The Financial Crisis Impact on the Composition of an Optimal Portfolio in the Stock Market - Study Applied to Portuguese Index PSI 20

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Abstract

In order to maximize their utility function, investors select some assets over others, choosing the portfolio that will allow them to maximize their wealth. Each asset is chosen considering the relationship between the risk of that particular investment (usually measured by variance) - and the profitability it can offer, as well as the risk between this and other assets (measured by covariance). The purpose of this study consisted of constructing the minimum variance portfolio, using data from the PSI-20 (2008-2016) representative asset quotation, where investors are risk reluctant and wish to minimize risk while maintaining the same level of profitability, or on the other hand, maintaining the same level of risk but maximizing expected profit. In order to do this, a comparison of the optimal portfolio in 2004-2017 was carried out, compared to the minimum variance portfolio after the financial crisis (2008-2016). The method used to estimate each asset's expected profitability that makes up the PSI-20 consists of extracting the obtained historical quotations. The optimal portfolio composition, in the period after the financial crisis, shows that the energy sector has an optimal portfolio weight reduction of 39.15%, that the big distribution sector (23.85%) was introduced into the portfolio and by last, the industrial sector stands its ground in the composition of the optimal portfolio.

Keywords: stock markets, portfolio, risk, profitability, financial crisis

JEL Classification: G10, G11, G14

1. Introduction

An investment portfolio is defined as a group of assets, financial or not, who belong to a certain investor that may be an individual or legal entity, constituting an investment portfolio, a particular asset. In a context of uncertainty and with limited resources, rational investors, that is, an investor facing alternative investments with the same level of risk, chooses the investment with the highest expected profitability, and risk reluctant. The investor may have (i) a high risk reluctance, and be strongly concerned about its safety, demanding considerable increases in profitability in the light of possible increase of risk, or (ii) an average risk reluctance, in which the aim is to achieve increases in profitability compatible with any observed risk increase, and (iii) a risk reluctance that tends to devalue the possible repercussions resulting from increased risk. This choice is called portfolio, or asset portfolio. The investor chooses the optimal portfolio taking the mean and profitability variance of the assets exclusively into account, and this notion of portfolio efficiency is based on the assumption that the individual's welfare increases with the expected profitability and decreases with risk.

This way, the behaviour is conditioned by two aspects: the profitability, which is the mean or the expected value of the profitability rate's probability distribution, associated with a security portfolio that matches the investment potential profitability, on the other hand, the risk, which is presented by the variance or the standard deviation of the profitability rate's probability distribution, associated to a security portfolio corresponding to the risk. We can still say that a portfolio is efficient only if, for the same level of risk, there is no other portfolio that allows an expected profitability higher than this.

2. Literature review

Harry Markowitz (1952) broadened the new financial theory horizons by linking the valorisation problem and stock selection to that of portfolio management, one of the most used models. Its approach to the selection of portfolios, which he called efficient, was based on the expected profitability of the shares and in the variability in obtaining this profitability, that is, the portfolio risk. The purpose of his model is to combine the shares in a portfolio to reduce risk into the same level of profitability. A rational investor will seek to optimize expected profitability and minimize risk. Before this, he faces a problem of choosing the combination of securities that make up the portfolio, in order to reach his goals. This way, the role of the investor is to identify the assets in which the investment is to be made, as well as the total investment proportions to be applied to each asset.

Markowitz (1959) argues that for the investor, the expected return and expected returns volatility are key aspects in attempting to establish an optimal portfolio. The expected portfolio of assets rate of profitability is obtained by the rate of profitability's expected mean values of the securities comprising it, considered according to the proportions of the amount to be invested in each of these securities.

$$E(R_p) = \sum_{i=1}^{n} E(R_i) \times w_i$$

Equation 1: Expected profitability

Therefore, E (Rp), represents the portfolio's expected rate of profitability, E (Ri), corresponds to the expected rate of return for security i; n, is the number of securities that make up the portfolio; wi, relative weight of security i in the portfolio, considering

$$\sum_{i=1}^{n} w_i = 1$$

Equation 2: Weight of assets in portfolio composition

The expected rate of profitability standard deviation is given by the square root of the covariance's sum between each pair of securities weighted by the proportion of the amount invested in each of them:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}}$$

Equation 3: Portfolio's standard deviation

"σ_p" represents the portfolio standard deviation expected rate of profitability, σ_i corresponds to the standard deviation of security i's expected rate of profitability; n, it's the number of the portfolio securities; w_i is the portfolio's i security relative weight; σ_{ij} is the covariance between the profitability rates of i and j securities.

Thus, it can be concluded that the risk of a financial assets portfolio does not depend exclusively on the risk of each of its securities, it is determined by the each security's risk, by the respective portfolio security weight and also by the respective correlation that may exist between the various securities that form the portfolio expected rates of profitability. It can also be verified that when there's a portfolio of securities that has a non-perfect linear correlation, its risk is lower than the average risk of the securities that make up the portfolio. This way, the risk of the portfolio will be reduced as the correlation coefficient approaches as closely as possible a perfect negative linear correlation. To this end, the rational investor will seek to pursue a diversification strategy in order to maximize its profitability and to minimize risk, which will depend on the diversification degree, that is, the greater the number of securities to be incorporated in a portfolio, the less the investor is subject to a specific risk corresponding to each of the securities representing the portfolio.

Since diversification will reduce risk, which leads to an increase in the portfolio's number of representative securities and a standard deviation reduction, in other words, the risk. However, when a certain number of securities is reached, this portfolio risk will no longer decrease since diversification allows us to reduce risk but does not eliminate portfolio risk, if n tends to infinity we can verify that the portfolio risk is market risk or systematic risk.

Considering a portfolio of n securities, and variance of the rate of return obtained by:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{i,j} = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n} w_i w_j \sigma_{i,j}$$

Equation 4: Variance of a portfolio composed of N assets

We can verify that the total risk of a portfolio can be reduced through diversification. However, a fully diversified portfolio has a certain level of risk, the market risk. Dividing the risk of the security into two components, the specific risk, which results from factors affecting only the considered security performance and not the totality of securities traded on the market. On the other hand, market risk will correspond to the risk that comes from behaviour affecting factors of all securities in the market. Taking into account the risk, we can affirm that the specific risk factors are distinct for each of the securities, by making the profitability of securities vary in a non-convergent direction and intensity, the specific risk can be reduced and can even be eliminated through diversification. Moreover, market risk is not eliminated through diversification, since the total risk of the portfolio cannot be eliminated.

Martins and Fernandes (2003, p. 221) show that "The construction of general equilibrium models allows one to obtain a relevant risk measure for each security as well as the ratio between expected profitability and risk for each asset when markets are in balance".The capital asset pricing model (CAPM) was developed by Sharpe, Lintner and Treynor. It's a model of general equilibrium of the capital market from which relations allows us to estimate the expected profitability for a bond in function of the expected capital market rate of return. The fundamental idea underlying the CAPM is that, in equilibrium, the market rewards investors according to the level of risk assumed in their investment. However, since part of an asset's total risk can be eliminated via diversification, only the part of the non-disposable risk is remunerated, so the risk prize of a particular security is directly related to the contribution of that security to an efficiently diversified portfolio.

Although anchored in the portfolio theory advanced by Markowitz and in the market model proposed by Sharpe, the formalization of CAPM, as we know it today, had the contributions of Sharpe (1964), Lintner (1965) and Mossin (1966). Ross (1976), presents the Arbitrage Pricing Theory model, which was established based on arbitrage and in this sense is not a general equilibrium model. Nonetheless, this model presents the same functional form as the CAPM, and may even be considered an extension of it, however, contrary to the CAPM model, when it was developed, no assumptions were made as to the usefulness of investors, nor on profitability empirical distribution of the of the assets or on the perfection of the markets for financial assets. The APT model expresses an asset's rate of profitability or portfolio of assets, according to several systematic factors, thus being a multifactor model, while the CAPM expresses this same rate of profitability as a function of only a systematic factor, that is, the copy of market portfolio.

The Arbitrage Pricing Theory establishes a linear relationship between the assets expected returns excess compared to the risk-free interest rate and a series of variables or factors. If it is considered a single factor, the relation established by the APT model will correspond to the same relation established by the CAPM. However, these models are conceptually distinct, since CAPM is a general equilibrium model, meaning that for its development arises the need to establish the assumptions regarding the expectations of the agents, regarding preferences, as well as provisions, on the other hand, the APT is established on the basis of arbitration arguments.

The idea underlying this APT model is that the constitution of a non-arbitrage portfolio, that is, a portfolio that does not involve any risk, not involving any risk to the investor, that is to say, specific or systematic risk and no initial investment involved, since the disposal of certain assets generates sufficient funds to acquire other assets, considering that this portfolio should have an average profitability of zero. After the initial study by Markowitz, many researchers have been conducting research in order to obtain a better optimization of the composition of portfolios, such as the goal programming model (Lee, 1972), using multi-objective decision making, and based on the concept of finding viable points as close to the goals as possible.

Konno and Yamazaki (1991) presented the mean absolute deviation model, which uses this as a risk measure and establishes that the uncertainties regarding the assets returns are what allows the construction of the optimum portfolio. On the other hand, Jorion (1997) presented the Value-at-Risk model, a measure with a high degree of acceptance, used by a wide variety of financial institutions in order to reduce market risk.

This model provides an estimate of what could be the maximum potential loss at a certain level of confidence to which an institution would be exposed for a pre-established period of time. Young (1998) developed the MiniMax model, that presents a portfolio optimization in which the candidate assets' returns are presented in a discrete way through the use of scenarios, and the result obtained by the worse return scenario is established as a measure of risk. In (2000), through Rockafellar and Uryasev, the Conditional Value-at-Risk model is presented, and is defined as the conditional expected value of the possible losses for a certain portfolio, since the losses to be assumed will be those that present higher value or equal amount to the VAR.

Cai et al. (2000) developed the minimax rule in portfolio selection and established a new model based on maximum absolute deviation as a measure of risk. The mean absolute deviation has been used as a risk measure in the selection of cardinality-restricted portfolios, where genetic algorithms provide efficient portfolios (Chang et al., 2009). From the perspective of classical modelling, a portfolio is optimal when it is intended to satisfy a certain balance between maximizing the desired return and minimizing investment risk, with the preferences of a given investor being present. The portfolio selection problem is an optimization problem having multiple objectives and/or additional constraints. In this paper, we propose the use of a multi-objective portfolio selection model (Ehrgott, Klamroth and Schwehm, 2004; Steuer, Qi and Hirschberger, 2007). Recently, some researchers have used more evolved multiobjective algorithms in order to deal more conveniently with this portfolio optimization problem (Anagnostopoulos and Mamanis, 2011; Liagkouras and Metaxiotis, 2014).

Solving the portfolio selection problem should seek to hold both the assets and the respective proportions that correspond to the optimal investment. Then, to determine the optimal portfolio, both optimization techniques and uncertainty quantification approaches must be considered. An alternative structure for portfolio selection decision-making analysis is based on the fuzzy set theory, which also allows for the incorporation of unknown knowledge into market behaviour from a modelling perspective. To address the problem of fuzzy portfolio selection, it is necessary to formulate how to address the uncertainty of the respective portfolio returns. Generally, the return of individual risk assets was approximated using fuzzy numbers with probability distributions (Carlsson, Fullér and Majlender, 2002; Vercher, Bermúdez and Segura, 2007). However, some researchers have recently sought to address these returns using credible distributions or on the assumption that returns on assets are random fuzzy variables (Hasuike, Katagiri and Ishii, 2009; Huang, 2008). On the other hand, fuzzy measures Value-at-Risk and fuzzy skewness were considered as viable for portfolio selection when symmetric returns on assets are not assumed (Li, Qin, Kar and Watada, 2011; Vercher and Bermúdez, 2013). Some investigations have been made regarding fuzzy multifaceted decision making for the portfolio selection problem, where different computational approaches have been considered to determine the respective efficient frontier (Bermudez, Segura and Vercher, 2012; Gupta, Inuiguchi, Mehlavat and Mittal, 2013; Jiménez and Bilbao, 2009).

Vercher, Bermúdez (2015), propose a heuristic procedure that combines a genetic algorithm and a decision support system in order to select the most efficient portfolios. The Pareto front obtained are widely distributed, showing good coverage in relation to two measures of risk: absolute mean half-deviation and VAR. on the other hand, tested their performance for a set of Spanish stock market data under different investment perspectives comparing with ex-post returns using the heuristic procedure.

3. Methodology

In order to verify the characteristics in terms of efficiency in the Portuguese market, it was attempted to analyse the listed companies in PSI-20 and verify the efficiency of the portfolio. The methodology used followed three steps: (1) identification of the market portfolio *proxie*: for this matter, the PSI-20 index was observed, namely the company quotations that comprise this index, in the period between May 2008 and May 2016, using the extracted weekly data. For this purpose, historical data was used in order to estimate the returns and expected risk for each of these companies that make up the PSI-20, because of the fact that there is a great subjectivity and inherent difficulties in its prediction, assuming that these historical data are relevant, which leads us to believe that these data corresponds to a reasonable representation of what may occur in the future.

4. Analysis and results discussion

4.1 Post financial crisis period

It should be noted that, at the time of analysis, the PSI-20 index was quoted by only 18 companies.

However, for our analysis, three companies were withdrawn because they did not present prices for some of the periods in question, such as CTT, EDP Renováveis and Montepio, which will not be subject of our analysis; (2) Determining, by the Markowitz theory of Efficient Frontier: For this point, and considering the determination of Markowitz efficient border, after simulated the minimum variance portfolio, for different levels of expected return, and having the maximization of the variable as its starting point. To do so, and in order to comply with the model's assumptions, we calculated the securities variance and the expected returns that are represented in the index under analysis, the covariance-variance matrix and, finally, the standard deviation and the presented portfolio expected profitability, which will be the subject of a more detailed description in the following sections; (3) analysis of the portfolio regarding efficiency and compliance with the diversification effect: at this point it was attempted to observe the characterization of the efficiency of the portfolio, first using equal weights for the respective securities, as well as for the analysis of the securities efficiency using the deliberations obtained through the optimization made with the Solver.

The database refers to the quotations of the PSI-20 companies in May 2016:

Table 1

PSI-20 companies description			
Symbol	Name	Symbol	Name
ALTR.LS	ALTRI	JMT.LS	JERONIMO MARTINS
BCP.LS	BCP R	MPIO.LS	CEMG
BPI.LS	BANCO BPI R	NOS.LS	NOS
COR.LS	CORTICEIRA AMORIM	PHR.LS	PHAROL
CTT.LS	CTT	RENE.LS	REN
EDP.LS	EDP-ENERGIAS R	SEM.LS	SEMAPA R
EDPR.LS	EDP RENOVAVEIS	SON.LS	SONAE R
EGL.LS	MOTA ENGIL	SONC.LS	SONAE CAPITAL
GALP.LS	GALP ENERGIA B	NVG.LS	NAVIGATOR COMPANY

The PSI-20 index is based on the 20 largest listed companies on Euronext-Lisbon. The data base consists of weekly returns from May 26, 2008 to May 23, 2016. The data presented will exclude companies that do not present stock prices for a period of more than 8 years. CTT, EDP Renováveis and Montepio, were removed from this analysis, as stressed previously. In order to obtain the portfolio composition that minimizes risk, the quotations data of each assets that compose the PSI-20 were obtained, using data obtained from Yahoo Finance (finance.yahoo.com), in which the assets expected returns that make up the PSI-20 were determined through the use of daily quotations in continuous returns, using the formula:

$$t = ln\left(\frac{x_t}{x_{t-1}}\right)$$

Equation 5: Determination of expected profitability

Based on the closing quotations of each Monday between May 26, 2008 and May 23, 2016.
The following returns which are presented in table 2 were obtained:

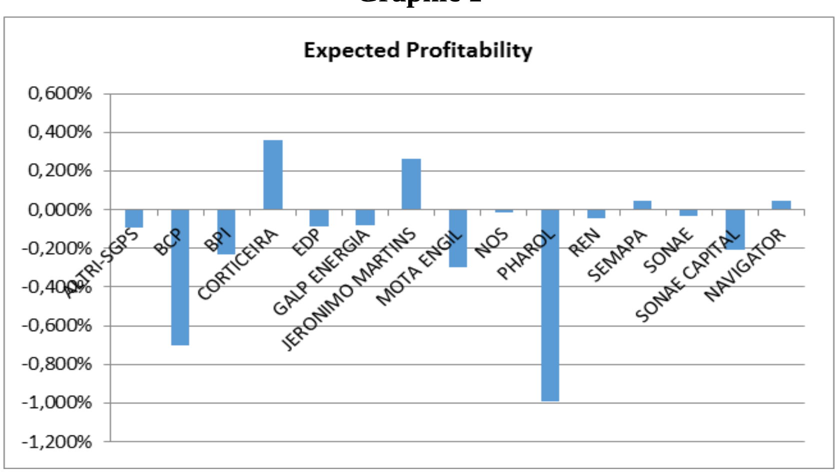
Table 2

Average profitability of listed companies in PSI-20	
Listed Companies	**Average Profitability**
ALTRI-SGPS	-0.09%
BCP	-0.70%
BPI	-0.23%
CORTICEIRA	0.36%
EDP	-0.08%
GALP ENERGIA	-0.08%
JERONIMO MARTINS	0.26%
MOTA ENGIL	-0.30%
NOS	-0.02%
PHAROL	-0.99%
REN	-0.04%
SEMAPA	0.05%
SONAE	-0.04%
SONAE CAPITAL	-0.21%
NAVIGATOR	0.05%

From the table above, we can see that of the 15 companies listed in the Portuguese PSI-20 index, only four of these companies had a positive average return over the analyzed period, to which we will now refer in descending order to the respective yields, Corticeira which presented the highest average yield in this period, about 0.36%, JERONIMO MARTINS with a 0.26%, profitability, and the remaining two companies show a positive profitability, however very close to zero, SEMAPA, with about 0.047% and NAVIGATOR, with an expected return of 0.046%. It should be noted that these companies which present average returns for the period under consideration are from quite different sectors.

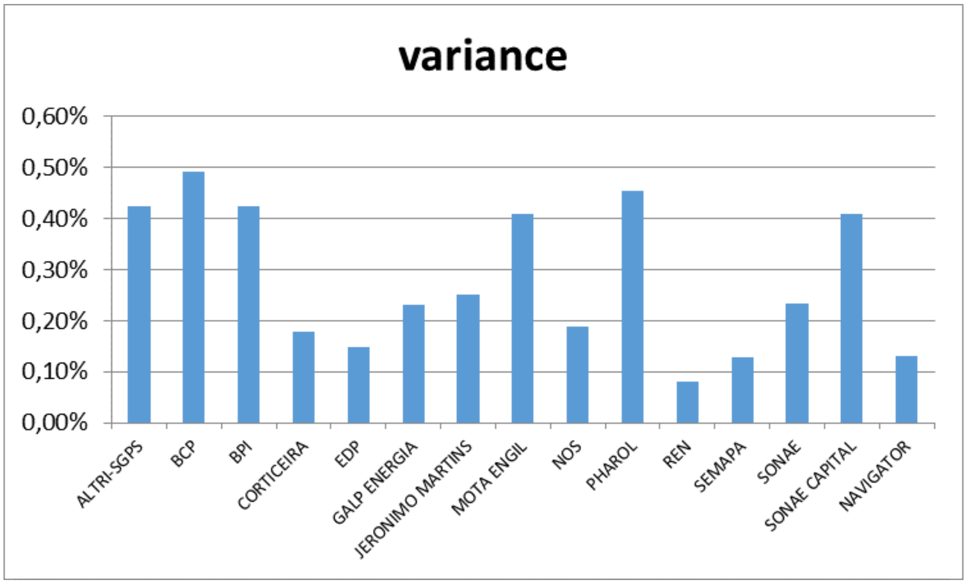
By exploring these data presented in Table 2, we will determine the portfolio that minimizes risk. The graphics 1 and 2 introduce the analysis of securities assigned to PSI-20, in terms of expected profitability and risk, as measured by variance. The graphics are presented for the period from May 2008 to May 2016. On the other hand, we can also see in graph 2 the risk for each of these companies.

Graphic 1



As seen in the previous chart, we can verify the expected profitability for each of the assets that make up the PSI-20.

Graphic 2



In a first observation of the presented data, we can verify that, in descending order, Corticeira, Jerónimo Martins, Semapa and Navigator, are those that present greater profitability in historical terms. At a first analysis of these data, and ignoring the correlation effect between the assets, these should be the titles with greater weight in the portfolio. On the other hand, we also find that Pharol and Altri have the lowest historical return, which are referenced to in Table 1 as the companies with the most negative expected returns. The portfolio was optimized in order to determine the composition of the respective weights to reach an expected return of 0.36% at the lowest possible risk. The results obtained considering this goal were, as expected, a return of 0.36%, however the risk measured by the variance was 0.18%, as well as the respective weights of the assets. The results are presented in Tables 3 and 4.

Table 3

Portfolio optimization	
Expected profitability	*0,36%*
Risk (standard deviation)	*4,21%*
Variance	*0,18%*

Table 4

Portfolio composition	
Companies	**Weights**
CORTICEIRA	*100.00%*
TOTAL	*100.00%*

Finally, with a rate of return of 0.25%, it was tried aiming at the minimum variance of the portfolio, and the solver presented a solution, where the risk measured by the standard deviation would be 2.93%. For this hypothesis we can observe that it presents a lower return, that is to say, 0.25% comparing to the previous, 0,36%. However, the risk measured by the previously observed standard deviation was 4.21%, and before this solution is 2.93%, with this hypothesis the risk was reduced by 43.7%. For this solution, it is necessary to invest in a portfolio composed of the following assets in their weights ascending order: Navigator (0.55%), Altri (0.56%), REN (6.79%), SEMAPA 98%), Jerónimo Martins (23.85%) and Corticeira (50.28%), as shown in Tables 5 and 6.

Table 5

Portfolio Optimization	
Expected Profitability	0,25%
Risk (standard deviation)	2,93%
Variance	0,09%

Table 6

Portfolio composition	
Companies	**Weights**
ALTRI-SGPS	*0,56%*
CORTICEIRA	*50,28%*
JERONIMO MARTINS	*23,85%*
REN	*6,79%*
SEMAPA	*17,98%*
TOTAL	*100.00%*

4.2 Pre-financial crisis period

In order to compare with the optimal portfolio data before the 2008 financial crisis, we optimized our portfolio to a profitability level of 0.34% at the lowest possible risk and obtained a risk measured by a 0.10% variance. In addition we can verify through Tables 7 and 8, this last one shows the weights to be invested in each of the assets.

Table 7

Portfolio optimization	
Expected Profitability	0,34%
Risk (standard deviation)	3,13%
Variance	0,10%

Table 8

Portfolio composition	
Companies	**Weights**
CORTICEIRA	*43,40%*
EDP	*42,02%*
JERONIMO MARTINS	*3,68%*
SEMAPA	*10,90%*
TOTAL	*100.00%*

Finally, it was tried to reduce the risk as much as possible, and the measured risk had a 0.05% variance and 0.33% expected return.

These results can be observed by looking at tables 9 and 10.

Table 9

Portfolio optimization	
Expected Profitability	0,33%
Risk (standard deviation)	2,16%
Variance	0,05%

Table 10

Portfolio composition	
Companies	**Weights**
CORTICEIRA	*45,46%*
EDP	*45,94%*
SEMAPA	*8,60%*
TOTAL	*100.00%*

5. Conclusions

As a theory, this study used the Markowitz model in the PSI-20 index, aiming to present the main concepts associated to investment portfolios, such as profitability, risk and investment, allowing us to analyse the optimal composition of the portfolio. From the obtained historical data, as data source, the profitability and risk measured by the standard deviation were analysed, in order to solve a quadratic programming problem, whose purpose would be to obtain the portfolio composition for a certain level of profitability.

It is concluded that the optimal portfolio would be composed of a decreasing order of weights to be invested in each of the companies, Corticeira (50.28%), Jerónimo Martins (23.85%), SEMAPA (17.98%), REN, 79%) and ALTRI (0.56%). Finally, the period before the financial crisis in the stock market was compared, and it was verified that the optimal portfolio at that time would be constituted by the investment in the following companies: EDP (45.94%), Corticeira Amorim (45.46%) And SEMAPA (8.60%).

It should be noted that the energy sector no longer has a significant weight in the optimal portfolio to be set up after the financial crisis of 2008, since before the crisis it was represented by EDP with a 45.94%weight and has now a weight of 6.79% with REN. On the other hand, it can be seen that the large distribution sector, which was not contemplated in the optimal portfolio, began to have a weight in the optimal portfolio of 23.85% with the company JERONIMO MARTINS after the crisis. Finally, it was verified that CORTICEIRA AMORIM is the only company that, after the crisis, maintains its preponderance in the optimal portfolio constitution. For future studies related to this topic it is suggested that another model is implemented in order to test these hypotheses, and to verify the optimal structure for the portfolio, or on the other hand, to carry out a comparison of the PSI-20 index with other European market indexes to verify the market's efficiency.

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