Modelling Volatility of Short-term Interest Rates in Kenya

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Abstract

There is an extensive theoretical and empirical literature that documents the link between short-term interest rate volatility and interest rate levels. This study sought to establish the link between the level of interest and the volatility of interest rates in Kenya using the Treasury bill rates from August 1991 to December 2007. The main variable for the study was the short term interest rate series. In Kenya, this is the Central Bank three-month Treasury bill rate. The interest rate volatility was studied using the general specification for the stochastic behavior of interest rates which is tested in a Stochastic Differential Equation (SDE) for the instantaneous risk free rate of interest as earlier defined by Chan et al. (1992). The study applied the monthly averages of the 91-day T-BILL rate for the period between August 1991 and December 2007 which were obtained from the Central Bank of Kenya. The results of the study were consistent with the hypothesis that the volatility is positively correlated with the level of the short term interest rate as documented by previous empirical studies. The key findings revealed that there exists a link between the level of short-term interest rates and volatility of interest rates in Kenya. Secondly, the study's key findings revealed that the GARCH model is better suited for modeling volatility of short rates in Kenya, as opposed to ARCH models. The study further establishes that GARCH models are able to capture the very important volatility clustering phenomena that has been documented in many financial time series, including short-term interest rates. The study recommends future research to examine if other forms of the GARCH process can produce similar results (i.e., EGARCH, PGARCH, GARCH, and FIGARCH).

Keywords: Volatility, interest rate, Kenya

1.1. Background to the Study

Traditional theories define interest rate as the price of savings determined by demand and supply of loanable funds. It is the rate at which savings are equal to investment assuming the existence of a capital market. The loanable fund theory argues that interest rate is determined by non-monetary factors. It assigns no role to quantity of money or level of income on savings, or to institutional factors such as commercial banks and the government. The liquidity theory, on the other hand, looks at the interest rate as the token paid for abstinence and inconveniences experienced for having to part with an asset whose liquidity is very high. It is a price that equilibrates the desire to hold wealth in the form of cash with the available quantity of cash, and not a reward of savings. Interest rate is a function of income. Its primary role is to help mobilize financial resources and ensure the efficient utilization of resources in the promotion of economic growth and development (Ngugi and Kabubo, 1998).

Short-term interest rates are charges levied by the lenders to the borrowers on loans that must be paid within a year such as Treasury bills and credit card loans. The Short Term Interest Rates are important variables in many different areas of the economic and financial theory. They are important in many financial economic models, such as models on the term structure of interest rates, bond pricing models and derivative security pricing models. They are also important in the development of tools for effective risk management and in many empirical studies analyzing term premiums and yield curves where risk free short-term rates are taken as reference rate for other interest rates. Besides, they are also a crucial feature of the monetary transmission mechanism. Duguay (1994) describes the monetary transmission mechanism as starting with a monetary authority's actions influencing short-term rates and the exchange rate, which then go on to ultimately affect aggregate demand of inflation. In order to understand the characteristics of the monetary transmission mechanism, it is therefore imperative to have a good model of the behaviour of short-term interest rates.

Empirical evidence documents a level effect in the volatility of short term rates of interest (Olan and Sandy, 2005; Turan and Liuren, 2005). That is, volatility is positively correlated with the level of the short term interest rate. Using Monte-Carlo simulations, Olan and Sandy (2005) examined the performance of the Engle-Ng (1993) tests which differentiate the effect of good and bad news on the predictability of future short rate volatility. The short-term interest rates being the US three month Treasury bills rates taken from the Federal Reserve Bank of St. Louis Economic database were sampled at a weekly frequency over the period of 5th January 1965 to 4th November 2003 yielding 2027 observations.

Their results established that the tests exhibit serious size distortions and loss of power in the face of a neglected level effect. The tendency for interest rates to be more volatile as short term rates rise is what is commonly referred to as 'level effects'. The dynamics of short-term treasury interest rates are central to the pricing of all fixed income instruments and their derivatives. Chan, Karolyi, Longstaff and Sanders (1992), hereafter CKLS compared a variety of single factor continuous-time models of the short-term risk-less rate over the period 1964 through 1989. They found that models that allow the volatility of interest changes to be sensitive to the level of the risk-free rate outperform other models. Longstaff and Schwartz (1992) presented a two-factor general equilibrium model, with the level and conditional volatility of short-term rates as factors. They showed that a two-factor model carries additional information about the term structure and leads to better pricing and hedging performance compared to a single factor model, which only uses the level of the short rate.

The factors that affect short-term interest rates include: the monetary policy, the Government fiscal policy, taxation, inflation, demand for capital, social values, and political trends. The monetary policy is used by the government to control the supply of money in the economy. When supply of money in the economy is low then the interest rates are expected to be high and vice versa. The volatility in money supply growth may lead to higher interest rates. Under the fiscal policy, the Government is supposed to finance all expenditure for the economy. In cases where expenditure exceeds revenue (budget deficit), the Government is forced to borrow from the local markets. This in turn affects the supply of money in the economy which in turn affects the trend of interest rates. Inflation on the other hand causes long-term interest rates to rise where investors sell-off their bonds in fear of inflation eroding their capital gains. Demand for capital influences interest rates when the demand/supply of funds is below or above the equilibrium levels. If there are fewer borrowers and the demand for funds is low then the interest rates will be low and vice versa.

In Kenya, the interest rates charged by banks are determined by: interest rate on deposits; cost of liquidity; cost of holding cash; and operational costs. The interest on deposits depends on the bank's cash ratio, its overall financial stability and the type of the bank for example whether it is a corporate bank or a network bank. The cost of liquidity covers both the cash, which is maintained by the banks with Central Bank as required cash ratio, and the cash maintained by the banks as the minimum amounts to meet unexpected demand from the customers. Cost of holding cash is derived from the cash held by the banks in form of liquid form to meet day-to-day customer's needs. The banks have to compare the costs of cash outs and the opportunity costs associated with the cash held in liquid form. Operational costs are mainly meant to cover the costs of running the bank and it includes capital costs, staff costs, and technology costs. The base rate charged by the banks takes into account all these factors. The bank can reduce the base rate by improving efficiency.

1.2. Short-term interest rates in Kenya

Prior to the implementation of Structural Adjustment Programme (SAP) in 1983, the financial sector in Kenya suffered from severe repression. Interest rates were maintained below market-clearing levels, and direct control of credit was the primary monetary control instrument of the authorities. Accompanying the SAP, interest rate deregulation took place. In September 1991 the maximum lending rate was increased from 10% to 14 %. The rediscounting rate for crop finance paper was raised to 11.25 %, while the minimum savings deposit rate was raised to 12.5 %. Between 1983 and 1987, the differentials between the interest rates of banks and non-bank financial institutions were narrowed. This improved the competitiveness of commercial banks. One of the first steps towards freeing interest rates was taken in 1989, when the government started selling Treasury Bonds through an auction.

In July 1991, interest rates were completely freed. Since then, interest rates have been following a steep upward ascent, with the gap between loan deposit rates shrinking (Naude, 1995). After the liberalization period, interest rates were liberalized and indirect monetary policy tools adopted.Steps were taken to establish financial markets, decontrol foreign exchange, liberalize trade and tighten prudential regulations. The role of the Central Bank was strengthened and monetary policy was tightened. All these were accompanied by declining economic performance. From the financial repression theory, a major achievement in the financial liberalization is the decontrol of interest rates. This has a positive impact on economic performance and also in indicating the direction the financial sector takes after the liberalization process (Ngugi and Kabubo, 1998).

High real short-term interest rates have reduced the demand for capital market instruments and crowded-out substantial domestic savings to short-term government securities (Kibuthu, 2005). This situation was particularly evident in 2001 when the Treasury bill (T-bill) rate was 12.6% compared to an inflation rate of 0.8%. However, the situation is being reversed as T-bill rates have fallen to about 8% resulting in increased demand for both equity and debt instruments (World Bank, 2002). Interest rate spreads are high and currently standing at about 13%.

Deposit rates are too low and lending rates too high thereby discouraging domestic savings and investment. The domestic savings are less than 10% of Gross Domestic Product (GDP) and thereby insufficient to meet investment needs and generate demand for equities and debt instruments (World Bank, 2002). Risk free interest rates play a fundamental role in finance. Theoretical models of interest rates are of interest both for the pricing of interest rate sensitive derivative contracts and for the measurement of interest rate risk arising from holding portfolios of these contracts. There is a vast literature focusing on modelling its dynamics. This study sought to specify a model for modelling volatility of short-term interest rates in Kenya.

1.3. Term Structure of Interest Rates

The term structure of interest rates concerns the relationship among the yields of default free zero coupon bonds that differ only with respect to maturity. Historically four competing theories of the term structure have attracted attention. These are expectation, liquidity preference, hedging pressure of preferred habitat and segmentation theories of the term structure of interest. According to the expectation theory, the shape of the yield can be explained by investors' expectations about future interest rates. The liquidity preference theory argues that short term bonds are more desirable than long term bonds because former are more liquid. The preferred habitat theory explains the shape of the term structure by the assumptions that if an investor is risk averse, he can be tempted out of his preferred habitats only with the promise of a higher yield. Market segmentation theory assumes that there are two distinct markets for the short and long term bonds. The demand and supply in the long term bond market determines the long term yield and the demand and supply in the short term bond Market determines the short rate. This means that the expected future rates have little to do with the shape of the yield curve.

Over the past few decades, theoretical development of modelling term structure dynamics has been mainly along the following two directions. One direction is, while keeping a simple, tractable, and parsimonious structure, to extend the model through more flexible specification in order to better describe the dynamics of state variables and project the term structure movements. Development along this direction is evidenced in various one-factor models (Merton, 1973; Vasicek, 1977; Dothan, 1978; Brennan and Schwartz, 1979; and Cox, 1980). Cox *et al*. (1985) defined the term structure of interest rates as the measure of the relationship among the yields on risk-free securities that differ only in their term to maturity. The yield is a rate at which the present value of all future payments of interest and principal is equated to the market price of the security. The yield curve is positively sloped implying that the yields of long-maturity securities are higher than the yields of short-maturity securities.

According to Litternman *et al*. (1991), the volatility of the short-term rate has two counteracting effects on the yield curve. Firstly, higher volatility of the short-term interest rates induces higher expected rates for the longer maturities (premium effects). Secondly, higher volatility of the short-term interest rates increases the convexity of the discount factor function and, therefore, reduces the yields for the longer maturities. According to Kimura (1997), the term structure of interest rates is the relationship between long-term and short-term interest rates. That is, it is the relationship between an interest rate and the maturity on security assuming that economic fundamentals such as inflation, unemployment, political environment remain unchanged. The term structure of interest rates shows the relationship between interest rate level and the term to maturity of a security.

1.4 Dynamics of Short-term Interest Rates

One of the most puzzling pieces of evidence on the term structure of interest rates is the weak link between the slope of the term structure and future changes in interest rates (Campbell, 1995). Mankiw and Miron (1986) related this evidence to the active targeting of interest rates on the part of the Federal Reserve. They argued that prior to the founding of the Federal Reserve System; the slope of the term structure of interest rates was a fairly accurate predictor of future changes in short-term rates. During this period, interest rates were quickly mean-reverting and highly seasonal, and therefore fairly easy to predict. In contrast, since the Federal Reserve's inception, the stabilization of interest rates was so successful that seasonal effects and volatility were greatly reduced (Mankiw, Miron and Well, 1987), and interest rates began behaving in a way similar to a random walk.

An important implication of Mankiw and Miron's (1986) analysis was that, by targeting the overnight-fed funds rate, the Federal Reserve effectively enjoyed a substantial amount of control over term-fed fund rates and longer-term yields. Goodfriend (1991) suggested that the targeting of the overnight-fed funds rate was implemented with exactly this goal, since longer-term rates were more strongly linked to macroeconomic goals such as unemployment and inflation.

The existing literature suggests that a Federal Reserve policy enforcing smooth interest rates was desirable to avoid “whipsawing” the bond market (Goodfriend 1991), to contain the variability of the inflation tax (Barro 1989), and to stabilize the macro economy (Mankiw, Miron, and Weil 1987). In their study, Pierluigi, Giuseppe, Silverio and Leora (1997) documented a new stylized fact concerning the relation between interest rate targeting and the dynamics of short-term rates. They showed that during the 1989-1996 period, the Federal Reserve was able to closely target the overnight-fed funds rate, and especially to reduce the persistence of its spreads from the target: these spreads averaged one basis point, and exhibited an autocorrelation of only 0.07, after one day. Still, term-fed funds rates of maturity up to three months fluctuated widely and persistently around the target. For example, the volatility of daily spreads of the three-month term-fed funds rate from the target was 36 basis point, and the autocorrelation of these spreads after 60 days was still 0.58. Both the volatility and the persistence of spreads of term-fed funds rates from the target were an increasing function of the maturity of the loan. This new stylized fact can be interpreted as evidence that, while central bank intervention is important in determining the shape and position of the term structure, even a tight targeting of the overnight-fed funds rate does not mechanically translate into a tight control of longer-term rates.

Some of the early work on term structure models focused on traditional factor analysis. Litterman and Scheinkman (1991) computed the principal components of yield changes and found that the first three principal components explained about 96% of the variation in yields. They referred to the three factors as “level,” “slope,” and “curvature.” The level factor referred to a parallel shift in the yield curve, the slope factors referred to a steepening or flattening, and the curvature factors referred to the twisting between intermediate term and short and long term yields. The level-slope-curvature factors were closely related to the latent factors that had been used for affine term structure models. Rather than using observed state variables, the state variables were backed out from the observable yields. This approach was later used in continuous time by Dai and Philippon (2005), and Dai and Singleton (2002) among others. The latent factors used in affine term structure models behave essentially like the level, slope, and curvature factors. The major drawback of this approach was that the factors were not observable, and so they did not lend themselves to good forecasting methods. They also did not provide any explanation of how macroeconomic variables affect the term structure.

To cover the anomalies identified in the factor analysis approach, Taylor (1993) and McCallum (1994) focused on using monetary policy rules to describe the dynamics of the short rates. These approaches have been very successful at describing monetary policy. However, these models assume a simple relationship between the short rate and the longer term yields. As a result, although the models describe short rates very well, they do not fit longer term yields very well. Some more recent work in the macro literature has focused on incorporating macroeconomic variables in the term structure model. Evans and Marshall (2001) used a vector auto regression (VAR) model of this form that includes factors for GDP and inflation. Their model, however, did not impose the restrictions of no-arbitrage. Even though the model did a better job of explaining the effects of macroeconomic variables on the full term structure, the lack of no-arbitrage restrictions means that the model was fundamentally missing out on important aspects of term structure dynamics.

In their study, Turan and Liuren (2005) performed a comprehensive analysis of the short-term interest-rate dynamics based on three different data sets and two flexible parametric specifications. They applied generalized autoregressive conditional heteroskedastic (GARCH)-type models with non-normal innovations to capture the potential impact of time-varying volatility and discontinuous interest rate movements. Estimates on both sets of models based on the three interest-rate series were performed using the quasi-maximum likelihood estimation method. They found that non-linearities were strong in the federal funds rate and the seven-day Eurodollar rate, but were much weaker in the three-month Treasury yield. They obtained similar findings when they estimated a two-factor diffusion model with stochastic volatility. They concluded that the conflicting evidence was partially due to the use of different data sets as a proxy for the short rate and the use of different parametric/ non-parametric specifications under which empirical studies perform the statistical tests.

1.5 Dynamics of Interest Rates in Kenya

The Treasury bill rates were stable from January 1983 to November 1990 where the lowest rate recorded was 11.5 1% and the highest was 15.79%. In December 1990, the Treasury bill rates shoot up to 16.68% and to 17.29% in January 1991 but remained stable in 1991 and 1992. In March 1993 the rates increased to 24.94% from 17.85% in February 1993 but shoot up drastically to 45.81% in April 1993. In July 1993 the rate was 84.60%, which was followed by a general decline reaching 23.37% in September 1994. The rates fluctuated in the range of 16.72% and 27.15 between October 1994 and November 1998.

In 2003 and 2004, the rates drastically declined to a level of 0.83% in September 2003, but in December the rate was 8.04%. Since January 2005 to date the rates have been fluctuating between 6% and 9%. According to the Central Bank of Kenya (2005), the stability of short term interest rates between 8% and 9%, have been vital to the financial sector stability and overall economic growth. The stability of domestic interest rates in Kenya has contributed to the predictable macroeconomic environment for investors and business people. This in turn has increased the level of confidence in the economy and has led to increased short term capital inflows.

Willem (1995) conducted a comparative empirical study between Ghana, Kenya, Zimbabwe and Nigeria. The sample comprised of four countries, two of the countries with the most advanced financial systems in Sub-Saharan Africa (Kenya and Zimbabwe), and two countries where structural adjustment had been an ongoing process for more than a decade (Kenya and Ghana). Willem applied short-term (less than 3 months) deposit rates and long-term deposit rates (longer than 12 months) from each of the four countries. The empirical findings from the sampled countries established that: (i) lending rates initially adjusted more slowly than deposit rates, creating initial periods during which the gap between lending and deposit rates narrowed, and even became negative in the case of Zimbabwe, and (ii) the level and volatility of interest rates increased after liberalization.

In the Kenyan case, the study established that interest rates in Kenya have been fairly stable and that a relatively constant gap had been maintained between lending and deposit rates for most of the period. However, it must be borne in mind that, although Kenya was one of the first African countries to implement a SAP, it was only in 1991 that full interest rate liberalization took place. Since then, interest rates have been following a steep upward ascent, with the gap between loan deposit rates shrinking after interest rate liberalization. Willem (1995) further revealed that for the Kenyan case, only changes in contemporaneous short-term interest rates seemed to have any effect on long-term interest rates, but the value of this parameter was smaller than 1 (0.69) which suggested a less than perfect correspondence between short and long rates. Furthermore, the acceptance that lags of short-term interest rates were insignificant, suggested that long-run interest rates do not adjust sluggishly to short-term rates.

1.6. Modeling sensitivity of volatility to the level of short-term interest rates

This section discusses the basic types of models that have been used to explain short-term interest rate dynamics. The first type of model is the diffusion model that is predominantly used in building term structure models. The second type of model is the Autoregressive conditional heteroskedasticity (ARCH) model that has proven useful in modeling the dynamics of the second moment of many financial time series. The third model is an extension of the basic diffusion model which allows for stochastic volatility.

1.6.1. Diffusion Models

Most term structure models assume that short-term interest rates evolve over time as some type of diffusion process. The beauty of the diffusion model is that the instantaneous change in the short rate can be characterized as a stochastic differential equation (SDE hereafter) and Itˆo calculus can then be utilized to characterize the term structure. This basic approach is used in both the arbitrage pricing and general equilibrium approaches to pricing the term structure. Chan et al. (1992) (CKLS hereafter) provided a general framework for modeling interest rate processes. They described interest rate volatility using the general specification for the stochastic behavior of interest rates. They asserted that the single-factor diffusion processes to be studied can be nested in the following Stochastic Differential Equation (SDE) for the instantaneous risk free rate of interest 'r_t' represented by equation (1):

$$dr_t = (\alpha + \beta r_t)dt + \sigma r_t^{\lambda} dZ \quad \text{.......................................} (1)$$

Where:

dZ denotes the standard Wiener process or Brownian motion

σr_t^{λ} is the instantaneous standard deviation of interest rate changes which is often referred to as 'volatility'

The dependence of the instantaneous standard deviation on r_t^{λ} is known as the 'levels effect'. The drift component of short term interest rates is captured by $\alpha + \beta r_t$ while the variance of unexpected changes in interest rates equals $\sigma^2 r_t^{2\lambda}$. While σ is a scale factor, the parameter λ controls the degree to which the interest rate level influences the volatility of short term interest rates. A λ of 1.0 indicates that the volatility of the interest rate is independent of its level and a λ above unity indicate that the volatility rises with the level of interest rates.

In equation (1), dZ is the single factor driving the evolution of the entire term structure. CKLS were concerned with calibrating this general SDE econometrically to evaluate the appropriateness of these competing models for the short rate. The exact functional form of the short rate SDE is of critical importance for models of the term structure. For example, Vasicek (1977) used an arbitrage argument to derive a partial differential equation for bond prices. His derivation was sufficiently general to allow for any diffusion type of SDE for the short rate and then proceeded to derive closed form bond process for the special case of an Ornstein-Uhlenbeck process for the short rate. To empirically calibrate the general SDE, CKLS employed a simple discretization of equation (1) to come up with a calibrated equation presented by equation (2):

$$\Delta r_t = \alpha + \beta r_{t-1} + \sigma r_{t-1}^{\lambda}\varepsilon_t \quad \text{......} \quad (2)$$

Where: r is interest rate at time t, $\Delta r_t = r_t - r_{t-1}$ is the change in the interest rate during the period t, and ε_t is a standard normal random variable. They estimated the parameters of this model by using the Generalized Method of Moments (GMM hereafter) estimation technique of Hansen (1982). They found out that the short rate is mean reverting, and that the elasticity of volatility parameter was 1.4999 (the standard error was 0.2519). The elasticity parameter indicates that the volatility of short-term interest rates is explosive. Other studies includes the work of Broze, Scaillet, and Zakoian (1995) who used maximum likelihood based procedures and the indirect inference technique of Gourieroux, Monfort, and Renault (1993) to account for the discretization bias which they found to be very small. Another approach due to Ait Sahalia (1996) estimated the implied density of discrete changes in the spot rate implied by various continuous time models, and compared these with the empirical distribution of the discrete changes in the spot rate.

1.6.2. GARCH Models

The ARCH model was introduced by Engle (1982) and later extended by Bollerslev (1986), who developed the generalized ARCH, or GARCH model. In a GARCH (1, 1) model (equation 3), the conditional mean and conditional variance of a time series process are modelled simultaneously.

$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t \quad \text{......} \quad (3)$$

Where the conditional volatility of ε_t is given by equation (4).

$$E\left[\varepsilon_t^2 / \phi_{t-1}\right] = h_t \quad \text{......} \quad (4)$$

and $h_t = \omega + \theta\varepsilon_t^2 + \psi h_{t-1}$ ...... (5)

$\alpha, \beta, \omega, \theta$ and ψ are regression constants.

r_t represents the interest rate series.

GARCH models are able to capture the very important volatility clustering phenomena that has been documented in many financial time series, including short-term interest rates (Bollerslev, Chou, and Kroner, 1992), as well as their leptokurtosis. Note that in GARCH models the volatility is a deterministic function of lagged volatility estimates and lagged squared forecast errors. One problem with GARCH models of the short-rate is that the parameter estimates suggest that the volatility process is explosive.

Bollerslev (1986) demonstrated that the variance process is covariance stationary when $\left|\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j\right| <$ 1. In this case, it is usually assumed that $\alpha_i, \beta_j \geq 0 \forall i, j$ to ensure that the conditional volatility is non-negative, so it is usually considered for the cases where $\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j < 1$. If this inequality is violated, then shocks to the volatility process are regarded as persistent or explosive. If the sum of the coefficients equals one, then the process is termed IGARCH (or integrated GARCH). If the sum of coefficients is strictly greater than one, then a shock to volatility is explosive, and $Lim_{j\to\infty} E\left[h_{t+j} / \phi_t\right] = \infty$. Parameter estimates of GARCH (1, 1) models fitted to short-term interest rates indicate an explosive process for the conditional volatility, or $\alpha + \beta > 1$. For example, Gray (1996) reported using weekly 30-day T-bill data that $\alpha + \beta = 1.0303$, and Engle, Ng, and Rothschild (1990) found that $\alpha + \beta = 1.0096$ for a portfolio of T-bills.

1.6.3. Stochastic Volatility Models

The stochastic volatility model allows log-volatility to itself evolve stochastically over time (Smith, 2000). This is in direct contrast with the GARCH type models which model volatility as a deterministic function of lagged squared forecast errors and lagged conditional volatility.

The stochastic volatility model is parsimonious and yet flexible, and has been successfully applied to a range of financial time series including short-term interest rates (Ball and Torous, 1999); exchange rates (Harvey, Ruiz, and Shephard, 1994); and stock prices (Sandmann and Koopman, 1998). Most stochastic volatility models are set in discrete time. Ball and Torous (1999) presented their stochastic volatility model as a simple extension of the discrete time diffusion models of the type presented in equation (2). Their extension of equation (2) is as shown in equation (6):

$$\Delta r_t = \alpha + \beta r_{t-1} + \sigma_t r_{t-1}^{\lambda} \varepsilon_t \quad \text{...} (6)$$

As the time subscript on σ in equation (6) indicates, the generalization employed allows the volatility to be time varying. The model allows log-volatility to evolve stochastically as a simple first-order autoregressive process represented in equation (7):

$$Log\sigma_t^2 = \xi + \kappa \log \sigma_{t-1}^2 + \eta_t \quad \text{..} (7)$$

Where ξ and κ are regression constants while $\eta_t \approx iidN(0, \sigma_n^2)$. The disturbance term η_t in (7) makes the process stochastic - the variance itself is subject to random shocks. This process is parsimonious and able to capture interesting dynamics. It can also be noted that GARCH models can be derived as the discrete time limit of a continuous time stochastic volatility model, but that the discrete time stochastic volatility model here considered are more direct.

One of the procedures available for estimating stochastic volatility models of this type is the quasi-maximum likelihood procedure of Harvey, Ruiz, and Shephard (1994). This approach uses a simple transformation of the residual in equation (6) to write the system in state-space form and then applies the Kalman filter to recursively build up the likelihood function. The transformation is employed on the residual $e_t = \Delta r_t - \alpha - \beta r_{t-1}$. Since $e_t = \sigma_t r_{t-1}^{\lambda} \varepsilon_t$ if the log of the squared residual is taken, a representation shown by equation (8) is obtained:

$$\log e_t^2 = \log \sigma_t^2 + 2\lambda \log r_{t-1} + \log \varepsilon_t^2 \quad \text{.........................} (8)$$

Equation (8) can further be simplified by introducing some new notation $y_t = \log e_t^2$ which is observable given the observed returns and the parameters α and β; and $x_t = \log \sigma_t^2$ is the state variable - log-volatility. Using this notation, the system of equations can be re-written in state-space form as shown by equations (9) and (10):

$$y_t = x_t + 2\lambda \log r_{t-1} + \log \varepsilon_t^2 \quad \text{..} (9)$$

$$x_t = \xi + \kappa x_{t-1} + \eta_t \quad \text{..} (10)$$

The Kalman filter is an iterative procedure that forecasts the state variable one period into the future by a linear projection and then updates this forecast when the observation on the variable y_t becomes available. If the disturbance terms are both Gaussian, then the linear projection is also the conditional expectation; and the conditional expectation and its mean squared error are all that is required to describe the conditional density. In this case, the Kalman filter enables the construction of the exact likelihood function, and then full maximum likelihood estimation. However, in this case the disturbance term for the observation equation (9) is non-Gaussian. In fact it is distributed as log-Chi squared random variable with one degree of freedom.

Harvey, Ruiz, and Shephard (1994) noted that $E[\log \varepsilon_t^2] = -1.2704$ and $Var[\log \varepsilon_t^2] = \frac{\pi^2}{2}$. They approximated the observation equation disturbance term by a normal random variable with the same mean and variance as $\log e_t^2$. The Kalman filtering equations and likelihood function are built in two steps. Step I involves the forecasting of log-volatility while step II involves updating of the forecasts. Since the Gaussian density is used to approximate the true density, this approach results in quasi-maximum likelihood parameter estimates. The central limit theorem of Dunsmuir (1979) is then used to establish the consistency and asymptotic normality of the resulting parameter estimates.

2.0 Research Methodology

2.1. Conceptual Model

In GARCH and Stochastic modelling, the volatility is regarded as a deterministic function of lagged volatility estimates and lagged squared forecast errors. This implies that for a short-term interest rate process: $r_t = f(r_{t-1})$ and $h_t = f(h_{t-1})$ where r_t is the short rate, and h_t is the conditional variance of the short rate.

2.2. Analytical Model

Chan et al. (1992) (CKLS) provided a general framework for modeling interest rate processes. They described interest rate volatility using the general specification for the stochastic behaviour of interest rates. They asserted that the single-factor diffusion processes to be studied can be nested in the following Stochastic Differential Equation (SDE) for the instantaneous risk free rate of interest 'r_t' represented by equation (11):

$$dr_t = (\alpha + \beta r_t)dt + \sigma r_t^{\lambda} dZ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(11)}$$

Where dZ denotes the standard Wiener process or Brownian motion and σr_t^{λ} is the instantaneous standard deviation of interest rate changes which is often referred to as 'volatility'. The dependence of the instantaneous standard deviation on r_t^{λ} is known as the 'levels effect'. The r represents the level of the short term interest rate. The drift component of short term interest rates is captured by $\alpha + \beta r_t$ while the variance of unexpected changes in interest rates equals $\sigma^2 r_t^{2\lambda}$. While σ is a scale factor, the parameter λ controls the degree to which the interest rate level influences the volatility of short term interest rates. A λ of 1.0 indicates that the volatility of the interest rate is independent of its level and a λ above unity indicate that the volatility rises with the level of interest rates. The estimate $\beta<0$ if significant suggest that the short-term rate is mean reverting. Equations (4) and (5) provide the conditional volatility of the error terms.

2.4. Diagnostic Tests

2.4.1. Lagrange Multiplier (LM) Test for Level Effects and Asymmetry

In developing a test for the joint null of asymmetry and levels effects an asymmetric GARCH model with a level effect provides a natural starting point given by the set of equations in (12):

$$\begin{aligned} &\Delta r_t = \varepsilon_t \\ &\varepsilon_t / \Omega_{t-1} \sim N(0, h_t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(12)} \\ &h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b r_{t-1}^{\lambda} + \alpha_2 \eta_{t-1}^2 \end{aligned}$$

Where $\beta + \alpha_1 < 1$, and β, α_i, $b > 0$ for i = 0, 1 and 2.

If η_{t-1} = Min $(0, \varepsilon_{t-1})$ then negative innovations have a greater initial impact of magnitude $\alpha_1 + \alpha_2$ on the volatility of the short rate change than a positive innovation of equal magnitude which has initial impact of size α_1. The level effect is captured by the dependence of the conditional volatility of the short rate change on the lagged short rate level. Its persistence is governed by the parameters *b* and λ. Implicitly the conditional mean of equations under (12) is equivalent to $\Delta r_t = \alpha + \beta r_{t-1} + \varepsilon_t$ under the restriction $\alpha = \beta = 0$. This restriction is consistent with the evidence provided by Chan, Karolyi, Longstaff and Sanders (1992), Longstaff and Schwartz (1992), and Brenner, Harjes and Kroner (1996).

The null hypothesis to consider is that of a symmetric GARCH (1, 1) while the alternative is an asymmetric GARCH (1, 1) with a level effect. This may be formulated as follows

$$H_0 : \alpha_2 = b = 0$$

$$H_1 : Either\ \alpha_2 \text{ and/or } b \neq 0$$

Where α_1, α_2, and *b* are regression coefficients derived from Equation (12) above. Sequential substitution for h_{t-1} and a first order Taylor series expansion about λ^* to linearize the level effect term in (12) yields

$$h_t = \sum_{i=1}^{t-1} \alpha_0 \beta^{i-1} + \sum_{i=1}^{t-1} \alpha_1 \beta^{i-1} \varepsilon_{t-i}^2 + \beta^{t-1} h_1 + \sum_{i=1}^{t-1} \beta^{i-1} b r_{t-i}^{\lambda^*} \left(1 - \lambda^* \ln r_{t-i}\right)$$
$$+ \sum_{i=1}^{t-1} \beta^{i-1} \phi r_{t-i}^{\lambda^*} \ln r_{t-i} + \sum_{i=1}^{t-1} \alpha_2 \beta^{i-1} \eta_{t-i}^2 \quad (13)$$

The null hypothesis of no level effect and no asymmetry may be reformulated as H_0: $b = \phi = \alpha_2 = 0$ where $\phi = b\lambda$. Under the assumption that the residual ε_t is conditionally normally distributed, the Lagrange Multiplier test statistic LM (λ^*) under the null hypothesis is approximately distributed as a Chi-square with three degrees of freedom.

2.4.2. Likelihood Ratio (LR) Tests

The likelihood ratio test (LRT) is a statistical test of the goodness-of-fit between two nested models (Hanfeng, Jiahua and Kalbfleisch, 2000). The LR tests was used to test for linear drift dynamics of the short-term rates. The form of the test as suggested by its name, is the ratio of two likelihood functions; the simpler model (s) has fewer parameters than the general (g) model. Asymptotically, the test statistic is distributed as a chi-squared random variable, with degrees of freedom equal to the number of maximum lags between the two models. The test procedure is algebraically represented as shown in equation (14).

$$LRT = -2\log_e \left(\frac{L_s\left(\hat{\theta}\right)}{L_g\left(\hat{\theta}\right)} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Where LRT denotes the Likelihood Ratio Test Statistic, Log_e denotes the natural logarithm, while L_s and L_g denote the likelihood functions from the simpler and the general models respectively.

2.4.3. T-Tests

The t-test was used to test the hypothesis that the regression coefficients are significant to the respective models. The test was performed at both 1% and 5% levels of significance.

2.5. Data sources and Sample

Empirical studies on the dynamics of short-rates have applied three different interest-rate data series namely: the federal funds rate (Conley et al., 1997), the seven-day Eurodollar deposit rate (Hong and Li, 2005; Jones, 2003), and the three-month Treasury bill rate (Stanton, 1997; Jiang, 1998; Chapman & Pearson, 2000; and Durham, 2003). The short term interest rate series in Kenya is the Central Bank three-month Treasury bill rate taken from the Central Bank of Kenya Database. The study applied the monthly averages of the 91-day T-BILL rate for the period between August 1991 and December 2007. Prior to 1983, the interest rates used to be controlled by the Government until the implementation of SAP in 1983. In July, 1991, the interest rates were fully liberalized. During this period, the factors influencing the interest rates were mainly the Market factors hence ideal for studying the volatility of the short-term interest rates in Kenya.

3.0 Data Analysis and Presentation of Findings

Figures 4.1 and 4.2 respectively present the level and the differenced series of the monthly averages of the sample short-term rates used in the study. Visual inspection of Figures 4.1 and 4.2 suggest that the short rate (i) was most volatile between January 1993 and December 2001 which includes the period of changes in the Kenyan monetary policies, (ii) that the volatility of the differenced series increases with the level of the short rate and (iii) that the differenced series of the short rate displays volatility clustering. Volatility clustering means that the volatility of the series varies over time.

Before performing the volatility tests, the original series were transformed into stationary series and modelling was performed based on transformed-stationary series. A special class of non-stationary process is the I(1) process (i.e. the process possessing a unit root). An I(1) process may be transformed to a stationary one by taking first order differencing. This was achieved by employing the Augmented Dickey-Fuller (ADF) unit root tests (Dickey and Fuller, 1979) to check for stationarity for the T-BILL rates data series. The null hypothesis, H_0 is that r_t has unit roots while the alternative hypothesis is that r_t is integrated of order zero, I (0). The hypothesis was tested at a critical level of 5% and 1%. (See Table 4.1)

4.1. Time Series Properties of the Sample Short-Term Rates

Figure 4.1: Level Form of Short-term rates Monthly Averages (Jan' 1991 – June 2008)

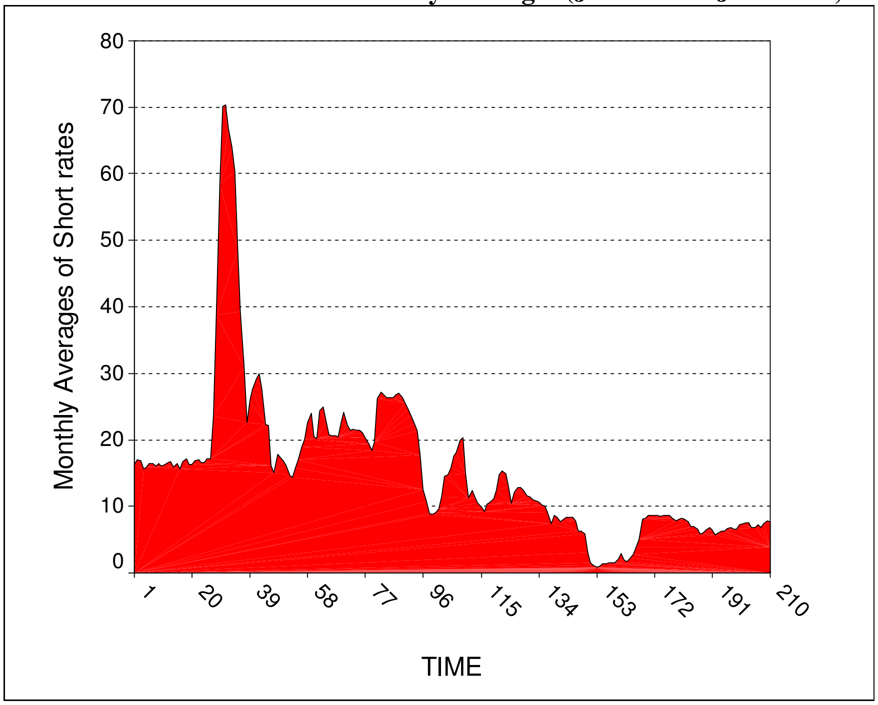

Figure 4.2: Differenced Series of Monthly Averages of Sample Short-term Rates (Jan 1991 – June 2008)

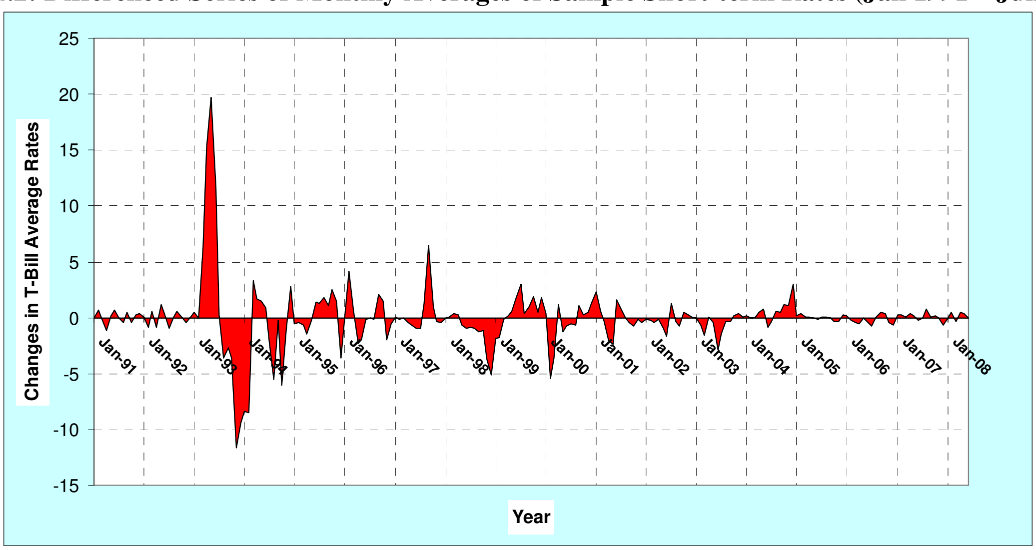

Table 3.1: Unit Root Test for the Sample Short-Term Rate

Variable	ADF	Critical Values (5%)	Critical Values (1%)	Decision
r_t	-2.387	-3.45	-4.04	Accept H_0
Δr_t	-5.305	-3.45	-4.04	Reject H_0

H_0: r_t has unit roots

The results of Table 4.1 were obtained by lagging the variables once. The results also indicate that the short-rate series was non-stationary at level form. This indicated that the series is an I(1) process and therefore differenced series was applied for modelling volatility. The decision rule was based on rejecting H_0: the series is non-stationary, if the ADF statistics are less than the critical values (Dickey and Fuller, 1979).

3.2. Modelling Volatility of Short-Term Rates

3.2.1. Lagrange Multiplier (LM) Test for Level Effects and Asymmetry

The residuals of the regressions of the differenced series were tested for level effects using the ARCH Lagrange Multiplier (LM) test and the results are presented in Table 4.2 below.

Table 4.2: ARCH LM test for Level Effects

Lags (*p*)	Chi-square statistic	Critical Values (5%)	Critical Values (1%)	d.f.	Decision
1	88.783**	3.481	6.635	1	Reject H_0
2	102.449**	5.991	9.210	2	Reject H_0
3	104.740**	7.815	11.345	3	Reject H_0

H_0: no Level effects vs. H_1: level effects disturbance present
* Denotes significance at 5% critical level (P-values < 0.05)
** Denotes significance at 1% critical level (P-values < 0.01)

The LM test was based on the null hypothesis that the differenced series had no level effects. The decision rule was based on rejecting the null hypothesis if the computed Chi-square statistics were greater than critical values of a known chi-square distribution at 95% and 99% levels of confidence. The findings are presented in Table 4.2. The results shows that the residuals developed for the T-BILL differenced short rate had level effects. Since the variance of the errors is not a constant, heteroscedasticity exists for the residuals of the short-term interest rate. Thus, though the serial correlation test, (Breusch-Godfrey LM test for autocorrelation, Table 4.3) show that ARCH model is a good fit for implicit yield on 91 day Treasury bill rate, the level effects are present and hence the model is not a good fit. The tests were based on procedures and decisions rules similar to those of LM test above. Hence, it is necessary to develop a better model to capture the ARCH level effects in the short-term interest rate series.

Table 3.3: Breusch-Godfrey LM tests for autocorrelation

Lags (*p*)	F-Statistic	Critical values		d.f.	Decision
		95%	99%		
1	92.320**	3.84	6.64	(1, 207)	Reject H_0
2	49.776**	3.00	4.61	(2, 206)	Reject H_0
3	33.194**	2.61	3.78	(3, 205)	Reject H_0

H_0: no serial correlation Vs. H1: Serial correlation present
* Denotes significance at 5% critical level (P-values < 0.05)
** Denotes significance at 1% critical level (P-values < 0.01)

3.2.2. Modelling Volatility Using ARCH/GARCH Models

The objective of modelling the stochastic volatility underlying 91-day T-BILL rate changes in Kenya is to allow for determination of better forecasting models by players in the Kenyan financial markets. Empirical evidence indicates that parameters for the models shift over time (Johnston and Scott, 1999), therefore it is more appropriate to calculate model parameters from time to time. Accurate descriptions of the short term distributions would allow for development of improved forecasting models. In this study, the parameters of the GARCH (1, 1) and ARCH (1, 1) models were calculated over the sample period, using maximum likelihood estimation. The findings derived of the maximum likelihood estimation are presented in Table 4.4 below.

Table 3.4: Modelling short-term interest rates using ARCH/GARCH Model (The variance equation)

Model	Coefficient	Value	Z-Statistic	P-values	Decision
ARCH (1,1)	Constant	0.6240267	1.74	0.081	Accept H_0
	Lag (1)	0.6932527	4.77**	0.000	Reject H_0
	Lag (2)	0.193107	1.48	0.140	Accept H_0
	Lag (3)	-0.4378276	-2.63**	0.008	Reject H_0
GARCH (1,1)	Constant	0.6240267	1.74	0.081	Accept H_0
	Lag (1)	-0.1852768	-1.56	0.120	Accept H_0
	Lag (2)	0.5983886	3.51**	0.000	Reject H_0
	Lag (3)	0.0868504	1.21	0.227	Accept H_0
LR Statistic = -386. 5642**					
Wald Chi-square Statistic $_{(d.f. = 1)}$ = 7.43E+11 **					

H_0: Value of Constants =0 vs. H_1: Otherwise
* Denotes significance at 5% critical level (P-values < 0.05)
** Denotes significance at 1% critical level (P-values < 0.01)

The findings of Table 3.4 above indicate that the residuals of the two models are in nonlinear form, that is, they have the volatility clustering effect and this is indicated by the significant coefficients of the ARCH(1) and GARCH(1) terms in the variance equation of the differenced 91 day Treasury bill rate. The sum of the significant coefficients on the lagged squared error and lagged conditional variance is less than one in all the cases. The sum equals 0.255426 for the ARCH (1,1) model (equivalent to lag 1 + lag 3 since lag 2 is not significant) and 0.5983886 for the GARCH (1,1) model (equivalent to lag 2 only since lag 1 & lag 3 are not significant). This sum is close to unity in the case of GARCH model indicating that shocks to the conditional variance will be highly persistent. A large sum of these coefficients implies that a large positive or a large negative return will lead future forecasts of the variance to be high for a protracted period. The variance intercept term 'constant' is very small (<1) as expected.

3.2.3. Likelihood Ratio Test

The likelihood ratio test (LRT) statistic presented in Table 4.5 indicate the significance of the goodness-of-fit between the two models as earlier identified by Hanfeng, Jiahua and Kalbfleisch, (2000). The form test represents the ratio of two likelihood functions for both the ARCH and GARCH series. Asymptotically, the test statistic is distributed as a chi-squared random variable, with degrees of freedom equal to the number of maximum lags between the two models. The test was based on the null hypothesis that there was no goodness-of-fit between the two models. The decision rule was to reject the null hypothesis if the absolute value of the computed statistic is greater than the critical values at the designated levels of significance. The null was thus rejected hence implying that there was significance of the goodness-of-fit between the two models at both 95% and 99% levels of significance.

Table 3.5: Likelihood Ratio Test (LRT)

Number of lags	LR Chi-square Statistic	Critical values		d.f.	Decision
		95%	99%		
3	-386. 5642**	7.815	11.345	3	Reject H_0

H_0: no Goodness-of-fit between the two models vs. H_1: Otherwise

* Denotes significance at 5% critical level (P-values < 0.05)

** Denotes significance at 1% critical level (P-values < 0.01)

3.2.4. ARCH Lagrange Multiplier Test for Level Effects

The residual series obtained from the estimated GARCH models of Table 4.4 above were tested for level effects to see if level effects are captured well in the estimated model. The findings are presented in Table 4.6 below.

Table 3.6: ARCH LM test – Residuals of the GARCH model

Lags (*p*)	F-Statistic	d.f.	Decision
1	0.034110	(1, 207)	Accept H_0
2	0.072686	(2, 206)	Accept H_0
3	0.240983	(3, 205)	Accept H_0

H_0: no ARCH level effects present vs. H_1: ARCH level effects disturbance present

* Denotes significance at 5% critical level (P-values < 0.05)

** Denotes significance at 1% critical level (P-values < 0.01)

The findings of Table 4.6 above indicate that the ARCH effects are not present in the model estimated after taking into account the GARCH terms. Thus, the GARCH model is better than the ARCH model for modelling volatility of short-term interest rates. However, the GARCH models estimated do not take into account the leverage effect and hence the E-GARCH models would be developed to test whether asymmetric effects are present.

3.2.5. Summary

The study identifies that the GARCH model is better suited for modelling volatility of short rates in Kenya, as opposed to ARCH models. The general specification is therefore of the form of a Stochastic Differential Equation (SDE) for the instantaneous risk free rate of interest 'r_t' represented by Equation (15) below

$$dr_t = (\alpha + \beta r_t)dt + \sigma r_t^{\lambda} dZ \quad \text{.......................} \quad (15)$$

Where dZ denotes the standard Wiener process or Brownian motion and σr_t^{λ} is the instantaneous standard deviation of interest rate changes which is often referred to as 'volatility'. The drift component of short term interest rates is captured by $\alpha + \beta r_t$ where the restriction applied was that $\beta + \alpha < 1$, and β_i, α_i, > 0 for i = 0, 1, 2 and 3. This restriction was found to be consistent with the evidence provided by Chan, Karolyi, Longstaff and Sanders (1992), Longstaff and Schwartz (1992), and Brenner, Harjes and Kroner (1996).

4.0 Conclusion

The aim of this study was to develop a general specification that can be used to model the sensitivity of volatility to the level of short-term interest rates in Kenya. The following research questions guided the study: Is there a link between the level of short-term interest rates and the volatility of interest rates in Kenya using the Treasury bills from August 1991 to December 2007. In answering this question, the study applied historical data for the monthly (average) 91-day T-BILL rates which were obtained from the Central Bank of Kenya. The key findings revealed that there exists a link between the level of short-term interest rates and volatility of interest rates in Kenya. Secondly, the study's key findings revealed that the GARCH model is better suited for modelling volatility of short rates in Kenya, as opposed to ARCH models.The results of the study were consistent with the hypothesis that the volatility is positively correlated with the level of the short term interest rate as documented by previous empirical studies (Olan and Sandy, 2005; Turan and Liuren, 2005). The key findings revealed that there exists a link between the level of short-term interest rates and volatility of interest rates in Kenya. Secondly, the study's key findings revealed that the GARCH model is better suited for modelling volatility of short rates in Kenya, as opposed to ARCH models.

The GARCH model is a more general case than the ARCH model. In their original form, a normal distribution is assumed, with a conditional variance that changes over time. For the ARCH model, the conditional variance changes over time as a function of past squared deviations from the mean. The GARCH processes variance changes over time as a function of past squared deviations from the mean and past variances. Overall results demonstrate that, although previous research indicates that volatility clustering plays a role in interest rate changes, it is not the primary factor generating these changes. GARCH models with normality assumptions provide a better description of exchange rate dynamics. Frequency distributions show independence still exists in the data after removing the ARCH effects. Likelihood ratio tests indicate the significance of the goodness-of-fit between the two models as earlier identified by Hanfeng, Jiahua and Kalbfleisch, (2000). The study further establishes that GARCH models are able to capture the very important volatility clustering phenomena that has been documented in many financial time series, including short-term interest rates (Bollerslev, Chou, and Kroner, 1992), as well as their leptokurtosis. Note that in GARCH models the volatility is a deterministic function of lagged volatility estimates and lagged squared forecast errors. One problem with GARCH models of the short-rate is that the parameter estimates suggest that the volatility process is explosive.

4.2. Further Research

Future research can examine if other forms of the GARCH process can account for the independence found (i.e., EGARCH, PGARCH, GARCH, and FIGARCH). They should also be tested to determine if they are superior to the ARCH/GARCH specification in regard to modelling volatility of short-term rates. Since all forms of the GARCH process are similar in form, focusing on volatility clustering, it would be interesting to see if they are an improvement. The study applied monthly observations, as opposed to daily or weekly observations. Therefore, further research can be done using weekly data on the 91-day T-BILL rate to ascertain if there would be any significant deviations from the findings of this study.

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