Estimation of one- and two-factor Vasicek term structure model of interest rates for the West African Economic and Monetary Union countries

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ABSTRACT

Term structure of interest rates has played an important role in the pricing of fixed-income securities. In this paper, we compare a one-factor with a two-factor Vasicek model of the term structure of interests. It is assumed that default-free discount bonds are determined by two factors that follow a joint Ornstein-Uhlenbeck process: the foreign interest rate and the spread between the domestic, proxied by the 7-day weighted interbank rate and foreign interest rates, represented by the Euribor 3-month rate. Assuming that the domestic interest rate is the sum of the spread and the foreign interest rate, a domestic bond pricing equation is derived and term structure interest rate is obtained. Empirical evidence of the model's performance in comparison with a one-factor Vasicek model is presented. The results show that the two-factor Vasicek does not perform better than the one-factor Vasicek model in predicting future movements of domestic interest rates.

Keywords: Interest rate term structure, one- and two-factor Vasicek model, equilibrium model, bond pricing, martingales, country risk.

1. Introduction

Since the beginning of the 1980s, the West African Monetary and Economic Union (WAEMU) which includes eight countries: Benin, Burkina Faso, Côte d'Ivoire, Guinea-Bissau, Mali, Senegal and Togo, and whose monetary institution is the Central Bank of West African States (BCEAO), gradually abolished restrictions on capital movements, lifted the administration of interest rates, bringing them closer to more liberal financial systems.

These different changes led to two major movements: greater integration of national financial markets and integration of the latter into international capital markets. These structural changes were not only supposed to increase the efficiency of the financial system in allocating resources and the efficiency of financial markets through competition and stronger substitutability between financial assets, but also improve the transmission channels of monetary policy.

These structural upheavals naturally increased the influence of external factors on domestic economic variables and the financial system.

The increasing integration of the international economy has made domestic macroeconomic performance more sensitive to foreign shocks. This is particularly true for domestic financial markets, given greater degree of integration that prevails in international asset markets compared to those for goods and services.

The yield curve, or the term structure of the interest rates, describes the relationship between the yield of a bond and its maturity. It is the most important concept in pricing all the fixed income securities and interest rate derivatives such as bond options, caps, swaps etc, whose payoffs are strongly dependent on the interest rates.

Its study is of great practical importance. It reveals agents' anticipations of the risks to come. Modelling future deformations of the yield curve is a major challenge in many areas of finance, both for managing the interest rate risks affecting banks' balance sheets, and for evaluating and hedging many financial products, especially financial products derivative assets, which the markets use to deal with interest rate and exchange rate risk. In particular, understanding the distortions of the yield curve makes it possible to establish a cash management strategy (choice of investment period, speculation on the structure of rates, hedging elements). The increased volatility of interest rates technically makes any progress in the direction of greater control of these problems very important.

Another reason is the debt policy. A good knowledge of the term structure of interest rates is a necessary condition for the development of a public debt management strategy, including the choice of the maturity of this debt. Indeed, governments, ors of new bond issues, need to know the yield curve to decide the maturity of bonds.

Current rates contain information about the future development of the economy (on this point one can consult the summary work of (Melino(1986)). Thus, the term structure of interest rates reflects market expectations concerning the future development of inflation and real interest rates. Knowledge of these expectations is important for the implementation of monetary policy. The short-term interest rate has become the preferred instrument of conduct of monetary policy. However, the decisions of economic agents depend much more on long-term interest rates. Knowledge of the yield curve can in this case help to understand the transmission of fluctuations from short- to long-term rates, and thus to better understand the effects of monetary policy on real economic activity. For example, there is a lot of interest in yield curve research in central banks, work that has demonstrated the existence of empirical links between the yield curve and fundamental economic data.

As a result, research on how to estimate a fitted yield curve has become a very important issue and captures both the academic and practical interests. Not surprisingly therefore, a great deal of effort has gone into studying the term structure of interest rates. The literature concerning the modelling of interest rates is abundant and diverse. It is one of the subjects most often treated and it is still very topical.

Although the literature on empirical tests of either single- or multi-factor is vast, most empirical studies have been conducted on developed countries data.

Despite the multiple applications of this curve, we note its non-existence in several African economic spaces. In the literature, the works relating to the construction of a risk-free rate curve in an African country are those of Moungala(2013) and Gbongue(2015) who use the models of Nelson and Siegel(1987) and Diebold and Li(2006). We can also cite Aling and Hassan(2011) and Muteba Mwanba et al.(2014) who use the CKLS model of Chan et al.(1992) to model the South African yield curve.

To our knowledge there is no paper that estimates this model for WAEMU countries with the proposed methodology. We intend to partially fill in this gap with data.

In this paper we calibrate a one-factor and a two-factor Vasicek model for WAEMU countries.

This work shall be structured as follows: the following section will be devoted to literature review. Then, in the next section, we shall present the theoretical model on which our analytical framework is based. Finally, we shall expose the results obtained in the last section.

2. Review of literature

Practitioners prefer an approach that is accessible, straightforward to implement and as accurate as possible. In general, there are two distinct approaches to estimate the yield curve of the interest rates.

The literature in the area of interest rate modelling is voluminous. One strand of the literature reviews historical interest rate movements in an attempt to determine general characteristics of plausible interest rate scenarios. Ahlgrim, D'Arcy, and Gorvett (1999) review historical interest rate movements from 1953 to 1999, summarizing the key elements of these movements. Chapman and Pearson (2001) provide a similar review of history in an attempt to assess what is known about interest rate movements (or at least what is commonly accepted) and what is unknown (or unknowable). Litterman and Scheinkman (1991) use principal component analysis to isolate the most important factors driving movements of the entire term structure. Some of the findings of these studies include:

- Short-term interest rates are more volatile than long-term rates.
- Interest rates appear to revert to some "average" level. For example, when interest rates are high, there is a tendency for rates to subsequently fall.
- While interest rate movements are complex, 99% of the total variation in the term structure can be explained by three basic shifts. Chapman and Pearson(2001) confirm that these three factors are persistent over different time periods.
- Volatility of interest rates is related to the level of the short-term interest rate.

Several popular models have been proposed to incorporate some of the characteristics of historical interest rate movements. The first approach includes the models of fitting the yield curves to the market data using statistical methods. The main purpose of these models is to find a smooth function between yields of bond prices and time to maturity. One of the yield curve estimation methods is the Bootstrap initiated by Bliss and Fama(1987) from discrete spot rates to fit a smooth and continuous yield curve to the market data. However, various curve fitting spline methods have been introduced. The most popular example of these procedures is the seminal work of McCulloch (1971,1975) which focused on estimating zero-coupon yields and discount factors using the polynomial splines.
He found that the discount function could be fitted very well by cubic or higher order splines and the estimated forward rates are a smooth function. Vasicek and Fong (1982) try to use a third order exponential spline to calibrate the discount function and show that these models have a better fitting performance than the polynomial splines models. Then, Nelson and Siegel(1987) and Svensson(1994,1996) suggested parametric curves that are flexible enough to describe a whole family of the observed yield curve shapes. However, this approach takes a static view targeting solely the shape of the yield curve of the interest rates. Models of this kind are usually not used for pricing derivatives since the models are not in line with the no-arbitrage principle.

The models in the second category are arbitrage-free models that are widely used for valuing derivatives and constructing hedging strategies. Of this category, the arbitrage-free models can also be approached by equilibrium and no-arbitrage principles. The essential difference between equilibrium and no-arbitrage models is that yields curve is an output from equilibrium model while it is an input to no-arbitrage ones.

The first one-factor models were those of Merton(1973), Cox(1975) and Cox and Ross(1976). This work marked the beginning of the application of stochastic processes to interest rates. These articles model the short-term interest rate. Despite their importance, these models were unrealistic when considering the term structures they could generate. Indeed, these models did not take into account the mean reversion phenomenon, a characteristic usually observed at the level of interest rates. This shortcoming was later corrected by Vasicek(1977) who proposed to use the Ornstein-Uhlenbeck process, which has the mean reversion characteristic, to model the short-term rate. A little later Cox, Ingersoll and Ross(1985) proposed a model using a square root process which ensures that the rates do not become negative. The first multifactorial model of equilibrium to emerge was that of Richard(1978). This model proposed to model the term structure of interest rates using the short-term rate and the rate of inflation. Then came the model of Brennan and Schwartz (1979), in which the dynamics of interest rates are modelled using the short-term rate and the long-term rate. Balduzzi et al. (1996) use short-term rate and stochastic mean. In the same line of multifactorial models came the models of Longstaff and Schwartz (1992), Schaefer and Schwartz(1984), Andersen and Lund (1997), with the level and conditional volatility of short-term rates as factors.

One of the primary advantages of equilibrium models is that bond prices and many other interest rate contingent claims have closed-form analytic solutions. Therefore, given a realized value for $r(t)$, rates of all maturities can be obtained.

The major benefit of these models is to provide a link between intertemporal asset pricing theory and the term structure of the interest rates that produces a frequently convenient closed form of solution for asset prices. However, all these models generally imply a term structure of the interest rates conflicting with the market yield curve. In fact, the drift and volatility of the interest rates and the market price of risk are considered as the single source of uncertainty to determine the dynamics of the term structure. This problem is solved by Ho and Lee(1986) and Hull and White(1990) who used different information to characterize the yield curve dynamics. The information set includes the spot interest rate, volatility and the functional form of the yield curve. The model of Heath, Jarrow and Morton(1992) also allows capturing the full dynamics of the entire yield curve in an arbitrage free framework. This is an extension of one factor model developed by Ho and Lee(1986) at a multi factor model by considering forward rates rather than bond prices. Hull and White (1994) presented a two-factor model that did not allow for arbitrage. The two factors in question are the short-term rate and an undetermined stochastic factor that can be seen as some kind of risk premium. They showed that a two-factor model carries additional information about the term structure and leads to better pricing and hedging performance compared to a single factor model, which only uses the level of the short rate.

Empirical research on the term structure models generally suggests that multi-factor interest rate models perform much better than single-factor models. Dai and Singleton (2000) show a substantial improvement in data fit offered by multi-factor models.

Unfortunately, the various models discussed above ignore the interactions of interest rates between countries. Frachot(1996) is the first to examine joint term structure dynamics in a one factor model setting. Lund (1999) estimates a two-factor model for EMU yield curves. Brand and Santa-Clara (2002) incorporate the exchange dynamics in the estimation procedure. The motivation for this type of models lies in the well documented observation that yields are correlated across countries. Corzo and Schwartz (2000) proposed a short-term rate model for a country before adopting Euro currency, which is based on the Vasicek model. The evolution of the European short-term rate is given by the one-factor Vasicek model. The domestic short rate is a similar process, but the drift depends on the current level of the European rate.

This model can be shown to be a special case of another published term structure model, a two-factor model described by Hull and White (1994). Svensson (1991) argued that the interest rate differential between two countries plays an important role in describing the term structure of interest rates for a country since it conveys some information on monetary policies and business conditions in these two countries. A developing country with an external borrowing constraint, rather than being able to borrow (or lend) at the World interest rate, faces an upward-sloping supply function of debt which measures cost, or risk, associated with lending to the economy. This assumption was first used by Bardhan (1967) and has received application by Bhandari et al.(1990). Along these lines Shoji(1994) propose a two-factor equilibrium Vasicek model for an economically dependent country where the determinant of interest rate in the leader country depends on its own economic conditions, while the determinant in the dependent country (or small open economy) will depend on the interest rate of the leader country as well.

The different countries that make up the WAEMU monetary zone can be considered, from a theoretical point of view, as small, open, dependent economies. This is why the analytical framework of Shoji (1994) seems to us well suited for this area.

This study presents and estimates a model to account for the dynamic interdependencies of term structures of interest rates across countries, extending the standard single-country term structure models.

3 Theoretical backgrounds

A default-free zero-coupon bond with maturity date T and face value 1 is a claim that has a non-random payoff of 1 for sure at time T and no other payoff before maturity. Let $P(t,T)$ be price of a zero-coupon bond with maturity date T at time t, with $0 \leq t \leq T$. We can write

$$P(t,T) = \exp(-R(t,T)\,(T-t)) \qquad (1)$$

Where $R(t,T)$ is the continuously compounded yield on this bond:

$$R(t,T) = -\frac{\log(P(t,T))}{\mathrm{T-t}} \qquad (2)$$

The zero-coupon yield curve or term structure of interest rates at time $t \geq 0$ is the function

$$\tau \to R(t,t+\tau)$$

The yield curve makes bonds of different times to maturity comparable with regard to their yields. Generally, a larger yield is expected for a longer investment horizon. However, this is not always the case. The instantaneous interest rate at time t, $r(t)$is defined as:

$$r(t) = \lim_{\mathrm{T}\to \mathrm{t}} -\frac{\log(P(t,T))}{\mathrm{T-t}}$$

Denote by $B(t)$ the value at time t of a bank deposit or money account. Assume that the instantaneous return from this bank deposit is time-varying but deterministic:

$$\frac{dB(t)}{B(t)} = \mathrm{r(t)dt}$$

Integrating the above equation and then taking the exponent we get

$$B(t) = B(0)\ \exp\left(\int_0^t r(s)ds\right)$$

The Fundamental Theorem of Finance states that under no arbitrage condition, there exists an equivalent martingale measure (risk-neutral) $\mathbb{Q}$ under which any security prices scaled by money market account are a martingale process. Such measure is unique if the market is both no arbitrage and complete (see Harrison and Kreps(1979), Duffie(1996)). According to the theorem, $P(t,T)/B(t)$ is a $\mathbb{Q}$-martingale:

$$\frac{P(t,T)}{B(t)} = E_{\mathbb{Q}}\left(\frac{P(T,T)}{B(T)}\right)$$

Where $E_{\mathbb{Q}}$ represents the conditional expectation under risk-neutral measure $\mathbb{Q}$. Using the bank deposit definition,the risk-neutral price of a zero-coupon bond with maturity date T and face value 1 is given by

$P(t,T) = E_{\mathbb{Q}}\left[exp\left(-\int_t^T r(s)ds\right)\right]$ (3)

The above equation determines the zero-coupon price completely. It tells that prices of zero-coupons only depend on the distribution of the short rate $r(t)$ under $\mathbb{Q}$. From all bond prices at a given time t, $P(t,T)$, one can reconstruct the whole zero-coupon interest-rate curve at the same time t, $R(t,T)$, so that the evolution of the whole curve is characterized by the evolution of the single quantity $r(t)$.

The Vasicek model is the interest rate model that allows the most explicit analytical analysis. We have therefore chosen to consider the one- and two-factor versions of it in this section.

3.1 The one-factor Vasicek model

A short-rate model for the term structure of interest rates is based on the assumption of a specific dynamics for the instantaneous spot-rate process $r(t)$. The Vasicek model introduced in Vasicek (1977) belongs to the one-factor affine linear short rate models. Due to its analytic tractability, it is still used nowadays although it can only reproduce the standard yield curve shapes. In the one-factor Vasicek model (see Brigo and Mercurio (2007),) the short rate $r_d(t)$ follows an Ornstein-Uhlenbeck process with constant coefficients. We assume that under the risk-neutral measure $\mathbb{Q}$ the dynamics are given by

$dr_d(t) = \kappa\,(\mu - r_d(t))dt + \sigma dW(t)$ (4)

The Vasicek model uses a mean-reverting stochastic process to model the evolution of the short-term interest rate. Mean reversion is one of the key innovations of the model. If the interest rate is bigger than the long run mean, then the coefficient down in the direction of makes the drift become negative so that the rate will be pulled and likewise when it drifts below the long-term rate it is pushed up. This feature of interest rates can also be justified with economic arguments: High interest rates tend to cause the economy to slow down and borrowers require fewer funds. This causes the rates to decline to the equilibrium long-term mean. In the opposite situation when the rates are low, funds are of high demand on the part of the borrowers so rates tend to increase again towards the long-term mean. (Zeytun and Gupta(2007, p. 2)).

Integrating equation (4) one can solve for r_d.

$$r_d(t) = r_d(u)e^{-\kappa(t-u)} + \mu(1 - e^{-\kappa(t-u)}) + \sigma\int_u^t e^{-\kappa(t-v)}dW(v) \qquad (5)$$

Let $P(t,T)$ be the discount bond price at time t of domestic country d that pays one-unit currency at maturity T, then under the risk neutral measure $\mathbb{Q}$ we can write:

$$P(t,T) = E_{\mathbb{Q}}\left[exp\left(-\int_t^T r_d(s)ds\right)\right] \qquad (6)$$

As r_d is normally distributed we can use the moment generating function for the normal distribution to rewrite equation (6) as

$$P(t,T) = exp\left[E_{\mathbb{Q}}\left(-\int_t^T r_d(s)ds\right) + \frac{1}{2}V_{\mathbb{Q}}\left(-\int_t^T r_d(s)ds\right)\right] \qquad (7)$$

Using the method described in Kwok(1998) and James and Webber(2000) we integrate equation (5) for r_d and get

$$P(t,t+\tau) = \exp\left(\left(\mu - \frac{\sigma^2}{2\kappa^2}\right)(B(\tau) - \tau) - \frac{\sigma^2}{4\kappa}B(\tau)^2\right) \qquad (8)$$

$$B(\tau) = \frac{(1 - e^{-\kappa\tau})}{\kappa}$$

We then deduce the term structure of interest rates using equation (2).

3.2 The two-factor Vasicek model

Along the lines of Shoji(1994) we consider in this study a two-factor equilibrium Vasicek model. Let r_e be the foreign short-term interest rate and r_d the domestic short-term interest rate. The interest rate differential (or spread) s is defined as:

$$s = r_e - r_d \qquad (9)$$

We follow Bardhan(1967) and consider that the spread measures the country risk. We assume that s and r_e satisfy the following stochastic differential equation under the risk neutral probability measure $\mathbb{Q}$:

$$\begin{cases} dr_e(t) = \kappa_e(\mu_e - r_e(t))dt + \sigma_e dW_e(t) \\ ds(t) = \kappa_s(\mu_s - s(t))dt + \sigma_s dW_s(t) \\ E(dW_s(t)dW_e(t)) = \rho dt \end{cases} \quad (10)$$

Where W_e and W_s are standard Wiener processes withρ the correlation coefficient and κ_s,μ_s,κ_e,μ_e,σ_s,σ_e are constant. Thus, $r_d(t)$ satisfies the following stochastic differential equation:

$$dr_d(t) = [\kappa_s(\mu_s - s(t)) + \kappa_e(\mu_e - r_e(t))]dt + \sigma_s dW_s(t) + \sigma_e dW_e(t) \quad (11)$$

The stochastic process of s and r_e are the Ornstein-Uhlenberg process used by Vasicek(1977) to derive a model of the term structure of interest rates. Integrating equation (11) one can solve for r_d.

$$r_d(t) = r_e(u)e^{-\kappa_e(t-u)} + \mu_e(1 - e^{-\kappa_e(t-u)}) + s(u)e^{-\kappa_s(t-u)} + \mu_s(1 - e^{-\kappa_s(t-u)}) + \sigma_e \int_u^t e^{-\kappa_e(t-v)} dW_e(v) + \sigma_s \int_u^t e^{-\kappa_s(t-v)} dW_e(v) \quad (12)$$

Let $P(t,T)$ be the discount bond price at time t of domestic country d that pays one-unit currency at maturity T, then under the risk neutral measure $\mathbb{Q}$ we can write:

$$P(t,T) = E_{\mathbb{Q}}\left[exp\left(-\int_t^T r_d(s)ds\right)\right] \quad (13)$$

As r_d is normally distributed we can use the moment generating function for the normal distribution to rewrite equation (13) as

$$P(t,T) = exp\left[E_{\mathbb{Q}}\left(-\int_t^T r_d(s)ds\right) + \frac{1}{2}V_{\mathbb{Q}}\left(-\int_t^T r_d(s)ds\right)\right] \quad (14)$$

Where $V_{\mathbb{Q}}$ denote the conditional variance under $\mathbb{Q}$. Using the method described in Kwok (1998) and James and Webber(2000) we integrate equation (5) for r_d and get

$$\int_t^T r_d(s)ds = (r_e(t) - \mu_e)\frac{1 - e^{-\kappa_e(T-t)}}{\kappa_e} + \mu_e(T-t) + (s(t) - \mu_s)\frac{1 - e^{-\kappa_s(T-t)}}{\kappa_s} + \mu_s(T-t) + \sigma_e \int_t^T \frac{1 - e^{-\kappa_e(T-t)}}{\kappa_e} dW_e(u) + \sigma_s \int_t^T \frac{1 - e^{-\kappa_s(T-t)}}{\kappa_s} dW_s(u)$$

Taking the expectation and variance of the result:

$$E_{\mathbb{Q}}\left(-\int_t^T r_d(s)ds\right) = -(r_e(t) - \mu_e)\frac{1 - e^{-\kappa_e(T-t)}}{\kappa_e} - \mu_e(T-t) - (s(t) - \mu_s)\frac{1 - e^{-\kappa_s(T-t)}}{\kappa_s}$$

$$V_{\mathbb{Q}}\left(-\int_t^T r_d(s)ds\right) = \frac{\sigma_e^2}{\kappa_e^2}\left[(T-t) - \frac{1 - e^{-\kappa_e(T-t)}}{\kappa_e} - \frac{1}{2\kappa_e}\left(\frac{1 - e^{-\kappa_e(T-t)}}{\kappa_e}\right)^2\right] + \frac{\sigma_s^2}{\kappa_s^2}\left[(T-t) - \frac{1 - e^{-\kappa_s(T-t)}}{\kappa_s} - \frac{1}{2\kappa_s}\left(\frac{1 - e^{-\kappa_s(T-t)}}{\kappa_s}\right)^2\right] + \frac{2\,\sigma_e\sigma_s\rho}{\kappa_e\kappa_s}\left[(T-t) - \frac{1 - e^{-\kappa_e(T-t)}}{\kappa_e} - \frac{1 - e^{-\kappa_s(T-t)}}{\kappa_s} - \frac{1 - e^{-(\kappa_e+\kappa_s)(T-t)}}{\kappa_e + \kappa_s}\right]$$

By substituting the expressions for the expected value and the variance in equation (14) we obtain the expression for the price of zero-coupons. We then deduce the term structure of interest rates using equation (2).

4 Calibration and estimation of model parameters

The foreign short-term interest rate used in this study is the weekly Euribor 3-month (European Interbank Offered Rate) obtained from Banque de France and the domestic short-term rate is the weekly 7-day Weighted Interbank Rate obtained from BCEAO, and cover the period spanning from 2008/10/06 to 2020/12/7 given a total of 635 observations.

In order to estimate the parameters of the model, we employ a discretization scheme due to Bergstrom (1984) and introduced to the interest rate modelling literature by Nowman(1997) in exact discrete analog of the model, which gives the corresponding exact discrete versions that holds for any size of Δt:

$$r_{e,t} = e^{-\kappa_e \Delta t} r_{e,t-1} + \mu_e (1 - e^{-\kappa_e \Delta t}) + \sigma_e \sqrt{\frac{1 - e^{-2\kappa_e \Delta t}}{2\kappa_e}} \epsilon_{et}$$

$$s_t = e^{-\kappa_s \Delta t} s_{t-1} + \mu_s (1 - e^{-\kappa_s \Delta t}) + \sigma_s \sqrt{\frac{1 - e^{-2\kappa_s \Delta t}}{2\kappa_s}} \epsilon_{st}$$

Vasicek(1977) model is equivalent to a first order autoregressive AR (1) model. The estimation of parameter vector $(\kappa, \mu, \sigma, \kappa_e, \mu_e, \sigma_e, \kappa_s, \mu_s, \sigma_s)$ is carried out using the OLS regressions. We use the following regressions:

$$r_{d,t+1} = \alpha + \beta r_{d,t} + \epsilon_t$$

$$r_{e,t+1} = \alpha_e + \beta_e r_{e,t} + \epsilon_t$$

$$s_{t+1} = \alpha_s + \beta_s s_t + \epsilon_t$$

To derive the parameters of the model, we transform the regression coefficients in:

$$\beta = e^{-\kappa \Delta t} \qquad \Rightarrow \kappa = -\frac{\ln(\beta)}{\Delta t}$$

$$\alpha = \mu(1 - e^{-\kappa \Delta t}) \qquad \Rightarrow \mu = \frac{\alpha}{1 - \beta}$$

$$\sigma = sd(\epsilon_t) \sqrt{\frac{2\kappa}{1 - e^{-2\kappa \Delta t}}}$$

and

$$\beta_e = e^{-\kappa_e \Delta t} \qquad \Rightarrow \kappa_e = -\frac{\ln(\beta_e)}{\Delta t}$$

$$\alpha_e = \mu_e(1 - e^{-\kappa_e \Delta t}) \qquad \Rightarrow \mu_e = \frac{\alpha_e}{1 - \beta_e}$$

$$\sigma_e = sd(\epsilon_t) \sqrt{\frac{2\kappa_e}{1 - e^{-2\kappa_e \Delta t}}}$$

and

$$\beta_s = e^{-\kappa_s \Delta t} \qquad \Rightarrow \kappa_s = -\frac{\ln(\beta_s)}{\Delta t}$$

$$\alpha_s = \mu_s(1 - e^{-\kappa_s \Delta t}) \qquad \Rightarrow \mu_s = \frac{\alpha_s}{1 - \beta_s}$$

$$\sigma_s = sd(\epsilon_t) \sqrt{\frac{2\kappa_s}{1 - e^{-2\kappa_s \Delta t}}}$$

Where $\Delta t = 1/52$. Table 1 reports the regression results. The parameters β which set the level of the restoring force are always very significant.

Table 1: OLS estimates

Regression coefficients	Transformations
$\beta = 0.892726$ (*P=0.0*)	κ =5.637796
$\alpha = 0.435435$ (*P=0.0*)	μ =4.051248
$sd(\epsilon_t)$=0.349392	σ=2.6511
$\beta_e = 1.00048$ (*P=0.0*)	κ_e=-0.02495401
$\alpha_e - 0.00436584$ (*P=0.4*)	μ_e=9.0955
$sd(\epsilon_t)$=0.056225	σ_e=0.405347
β_s=0.960760 (*P=0.0*)	κ_s=2.081593

$\alpha_s = 0.0449533$ (*P=0.0*)	μ_s=1.145599
$sd(\epsilon_t) = 0.358621$	σ_s=2.637983
ρ =corr$(r_e, s) = -0.13898$	

Returning to equations (3), we see that to obtain simulations of trajectories from the equation driving the short rates, we need to make sure that the correlation property is correctly represented. In this order, we will write the second Wiener process W_s as

$$W_s(t) = \rho W_e(t) + \sqrt{1-\rho^2}\, W(t)$$

Where ρ is correlation coefficient between $W_e(t)$ and $W_s(t)$, and $W_e(t)$ and $W(t)$ are two independent Wiener processes. Figure 1 shows the simulations results of the two models. We add the original series for comparison. Figure 2 reports the simulation errors, that the difference between the market and the simulated variable. We note the one-factor Vasicek performs better than the two-factor model.

In general, the results of both models are not very good. Although we can see that the one-factor model performs better than the two-factor one. The two models behave particularly badly between 2016 and 2019. Thus, the external factor does not seem to play any significant part in the dynamics of the domestic interest rate.

We can plot the yield curve of the two-factor Vasicek model in September 2020 with the market yield curve and the yield curve implied by the Vasicek model together for comparison. From Figure 2, we can see that the one-factor Vasicek yield curve fits better to the market than the two-factor Vasicek. Note that for the two-factor Vasicek model, the spread rate is set to the current yield in September 2020 and the Euribor rate is assumed to be 3.07% corresponding to its initial value at 13/08/2008.

5 Conclusions

In this article, we have calibrated the one- and two-factor Vasicek term structure model using WAEMU weekly data over the period 2008/10/06 to 2020/12/7. The second factor, the spread between foreign and domestic interest rate, represents country risk. We find that the one-factor performs better than the two-factor for this monetary union.

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6 Appendix

Model 1: OLS, using observations 2008-10-13:2020-12-07 (T = 635)
Dependent variable: eur3m

	coefficient	std. error	t-ratio	p-value	
const	−0.00436584	0.00554592	−0.7872	0.4315	
eur3m(-1)	1.00048	0.00173133	577.9	0.0000	***

Mean dependent var 2.929641 S.D. dependent var 1.291583
Sum squared resid 2.001050 S.E. of regression 0.056225
R-squared
0.998108 Adjusted R-squared 0.998105
F(1, 633) 333931.2 P-value(F) 0.000000
Log-likelihood 927.7591 Akaike criterion −1851.518
Schwarz criterion −1842.611 Hannan-Quinn −1848.060

rho 0.721447 Durbin's h 18.19722

Model 2: OLS, using observations 2008-10-20:2020-12-07 (T = 634)
Dependent variable: s

	coefficient	std. error	t-ratio	p-value	
const	0.0449533	0.0191219	2.351	0.0190	**
s(-1)	0.960760	0.0110040	87.31	0.0000	***

Mean dependent var	1.158941	S.D. dependent var	1.295070
Sum squared resid	81.28094	S.E. of regression	0.358621
R-squared	0.923441	Adjusted R-squared	0.923319
F(1, 632)	7623.027	P-value(F)	0.000000
Log-likelihood	−248.4455	Akaike criterion	500.8910
Schwarz criterion	509.7951	Hannan-Quinn	504.3486
rho	−0.270317	Durbin's h	−7.083737

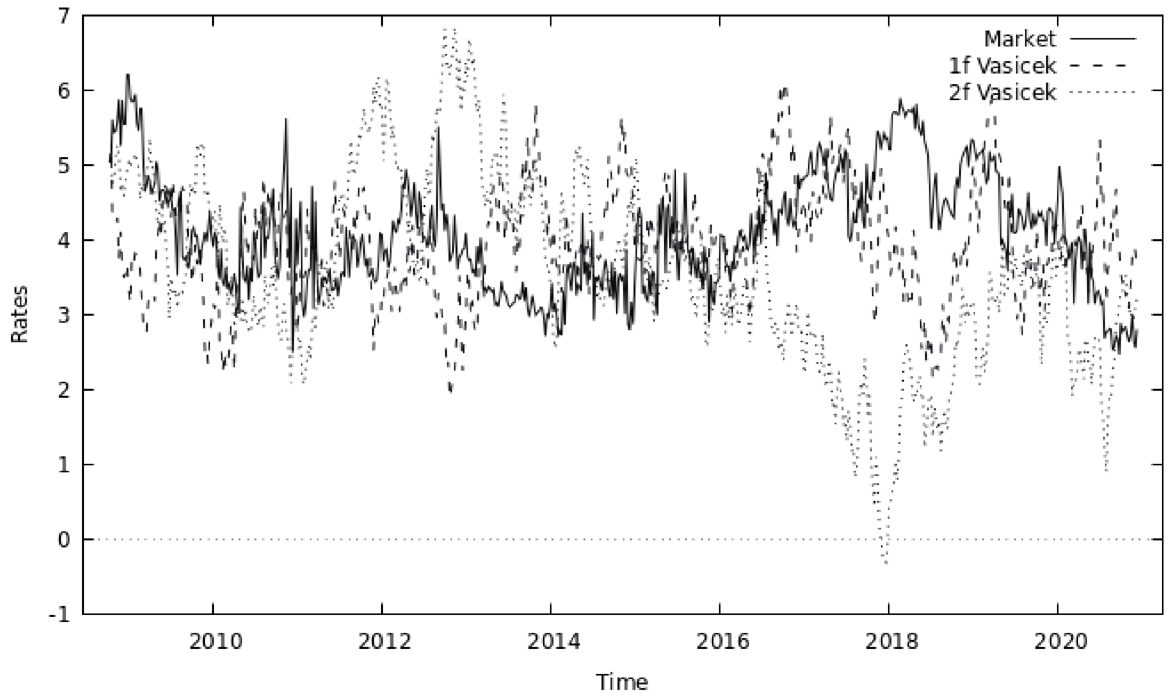

Figure 1: Simulated domestic short-term interest rate

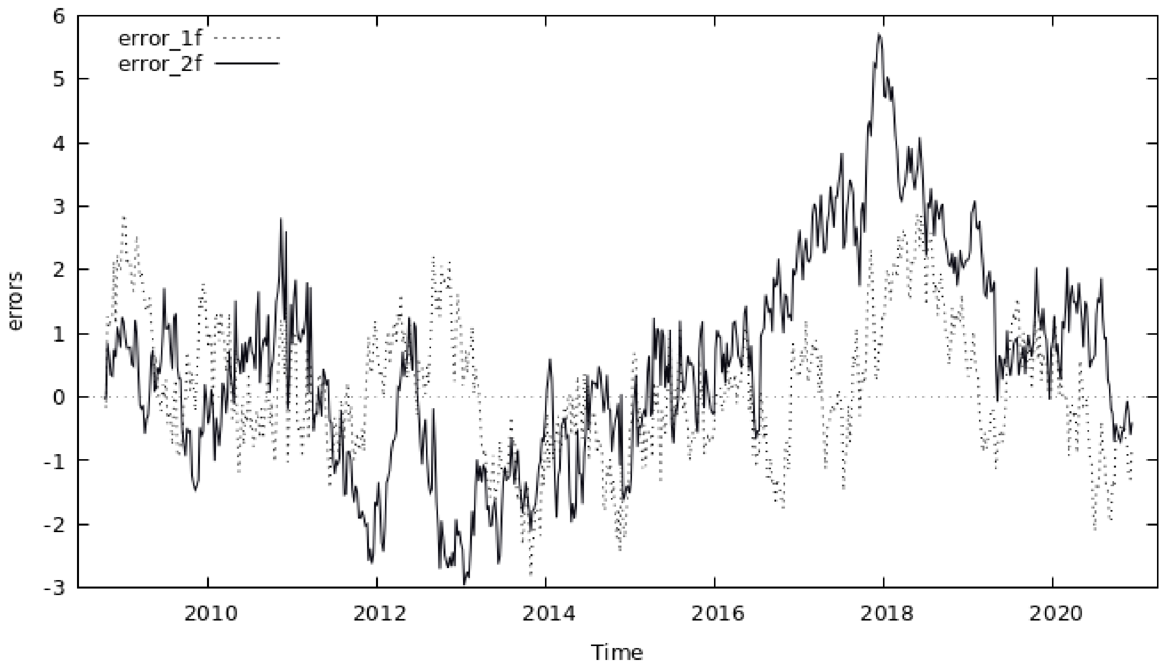

Figure 2: Simulation errors

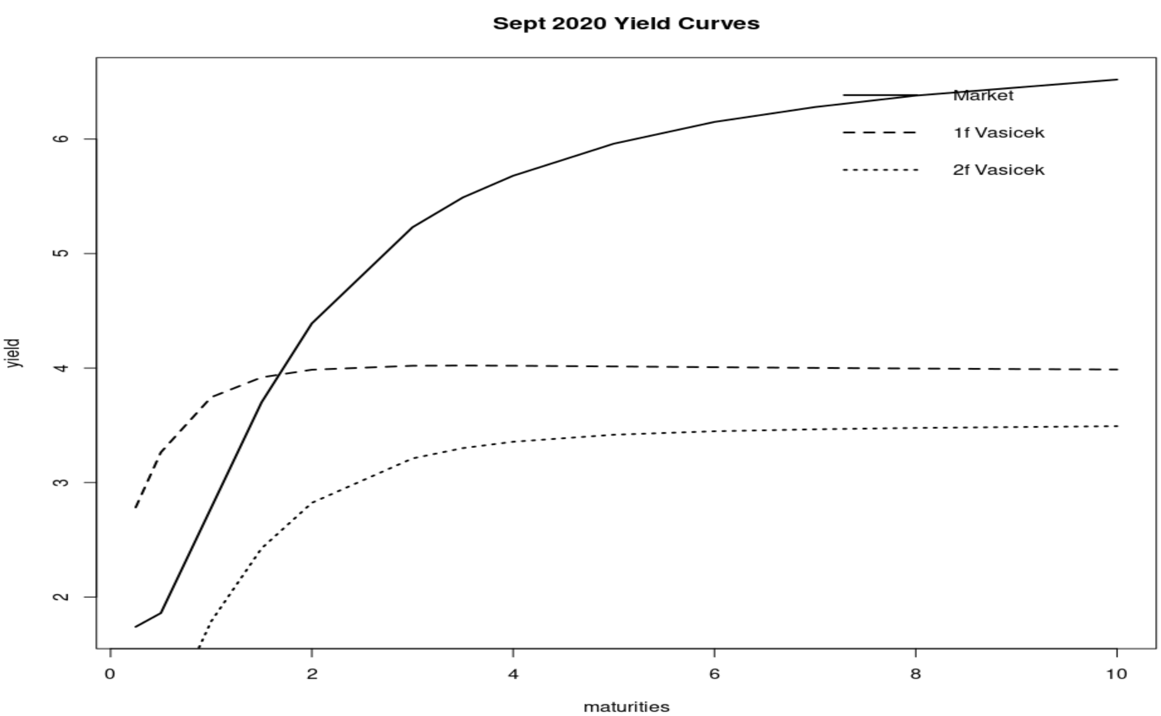

Figure 3: Comparison of the one-factor Vasicek, two-factorVasicek, and the market