Development and Evaluation of an Instrument to Assess Prospective Teachers' Dispositions with Respect to Mathematics

James Beyers[a]
School of Education
The College of New Jersey
Ewing, NJ 08628, USA.
E-mail: Beyers@tcnj.edu

Abstract

Presented here is a survey for the assessment of prospective elementary teachers' dispositions with respect to mathematics. From a review of the literature, it was found that many, and at times inconsistent, operationalizations of the disposition construct were being used to examine student dispositions with respect to mathematics. Presented here is a conceptual framework composed of three modes of dispositional functioning, cognitive, affective, and conative, comprising ten subcategories of dispositional functions. The proposed framework offers consistency to language used in previous work, as well as a 'focused' lens for examining issues related to student dispositions. The proposed framework was used to guide the development of a survey, the Mathematics Dispositional Functions Inventory. Measures taken to strengthen the validity and reliability associated with the survey are reported. Results provide empirical support for the proposed ten-factor structure used in the instrument. Estimates of internal consistency reliability are also given.

1. Introduction

Dispositions with respect to mathematics, sometimes called mathematical dispositions (Macintosh, 1997; Moldavan & Mullis, 1998; NCTM, 1989, 2000; Royster, Harris, & Schoeps, 1999) or dispositions toward mathematics (NRC, 2001), are labels given to a construct which has had many varied, and at times inconsistent, conceptualizations and definitions in research literature (Boaler, 2002). Consequently, a comprehensive measure of the disposition construct has not been available. However, in order to be able to assess dispositions comprehensively, an improved, well-structured conceptual framework should be in place. One which can offer clarity to the field's conceptualization of the disposition construct as well as inform the development of an instrument which enables us to conduct such a comprehensive assessment. In response to this absence of such an instrument, I developed the Mathematics Dispositional Functions Inventory (MDFI[1]). There is a need for such an instrument because as research has shown, elements of student dispositions can affect their learning of mathematics content, in that, those dispositions can influence the way students may or may not take advantage of opportunities to learn mathematics (NRC, 2001). An additional benefit of a comprehensive measure of student dispositions such as the MDFI is that it can help researchers and educators to begin to understand the subtle nuances of the relationships between various elements of student dispositions and learning in mathematics.

As students, prospective elementary teachers' dispositions with respect to mathematics warrant considerable attention, as those dispositions can influence the nature of the mathematical knowledge they develop while enrolled in a teacher preparation program. For example, consider a prospective elementary teacher who tends to experience math anxiety when engaging mathematical tasks. He may avoid the perceived source of that anxiety and consequently not take advantage of the opportunity to learn mathematics offered by the task at hand. Or, consider a prospective elementary teacher who does not tend to try and make connections among mathematical ideas. She may not develop a deeper understanding of the conceptual underpinnings of those related mathematical topics. An instrument, which facilitates a comprehensive measurement of prospective teachers' dispositions, could enable teacher educators to explicitly address specific aspects of dispositional issues in mathematics content courses for teaching, which could, in turn, have the potential to support their learning of mathematics.

2. Motivating the Framework for the Instrument

In light of concerns with students' lack of comprehension in mathematics (Lee, Grigg, & Dion, 2007) increased attention has been given to students' dispositions with respect to mathematics and how they may influence learning of mathematics.

[1] For a complete copy of the instrument, please contact the author.

Several conceptualizations and operationalizations of the disposition construct have been utilized in order to help guide investigations regarding student dispositions in the context of mathematics.[2] Two umbrella terms, mathematical dispositions (Macintosh, 1997; Moldavan & Mullis, 1998; Royster, Harris, & Schoeps, 1999) and dispositions toward mathematics (Fernandez & Cannon, 2005; NRC, 2001), are commonly used in mathematics education literature to describe students' disposition with respect to mathematics. Even though similar labels are given to constructs, i.e., mathematical dispositions or dispositions toward mathematics, there are still a plethora of interpretations and operationalizations of what constitutes the disposition construct and its components. NCTM, in its *Curriculum and Evaluation Standards for School Mathematics* (1989), defines students' mathematical dispositions as "not simply attitudes but a tendency to think and to act in positive ways." (p. 233). This definition is very broad, and consequently, it is open to a wide variety of interpretations. Nonetheless, there are consistencies between the NCTM definition and other qualifications of students' mathematical dispositions (See, AUTHOR 2011 for a complete discussion of this). The two umbrella terms highlighted previously offer varying degrees of specificity in asserting what constitutes student dispositions in the context of mathematics. Consequently, substantially different operationalizations of the disposition construct exist.

To provide a common language and framework for discussing and organizing conceptualizations, I propose the categories of cognitive, affective, and conative mental functions. These categories are useful as they can serve as a lens for exploring the nature of students' dispositions in relation to mathematics that other conceptions do not. Perhaps consideration for elements of cognition, affection, and conation can help improve the clarity and structure of the field's conceptualizations and definitions of the dispositions construct.[3] For example, consider one feature of students' mathematical dispositions, 'discerning mathematically acceptable explanations' as described by Yackel and Cobb (1996). Although how someone might discern the mathematical acceptability of explanations could vary, e.g., employing heuristics to govern all comparisons or approaching each case as a unique comparison, the process of discerning mathematically acceptable explanations is arguably a mental function, which generates awareness, in the individual, of new knowledge. Thus, discerning mathematically acceptable explanations might be thought of as a cognitive mental function. Also, consider the prevalent references to students' attitudes toward and beliefs about mathematics as features of students' dispositions with respect to mathematics (e.g. NCTM, 1989, Royster, et al., 1999). Attitudes and beliefs, in the context of the affective domain, can be thought of as general reactions toward something, the essential quality of an emotion, feeling, mood, or temperament (McLeod, 1992), and thus, could be considered affective mental functions.

Lastly, consider the notion of persistence or diligence, each highlighted in different contexts as components of students' dispositions with respect to mathematics (NCTM, 1989; Kilpatrick, et al., 2001). Conative mental functions are described as "that aspect of mental process by which it tends to develop into something else; an intrinsic unrest of the organism…almost the opposite of homeostasis. [An impulse] to act, a conscious striving…It is now seldom used as a specific form of behavior, rather for an aspect found in all. Impulse, desire, volition, purposive striving all emphasize the conative aspect" (English & English, 1958, p. 104). Persistence and diligence both imply a directed, purposeful action, and consequently could be thought of as conative mental functions.

By considering features of students' dispositions with respect to mathematics as elements of cognitive, affective, or conative modes of mental functioning, one may be afforded a clearer, more systematic way for organizing and conceptualizing students' dispositions with respect to mathematics as dispositional cognitive, affective, or conative functions. Now, operating on the assumption that the proposed framework is at least useful for examining and discussing student dispositions in the context of mathematics, I will turn to the benefit of the instrument - Mathematics Dispositional Functions Inventory [MDFI]. A considerable body of literature attends to the examination of student dispositions with respect to mathematics (e.g., Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996; Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski, & Smith, 2000; Royster, et al., 1999; Yackel & Cobb, 1996).

[2] It should be noted that the term disposition has been widely used throughout both education and mathematics education literature in conjunction with both teachers and students; however, the intent of this discussion is to explore the dispositions of students. Moreover, a comprehensive discussion of all conceptualizations of students' dispositions, e.g. cultural and ethical dispositions (Schussler, 2005), is outside the scope of this paper, as those conceptualizations of students' dispositions do not specifically focus on the relationship between the student and the domain of mathematics.

[3] A comprehensive discussion of the terms introduced (and alluded to) here will occur in the conceptual framework portion of this paper.

This body of research contains a multitude of research methodologies employed to examine student dispositions. My intention is not to critique or review all of those methods, but simply to suggest that previous studies have not examined all categories of students' dispositional functioning with respect to mathematics simultaneously or comprehensively. Regardless of the methodology employed, e.g., observations by McClain and Cobb (2001) or via survey (Royster et al., 1999), it is apparent from the earlier discussion that previous operationalizations focus on isolated components of dispositional functions. It is not my assertion that these examples represent poor or deficient research; I offer, simply, that they do not capture all of the nuances of student dispositions with respect to mathematics. And it is my belief that examining student dispositions in a comprehensive manner can provide a substantial contribution to the progress of our field because we will be better equipped to explore and understand the many nuances of student dispositions.

One might also ask why not use an already developed instrument, e.g., the Fennema–Sherman Mathematics Attitudes Scales (1976) or the Mathematics Disposition Survey (Royster, et al., 1999), or some other instrument used in disposition related research. Simply stated, these instruments do not adequately capture the nuances of dispositions with respect to mathematics. The Fennema-Sherman Mathematics Attitude Scales (1976) were developed in 1976, and it has become one of the most popular instruments used in mathematics education research over the last three decades (Tapia & Marsh, 2004). The Fennema-Sherman Mathematics Attitude Scales consist of a group of nine instruments: (1) Attitude Toward Success in Mathematics Scale, (2) Mathematics as a Male Domain Scale, (3) and (4) Mother/Father Scale, (5) Teacher Scale, (6) Confidence in Learning Mathematics Scale, (7) Mathematics Anxiety Scale, (8) Effectance Motivation Scale in Mathematics, and (9) Mathematics Usefulness Scale. Upon examination of the scales it is clear that, although they were relevant to that investigation at the time, some are not relevant to this current study of student dispositions with respect to mathematics, e.g., (3), (4), and (5). While an examination of the items from the survey (Mathematical Disposition Survey) designed by Royster et al. (1999) shows that the instrument does not adequately reflect the complexity of student dispositions with respect to mathematics as demonstrated in the previous section regarding the researchers' operationalization of the disposition construct.[4]

Similarly, when looking at other instruments employed in research on issues related to students' dispositional functions, e.g., Kloosterman & Stage (1992), an examination of the items reveals that the instrument focuses on only a limited aspect of students' dispositional functioning. In this case, however, that is to be expected as the afore mentioned research (e.g., Kloosterman & Stage, 1992) is not explicitly attending to student dispositions, that study focuses on issues related to students attitudes and beliefs about mathematics, affective dispositional functions. When considering the intended focus of any study, which employs a survey or questionnaire, it is understandable that the instrument being used may only focus on some subset of student dispositions with respect to mathematics. However, the instrument described here, the MDFI, is intended to facilitate a comprehensive examination of student dispositions with respect to mathematics, a task not undertaken in previous research.

3. Conceptual Framework

I adopted a psychological perspective for the exploration of prospective elementary teachers' dispositions. Consequently, dispositions with respect to mathematics can be considered those cognitive, affective, and conative functions, which a student of mathematics is inclined to engage in a mathematical context (e.g., doing and/or learning mathematics, etc.). I will now describe these mental functions and their connection to dispositions in the context of mathematics.

3.1 Educational Psychology and Modes of Mental Functioning

Three modes of mental functioning, cognition, affect (also called affection), and conation (also called volition) have been used to distinguish three categories under which all mental processes are classified (Snow, Corno, & Jackson III, 1996). NCTM (1989) suggests that a mathematical disposition should be thought of as "not simply attitudes but a tendency to think and to act in positive ways." (p. 233). Considering the three modes of mental functioning together with NCTM's assertion, one could infer that there are dispositional cognitive, affective, and conative mental functions, which contribute to a student's mathematical disposition. Cognitive mental functions are defined to be "process[es] whereby an organism becomes aware or obtains knowledge of an object…It includes perceiving, recognizing, conceiving, judging, reasoning…[I]n modern usage sensing is usually included under cognition" (English and English, 1958, p. 92).

[4] Recall, Royster et al. (1999) focused on isolated components of students' affective dispositional functioning.

For the purposes of this discussion, a cognitive mental function is then considered dispositional, i.e., a *dispositional cognitive function* with respect to mathematics, if a person has a tendency or inclination to engage (or not) in a particular cognitive mental process associated with perceiving, recognizing, conceiving, judging, reasoning, and the like in mathematics. For example, if a student were learning about the standard algorithm for division of rational numbers, the student could be inclined to reason why the algorithm calls for multiplying by the reciprocal of the divisor. Another student, as many do, could simply accept the algorithm at face value and have no inclination to engage mathematical reasoning in order to understand how the algorithm works. Consequently, reasoning may be considered a dispositional cognitive function.

Affective mental functions are said to be "a class name for feeling, emotion, mood, temperament…a single feeling response to a particular object or idea…the general reaction to something liked or disliked…the dynamic or essential quality of an emotion; the energy of an emotion" (English & English, 1958, p. 15). McLeod (1992) suggests that attitudes toward mathematics, beliefs about mathematics as well as about one's self (in relation to mathematics), and emotions, e.g., joy or aesthetic responses to mathematics, reside within the affective domain. An affective mental function is said to be dispositional, i.e., a *dispositional affective function* with respect to mathematics, if a person has a tendency or inclination to have or experience particular attitudes, beliefs, feelings, emotions, moods, or temperaments with respect to mathematics. Albert and Haper (1960) suggest that students who experience debilitative math anxiety avoid mathematical tasks, and thus the perceived source of the anxiety. Such persons have a tendency to experience angst when engaged in mathematical activity, and consequently, the affective function of anxiety can be thought of as dispositional.

Conative mental functions are said to be "that aspect of mental process by which it tends to develop into something else; an intrinsic unrest of the organism…almost the opposite of homeostasis. [An impulse] to act, a conscious striving…It is now seldom used as a specific form of behavior, rather for an aspect found in all. Impulse, desire, volition, purposive striving all emphasize the conative aspect" (English & English, 1958, p. 104). A conative mental function is said to be dispositional, i.e., a *dispositional conative function*, if a person has a tendency or inclination to purposively strive, exercise diligence, effort, or persistence in the face of mathematical activity. As educators, we have experienced situations where students are faced with difficult mathematical tasks and seen different levels of student engagement in those tasks. Students may tend to exhibit high or low levels of persistence or effort, and be less likely to purposively strive in the face of challenging mathematical tasks, supporting the assertion that conative functions can be thought of as dispositional.

3.2 Categories and Subcategories of Dispositional Functions

The MDFI was designed to assess these three categories of dispositional functions under which subcategories of elements of student dispositions with respect to mathematics can be classified and it is composed of forced-response items. Within the *Cognitive Dispositional Functions* scale lie items designed to assess *connections* and *argumentation* dispositional functions. Within the *Affective Dispositional Functions* scale lie items designed to assess *nature of mathematics*, *usefulness*, *worthwhileness*, *sensibleness*, *mathematics self-concept*, *attitude*, and *math anxiety* dispositional functions. Because effort, persistence, etc. are considered synonymous, there is only one category of dispositional functions within the *Conative Dispositional Functions* scale. The three primary scales as well as the ten[5] subcategories of dispositional functions were designed using a rationale/correspondence method (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 1999), i.e., the construction of each of the scales and subsequent categories is based on the conceptual framework derived from my review of the literature, not through statistical means such as factor analysis. Table 1 below shows a description of each scale, the subcategories of dispositional functions, as well as sample items. The subcategories of dispositional functions are derived from various conceptions of the disposition construct found in relevant literature and are based on those described earlier in this paper.

Insert table (1) about here

3.3 Subcategories within cognitive dispositional functioning

The connections subcategory of dispositional functions is defined as a tendency to make connections within or across mathematical topics. Boaler (2002) suggests that some students could have extensive knowledge of multiple areas in mathematics, but not have a tendency to make any mathematical connections among those areas.

[5] The conative dispositional functioning scale is also being called a subcategory of dispositional functioning.

Noss, Neally, and Hoyles (1997) argue that mathematical knowledge can be derived from making mathematical connections. Given that some students may not be inclined to make mathematical connections and such connections can serve as a basis for the development of new mathematical knowledge, it is reasonable to suggest that making connections can be considered a cognitive dispositional function. The argumentation subcategory of dispositional functions is defined as a tendency to evaluate the mathematical correctness of statements, make mathematical arguments, and justify mathematical statements. McClain and Cobb (2001) operationalized the disposition construct as students' tendency to discern mathematically acceptable explanations from unacceptable explanations as well as a tendency to determine the mathematical basis and difference of why students' explanations or strategies. Although how someone might discern the mathematical acceptability of explanations could vary, e.g., employing heuristics to govern all comparisons or approaching each case as a unique comparison, the process of discerning mathematically acceptable explanations is arguably a mental function which generates an awareness, in the individual, of new knowledge, i.e., discerning mathematically acceptable explanations can be thought of as a dispositional cognitive function.

3.3.1 Subcategories within affective dispositional functioning.

The nature of mathematics subcategory of dispositional functions is defined as a tendency to believe that mathematics is composed primarily of unrelated procedures as opposed to being composed of related concepts. Researchers have described students' beliefs about the nature of mathematics as a continuum between a perception of mathematics as 1) a system of unrelated facts and procedures or 2) a system of connected concepts that can be constructed (Kloosterman, 2002; Kloosterman & Stage, 1992).

The usefulness, worthwhileness, sensibleness, and mathematics self-concept subcategories of dispositional functions are derived from the NRC (2001) definition of a productive disposition as well as an earlier interview study conducted by the researcher (AUTHOR, 2005) in which prospective teachers were asked about their beliefs about the usefulness, worthwhileness, and sensibleness of mathematics, as well as their beliefs about diligence in relation to learning and doing mathematics, and their mathematics self-concept. The usefulness subcategory of dispositional functioning is being defined as the tendency to believe that mathematics is useful for meeting current or future needs in or out of school, for your career, etc… The worthwhileness subcategory of dispositional functioning can be thought of as a tendency to believe that the work that the student has done to learn mathematics is worth it to the student, i.e., it is a value judgment made by the student about whether the 'payoff' for doing the work it takes in order to learn mathematics is ultimately worth it to them. The sensibleness subcategory of dispositional functions can be thought of as a tendency to believe that mathematics is composed of ideas that can be made sense of by the student.

The mathematics self-concept subcategory of dispositional functioning can be thought of as the student's tendency toward particular beliefs about himself as a learner of mathematics, i.e., whether the student is inclined to believe that he is able to learn mathematics successfully. The attitude subcategory of dispositional functioning is derived from McLeod (1992) and can be thought of as a student's inclination toward particular emotional reactions to mathematical activity in or out of school, e.g., like, hate. The last subcategory within affective dispositional functioning, math anxiety, is also derived from McLeod (1992) and can be thought of as whether the student tends to feel anxious in the face of mathematical activity.

4. Methods

4.1 Instrument Development Process

The instrument development process followed two phases: instrument formation and instrument validation. Figure 1 (below) shows the relevant activities for each phase of instrument development.

Insert figure (2) about here

4.2 Phase 1: Instrument formation

This phase had five components: 1) interviewing prospective teachers who were demographically similar to those that would ultimately participate in the current study, 2) performing a review of relevant literature[6], 3) generating items, and 4) evaluating of proposed items by colleagues in mathematics education and educational measurement, as well as, 5) pilot testing the draft of the instrument.

[6] The review of the literature occurred concurrently with phases one and three.

Prior to completion of the theoretical framework, an interview study was conducted (AUTHOR, 2005), with N = 9 participants. The purpose of this study was two-fold: to explore possible relationships between the dispositions of prospective teachers and their performance in mathematics, and to gather information about the ways that prospective teachers think and talk about their cognitive, affective, and conative dispositional functions in order to inform the development of items for the questionnaire. The specific language used and ideas expressed by the interviewees captured in transcriptions informed the construction of relevant items to be included in later phases of instrument development. The language and themes included in several items were chosen to ensure that they were germane to specific issues raised by prospective teachers. For example, transcripts of respondents' interviews revealed that students' dispositions were dependent on the level of mathematics content being discussed. For example, when asked about how mathematics was useful for them, participants indicated that mathematics learned at the elementary school level was more useful to them in a pragmatic sense, e.g., making calculations or measurements, than the mathematics learned at the secondary level (AUTHOR, 2005). In addition, respondents indicated that they believed the mathematics they are learning in their teacher preparation program would be useful for meeting their professional or career needs than mathematics learned in earlier grades, while the mathematics learned in high school would not be useful in that sense.

In order to develop items for a questionnaire designed to assess student dispositions in the context of mathematics, a review of the relevant literature concerning mathematical dispositions (NCTM, 2000) and dispositions toward mathematics (NRC, 2001) was conducted. From this review, the theoretical framework was developed, which includes the three primary categories of dispositional functions outlined earlier, i.e., cognitive, affective, and conative dispositional functions. Dispositional functions in the context of mathematics were consequently defined to be the composition of those cognitive, affective, and conative functions, which a student of mathematics tends, or is inclined, to engage or espouse habitually in a mathematical context. Within the cognitive scale lie two subcategories, connections and argumentation, and within the affective scale lie seven subcategories of dispositional functions, nature of mathematics, usefulness, worthwhileness, sensibleness, mathematics self-concept, attitude, and anxiety. For the purposes of this study, effort, persistence, etc. are considered synonymous, and consequently, there are no subcategories of dispositional functions within the conative scale.

Each of the ten categories of dispositional functions was explicitly defined prior to item construction. Item generation was done in consideration of the theoretical framework as well as the interview study (AUTHOR, 2005). Consequently, for all of the subcategories of dispositional functions a pool of approximately 80 items, inspired by relevant literature outlined in the "Categories and Subcategories…" section above, and the interview study, was generated. All items were coded along a five-point response protocol (agree, somewhat agree, neither agree nor disagree, somewhat disagree, or disagree) to ensure higher reliability, as the use of five to seven response categories generally results in higher reliability (Streiner & Norman, 1995).

The initial pool of items was examined for face and content validity over the course of several 90-minute seminars attended by mathematics education faculty and doctoral students and several one-on-one meetings with educational measurement faculty and doctoral students. Persons included in this phase of the process were asked to evaluate items for relevance, completeness, wording, and appearance bias. In addition, they were asked to examine accuracy and completeness of items in relation to the appropriate category of dispositional functioning, i.e., they were asked to assess whether the items associated with each category of dispositional functions accurately and completely reflect the meaning the relevant construct. Finally, to ensure that all dimensions of each construct were adequately represented, readers were asked to suggest additional and/or alternative items related to the constructs. An analysis of feedback led to changes in wording for some items to ensure greater clarity, removal of phrases that made some items have multiple questions, as well as the deletion of redundant items (See Table 2 below for examples of typical revisions). However, content analysis of their suggested items did not result in the addition of any new items. This phase led to a pool of 59 items deemed adequate for pilot testing.

Insert table (2) about here

The draft of the 59-item instrument was pilot tested with a convenience sample of 22 prospective teachers. The objectives of the pilot testing were to further establish content validity for the instrument and to verify interpretability and completeness of the instructions, items, and scaling format (the 5 levels of agreement).

To further establish content validity for the instrument, the 22 participants were given questionnaires instructing them to explain why they chose the responses they gave to various items on the survey, to identify any items that were problematic for them, and to explain why they were problematic. In addition, internal consistency reliability estimates for each scale were found using Chronbach's alpha. The pilot study resulted in minor revisions of the wording for three items, either misinterpreted or found to be unclear by some participants as indicated by the explanations given on their questionnaires. The example revision seen in Table 3 below is typical of the revisions made.

Insert table (3) about here

4.3 Phase 2: Instrument validation

Statistical methods. For the current study, there were N = 107 out of a possible 137 (78%) participants. Data were computed using the software package Statistical Package for Social Sciences, SPSS for Windows. When computing, scores for negatively worded items were reversed so that all items were scored in the same direction. Cronbach's alpha was used to estimate the internal consistency of each of the scales as well as the MDFI as a whole. In addition, reliability estimates were calculated if items were deleted. Confirmatory factor analysis was used to examine the three-factor model to determine whether the three dispositional functions scales could be discriminated from each other. Maximum likelihood was used as the estimation method and all analyses were conducted on covariance matrices. Listwise deletion of missing data was used to determine covariance matrices.

In evaluating the goodness of fit of the model the researcher followed previous recommendations (Hoyle & Panter 1995; Hu & Bentler, 1995; Jaccard & Wan, 1996) and used multiple indices of fit. More specifically, in addition to reporting the chi-square test statistic, the researcher reported the Goodness-of-fit index (GFI), and the Tucker-Lewis index (TLI). Each of these indices evaluates the fit of the model slightly differently (see Hu & Bentler, 1995) and therefore an indication of good fit from these various indices increases the confidence in the model. The critical value under which a model is considered to have a questionable fit recommended for last two indices is .90, while a chi-squared statistic between .70 and .80 represents a "good" fit. Responses were subjected to a factor analysis using the maximum likelihood method of extraction and a varimax, orthogonal, rotation.

5. Results

5.1 Confirmatory Factor Analysis

Items from each of the ten subcategories of dispositional functioning are hypothesized to load on only on their respective latent variables, i.e., on one of the ten categories of dispositional functioning. The fit for this model was $\chi^2 = 3818.34$ ($df = 1711$, N = 107), $p < .001$; GFI = .92; TLI = .91. The proposed ten-factor model accounted for 59.7% of the variation (see Table 4 below).

Insert table (4) about here

The confirmatory factor analysis, therefore, provides some empirical support for the proposed theoretical structure of students' dispositions with respect to mathematics as measured by the MDFI. However, an examination of the cross-loadings of items on factors indicates that some items load on more than one factor.[7] Upon further analysis of the items, which load on multiple factors, it seems that the wording of some of the items suggests that the items are inadvertently assessing multiple ideas (See Table 5 below). Table 5 below shows examples of a item, which could be assessing more than one idea, which in turn could explain why an item is loading on multiple factors. A point for future consideration is whether the proposed wording (seen below) would reduce the likelihood that such items would load on multiple factors. For example, the first item given in Table 5 is assessing a belief about the connectedness of mathematics and whether mathematics is sensible. The respondents could be indicating agreement with either notion, but not necessarily both, leading the item to load on multiple factors. Additional revisions have been made based on the confirmatory factor analysis and can also be found in Table 5 below. The revised items will be included in the MDFI and a follow-up study with a larger population will be conducted to further corroborate the effectiveness of the instrument.

Insert table (5) about here

5.2 Internal consistency reliability

A measure of internal consistency for each of the three primary scales and the MDFI were calculated using Cronbach's alpha (See Table 6 below).

[7] For a complete view of the factor loadings for specific items, please contact the author.

In addition, internal consistency reliability was determined if each item were deleted. The corrected item-total correlations did not improve for any item being deleted.

Insert table (6) about here

6. Discussion

As noted earlier, several varied and at times inconsistent conceptualizations and operationalizations of the disposition construct have occurred throughout the literature (e.g., Rosyter, Harris, & Schoeps, 1999; NCTM, 1989; NRC, 2001). The conceptual framework forwarded here offers three categories of dispositional functioning, cognitive, affective, and conative, as an attempt bring consistency to the language and interpretations of the disposition construct in the context of mathematics. A substantial body of research exists suggesting that students' dispositions are an integral factor influencing the way students engage in mathematical activities. However, there are opportunities for additional work to be done explicating the nature of students' dispositions and its impact on students' mathematical activity, because even though the same labels are given to constructs, i.e., mathematical dispositions or dispositions toward mathematics, there is still a wide range of interpretations and operationalizations of what constitutes the disposition construct and its components. The framework discussed here offers some guidance toward furthering our collective understanding of students' dispositions in relation to mathematics, in that it helps us to develop a shared sense of what is meant by students' dispositions as well as a systematic way to map out the vast domain of students' dispositions with respect to mathematics.

It is useful to organize elements of the disposition construct identified in the literature, in terms of the cognitive, affective, or conative modes of mental functioning. This categorization can be useful as it can help provide a common language and framework for discussing and organizing conceptualizations as well as a lens for exploring the nature of students' dispositions in relation to mathematics. The conceptual framework also enabled the development of a comprehensive assessment of student dispositions with respect to mathematics, namely the Mathematics Dispositional Functions Inventory (MDFI), which facilitates the comprehensive assessment of student dispositions and allows for consideration of the relative contributions of various components of student dispositions to the relationship with achievement in mathematics. The confirmatory factor analysis provides empirical support for the proposed ten-factor model for assessing student dispositions in the context of mathematics as captured in the MDFI. Multiple indices support the use of factor analysis comprising the ten categories of dispositional functioning in the MDFI.

The MDFI, designed in light of the conceptual framework forwarded here, contains three primary scales capturing the many nuances of student dispositions in terms of the 10 subcategories of dispositional functioning: argumentation, connections, nature of mathematics, usefulness, worthwhileness, sensibleness, attitudes, math anxiety, mathematics self-concept, and conation. Use of reliability estimates suggests that there is an acceptable degree of internal consistency overall within the MDFI as well as two of its primary scales, cognitive and affective dispositional functioning. However, the third scale, conative dispositional functioning did not demonstrate such high internal consistency. There are at least two plausible explanations for this: the scale was not long enough (only 5 items) or the nature of the items was focused on too many ideas. The intent of the conative dispositional functioning scale was to assess the degree to which students felt effort and persistence mattered when they were engaged in mathematical activity. However, the items assessed the degree to which the participants valued persistence or effort at various levels of their educational experience with mathematics. So it is possible that although students valued effort at the high school or collegiate level, they may not have valued it at the elementary level.

Any permutation of disparate values about the role of effort or persistence within a respondent's choices related to various levels of their education could result in lower internal consistency. For example, they could strongly agree about the importance of effort at the high school, but not at the elementary level, because perhaps the mathematical activities were more difficult for them and required that they exert more effort. Generally speaking, longer assessments are more reliable (Wiersma, 1995), therefore, a point for future consideration specifically about the MDFI is whether multiple items assessing the degree to which students value effort or persistence might result in a higher level of internal consistency.

6.2 Future considerations and uses for the MDFI

The MDFI assesses student dispositions with respect to mathematics along three dimensions.

The MDFI allows for exploration of many relationships, like between students' cognitive dispositional functioning and problem solving behaviors, or students' dispositions toward mathematics (NRC, 2001) and achievement in mathematics. Or consideration for issues associated with the stability of student dispositions over time. With the MDFI and the resultant subcategories of student dispositions easily identifiable, research can be conducted examining how particular aspects of student dispositions may evolve or change over time as a result of a targeted intervention directed toward effecting change or simply as a consequence of students' collective experiences in school. Many new opportunities to explore the nature and development of students' dispositions with respect to mathematics are now upon us in light of the MDFI.

Future research should not necessarily be limited to research on prospective elementary teachers' dispositions with respect to mathematics. Although the current study the MDFI was used for focused on prospective teachers, the content of the MDFI can be applied to different populations of mathematics students, such as prospective secondary mathematics teachers, any post-secondary students taking mathematics courses, or even high-school students taking mathematics courses. The potential to explore the nature of other students' dispositions with respect to mathematics is possible, because as indicated earlier, the MDFI was designed in light of the literature on student dispositions, so these issues are relevant not only to prospective elementary teachers, but also any student of mathematics. On example of a future study could include an examination of how students' disposition evolve or varying periods of time, addressing the stability issue associated with students; dispositions with respect to mathematics. Administering the MDFI over multiple periods of time, to the same population could offer some insights into addressing that question.

6.3 Limitations

The MDFI does contain some language specific to mathematics content, which could pose a problem to some potential populations of interest, such as elementary-aged students or ESL learners. Also, some of the items are related to experiences that occurred in relation to mathematics courses taken at the post-secondary level, such as math content course taken in a teacher preparation program. However, it seems that it may be acceptable to reword those items to reflect the current experiences of the population of interest. This issue should be approached with caution, though, as it is not clear how changing items in such a manner could affect the overall reliability of the instrument. Also, if one were interested in investigation involving dispositions of elementary-aged students, such investigations should be approached with some degree of caution, as it is not clear how accessible the wording of items would be to those students. Perhaps, the survey items would need to be read to students in order to ensure there are no issues associated with reading level. It is possible that students at any level whose primary language is not English (ESL) could experience some difficulties in taking this survey, as some of the vocabulary is specific to mathematics. Furthermore, issues associated with reading levels, could become particularly relevant as it is not entirely clear whether the vocabulary (even the non-mathematical 'jargon') is accessible to all potential populations of interest.

It is the belief of the author that, although many of the 10 subcategories of dispositional functions demonstrated high levels of internal consistency (not reported here, but available upon request to the author), the instrument should not be dismantled to the extent that only a single subset of categories of dispositional functions is used without further evidence that such a parceling would be empirically sound. However, one possible consideration for future research is to combine a subset of the ten subcategories of dispositional functions, worthwhileness, sensibleness, usefulness, mathematics self-concept, and conation, the author's operationalization of the elements of a 'productive' disposition toward mathematics as outlined by the NRC (2001).

6.4 Conclusions

The Mathematics Dispositional Functions Inventory provides researchers an instrument, which now allows for a comprehensive examination of student dispositions with respect to mathematics. The development of the instrument was a consequence of a proposed conceptual framework for students' dispositions with respect to mathematics in terms of dispositional cognitive, affective, and conative functioning. The field employed many, and at times, inconsistent operationalizations of the disposition construct, which perhaps resulted from a 'fuzzy' or imprecise notion of what mathematical dispositions comprise. Nonetheless, a well-defined notion of the disposition construct was lacking (Boaler, 2002). It seems that the framework, as well as the MDFI being forwarded here, are useful devices which can provide the field a point of departure toward a better and shared sense of what constitutes students' dispositions with respect to mathematics, enabling us to continue developing and strengthening our understanding of the students' dispositions.

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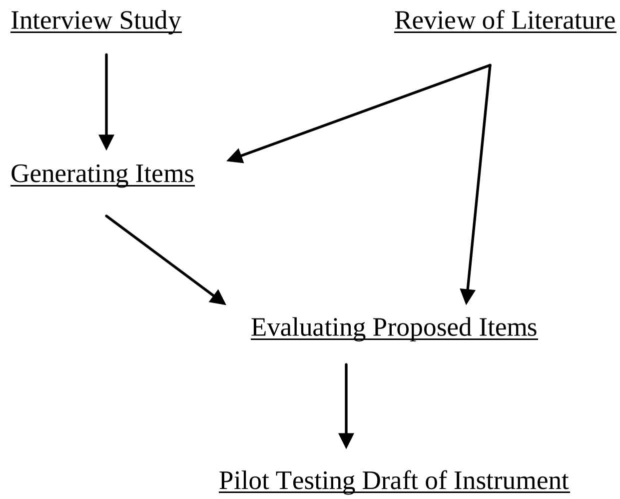
Table 1. Descriptions of the categories (and subcategories) of dispositional functions with a sample item.

Scale	Subcategory of dispositional function	Description of Subcategory	Sample Item
Cognitive	Connections	A tendency to try and connect ideas with or across mathematical topics.	In general, I try to see how mathematical ideas in different math classes are connected to each other.
	Argumentation	A tendency to evaluate the mathematical correctness of statements, make mathematical arguments, justify mathematical statements, etc.	Even if I'm not asked to, I try to develop and evaluate mathematical arguments to explain things in math classes.
Affective	Nature of Mathematics	A belief about mathematics being more procedural or conceptual in nature.	In general, mathematics is made up of procedures and algorithms.
	Usefulness	A belief about the usefulness of mathematics for meeting current or future needs in or out of school, for your career, etc.	I need to learn math because, if I want to be a teacher, I need to know math.
	Worthwhileness	A value judgment that the work put forth in learning mathematics has been worth it to the student.	All the work I have had to put into learning math has been worth it to me.
	Sensibleness	A belief that mathematics is composed of ideas that can be made sense of.	In general, math is a connected system that can be made sense of.
	Mathematics Self-Concept	What the student believes about him or herself as a learner of mathematics.	In general, math is too challenging for me to really understand it well.
	Attitude	The respondent's emotional reactions to mathematical activity in or out of school, e.g., like, hate, etc	I like doing math in school.
	Math Anxiety	Whether or not the student experiences anxiety in relation to mathematics.	In general, I get stressed out when I have to take a math test.
Conative	Effort/Persistence	A tendency to persist or exert effort if necessary.	If someone is having difficulties in math, they can eventually do well if they persist.

Figure 1. Shows the phases of the development of the instrument.

Phase 1: Instrument formation



Phase 2

Instrument Validation

Table 2. Shows how some items were revised in light of feedback from colleagues.

Before	**Feedback**	**After**
When I am stuck on a math problem, I look for similar problems that have a solution and follow the steps.	The introductory phrase is distracting and the response to the item shouldn't depend on whether the student is "stuck." "Similar problems that have a solution" doesn't necessarily communicate worked out sample solutions (the intention of the phrase).	When I'm doing a math problem, I look for solutions to similar problems and follow the steps from those solutions to find an answer to my problem.
Learning basic math skills is more important than doing problems.	What do you mean by basic math skills? 'Basic' to some may be different than 'basic' to others. Important? 'Doing problems' is too ambiguous.	In general, learning computational skills, like addition and multiplication, is more useful to me than learning to solve math problems.

Table 3. Shows typical revision based on the feedback of participants in the pilot study.

Before	Feedback	After
Even if I'm not asked to, I try to develop arguments to explain things in mathematics.	I disagreed because I am not one to argue in class.	Even if I'm not asked to, I try to develop *mathematical* arguments to explain things in mathematics.

Table 4. Percentage of variance accounted for by the proposed ten-factor model.

Rotation Sums of Squared Loadings

Factor	Eigen Values	% of Variance	Cumulative % of Variance
1	9.183	15.564	15.564
2	4.473	7.581	23.145
3	3.350	5.677	28.822
4	3.286	5.570	34.393
5	3.249	5.507	39.900
6	2.813	4.768	44.668
7	2.574	4.363	49.031
8	2.561	4.341	53.372
9	1.888	3.201	56.573
10	1.848	3.132	59.704

Table 5. Examples of problematic items and proposed revisions.

Current Phrasing of Item	Proposed Wording for future consideration
In general, math is a *connected system* that *can be made sense of*.	In general, I can make sense of mathematical ideas
Even if I'm not asked to, I try to *make and investigate* mathematical conjectures in math classes.*	Even if I'm not asked to, I try to make mathematical conjectures in math classes. Even if I'm not asked to, I try to investigate mathematical conjectures in math classes.
Even if I'm not asked to, I try to *develop and evaluate* mathematical arguments *to explain things* in math classes.*	Even if I'm not asked to, I try to develop mathematical arguments in math classes. Even if I'm not asked to, I try to evaluate mathematical arguments in math classes.
In general, people *don't need to learn* mathematics beyond basic arithmetic.	In general, mathematics beyond basic arithmetic is not really useful for people.

Note: The italicized portion of the item represents the possible point of contention, which may have led the item to load on multiple factors.
*Indicates an item that may be assessing multiple notions within the appropriate category of dispositional functioning. Consequently, the item was separated into two items within that category.

Table 6. Internal consistency reliability estimates using Cronbach's alpha.
Estimates of Internal Consistency Reliability Using Cronbach's alpha

MDFI	$\alpha = .938$
Cognitive Dispositional Functioning	$\alpha = .852$
Affective Dispositional Functioning	$\alpha = .911$
Conative Dispositional Functioning	$\alpha = .552$