} = \frac{2[t_L + t'(L)L] + 2w'(L)}{3}$$

$$\frac{\partial D_S}{\partial P_L}\frac{\partial P_L}{\partial L} = \frac{2[t_L + t'(L)L] + 2w'(L)}{3(t_L L - t_S S)};$$

$$\frac{\partial D_S}{\partial t_L} = -\frac{L(P_L - P_S)}{(t_L L - t_S S)^2} = -\frac{LU}{(t_L L - t_S S)}; \qquad \frac{\partial t_L}{\partial L} = t'(L)$$

$$D'(t_L)t'(L) = -\frac{Ut'(L)L}{t_L L - t_S S};$$

Therefore:

$$\frac{\partial \pi_S}{\partial L} = (P_S - w_S)\left[\frac{2[t_L + t'(L)L] + 2w'(L)}{3(t_L L - t_S S)} - \frac{U\left[t_L + 2t'(L)L\right]}{(t_L L - t_S S)}\right]$$

$$= (P_S - w_S)\left[\frac{2[t_L + t'(L)L]}{3(t_L L - t_S S)} + \frac{2w'(L) - 3U\left[t_L + 2t'(L)L\right]}{3(t_L L - t_S S)}\right]$$

It is required that the second term in square brackets be greater than zero in order to guarantee maximal product differentiation. A1 ensures that this outcome is obtained where the large firm further distinguishes itself from the small firm, the small firm's profit increases.

$$\Leftrightarrow \qquad 2w'(L) - 3U\left[t_L + 2t'(L)L\right] > 0$$

$$\Leftrightarrow \qquad U < \frac{2w'(L)}{3[t_L + 2t'(L)L]} \qquad Q.E.D.$$

The results derived from propositions (5) and (6) which are guaranteed by assumption (1) ensure that the principle of maximal product differentiation is sustained. Equations (11) and (12) show that the firms' profitability is increasing the more distinct they are in size. Following from this, both firms will have an incentive to distinguish themselves in their choices of size, such that there is a unique sub-game perfect equilibrium. The uniqueness of the equilibrium is the result of the no leap-frogging argument and formal proof. These results confirm the outcome of maximal product differentiation.

Proposition 7

Under A1 the large firm is more efficient at producing quality than the small firm – there is no leap frogging.

Proof of the no Leap frogging result:

To ensure that our solution is unique and for maximal product differentiation to hold, firms must not have any incentives to change their type, hence $w'(L)$ < w'(S)$2w'(S)$. From A(1)

$$\frac{2w'(S)}{3[t_S + 2t'(S)S]} < U < \frac{2w'(L)}{3[t_L + 2t'(L)L]}$$

Where; $U \sim u[0,1]$ and let ξ be the smallest continuous unit change in price (for undercutting) or explicitly changing the position of the indifferent consumer. Then:

$$\frac{2w'(S)}{3[t_S + 2t'(S)S]} - \xi < U < \frac{2w'(L)}{3[t_L + 2t'(L)L]} - \xi$$

Since $U > 0$;

$$\Leftrightarrow \frac{2w'(S)}{3[t_S + 2t'(S)S]} - \xi > 0$$

$$\Leftrightarrow \qquad w'(S) > \frac{3\xi[t_S + 2t'(S)S]}{2}$$

Also since $U < 1$; Then:

$$\Leftrightarrow \frac{2w'(L)}{3[t_L + 2t'(L)L]} - \xi < 1$$

$$\Leftrightarrow \qquad w'(L) < \frac{3\xi[t_L + 2t'(L)L]}{2}$$

But we know that $t_S > t_{L,}$ and based on the convexity of the growth path of small firms and the concavity of the growth path of large firms it is true that $t'(L)L < t_{L,} < t_S < t'(S)S,$ which implies that $t'(L)L < t'(S)S$. Therefore:

$$w'(S) > \frac{3\xi[t_S + 2t'(S)S]}{2} > \frac{3\xi[t_L + 2t'(L)L]}{2} > w'(L)$$

Therefore $w'(S) > w'(L)$.

4. Discussions

The idea of signaling is predominant in the model. It is clear that firms are aware of how efficient they are, guiding their size choice. A firm will be faced with the decision to choose size knowing that the current decision will determine future payoff. In the event that both firms, though faced with varying marginal cost, have the same level of efficiency then the choice would be dependent on whether current profit will be sacrificed for large size in the initial stage or higher profit now and small size. For such a scenario, a firm that chooses large size is more willing to sacrifice current profits for future profits.

However, the situation modeled here considers firms that are aware from the initial stage which size choice to make. It is shown that once this choice is made it is not beneficial to deviate. More pointedly, in accordance with the literature, small firm's growth potential is greater and in this environment it is shown that due to greater responsiveness of marginal cost to size increases (low efficiency) the firm will remain small.

In other words, a small firm will have no incentive to exercise the ability to grow and as such will remain small, charging a lower price. The question might be asked as to why then is the small firm in the market or what role does it play? In reality, the wider populace is among middle to low income groups. It is therefore expected that faced with a particular budget constraint such economic agents can afford only the low quality product provided by the small firm. Given that this argument holds, then it can easily be seen that all firms will face positive demand and a small firm with its low quality will be able to capitalize on its distinct size, thereby holding a share of the market. Furthermore, the small firm may hold a larger share of the market relative to its larger, more expensive counterpart.

It is shown that large firms manage to make a larger profit afforded by the higher price charged, their greater efficiency and the lower response rate of marginal cost as firm size increases. The idea that the large firm is more efficient at producing quality than the smaller firm is obtained from A1 and is the final proposition proven in the paper. Intuitively, it shows that the cost effect of profit for the small firm is greater than the growth effect on profit such that the small firm has no incentive to grow even though its growth rate is greater than the growth of the large firm. In retrospect, the above discussion highlights that modeling a different scenario from the ones explored by Shaked and Sutton (1983) and Thomas (2010) yields the same maximal product differentiation results.

5. Conclusions

This paper shows that firms choose size (large or small) and they will maintain their decision throughout any stage of the game. Though small firms have the potential to grow faster than large firms they will not exercise that choice because of the greater efficiency of large firms to produce quality. It is shown that both large and small firms have no incentive to deviate from the size choice. This property ensures that the principle of maximal product differentiation is maintained. The model allows marginal cost to vary with size in a similar fashion to that done in Thomas (2010). An additional feature brought out in the model is the growth rate of firms. This feature also supports the principle of maximal product differentiation and highlights an interesting result. That is, a small firm for example will choose to supply low quality to grow fast. In fact, the payoff from remaining small outweighs the payoff from its growth potential since exercising this option is accompanied by heavy cost. As a result, with the assumption that firms have positive demand, each firm will benefit if either and/or both firms make a distinction in quality by distinguishing its size.

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