Capital Structure in an R&D Duopoly: a Differential Game Approach with Randomly Generated Polynomials

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Abstract

This paper considers financial distress and optimal capital structure given a duopoly market characterized by an upstream firm that is primarily engaged in research and development (the R&D firm) and whose value comes from the market valuation of these activities, and a downstream firm primarily engaged in distribution and marketing (the D&M firm). This is within the context of a differential game with a finite planning horizon. A numerical solution is obtained by randomly generating parameters for second degree polynomials for the decision variables of the R&D and D&M firms and from this derives an estimate of the state equation. The results suggest that optimal capital structure for the R&D firm does not come from converging to a target debt-to-equity ratio as in tradeoff theory, but evolves over time as it responds strategically to decisions made by the D&M firm.

Keywords: capital structure; differential game; research and development; numerical methods; firm value; intertemporal choice

JEL:C7 – Game Theory and Bargaining Theory; D9 – Intertemporal Choice and Growth; G32 – Financing Policy, Financial Risk and Risk Management; Capital Ownership Structure, Value of Firms, Goodwill; O32 - Management of Technological Innovation and R&D; L1 – Market Strategy, Firm Strategy, and Market Performance.

1.0 Introduction

This paper considers financial distress and optimal capital structure given a duopoly market characterized by an upstream firm that is primarily engaged in research and development (the R&D firm) and whose value comes from the market valuation of these activities, and a downstream firm primarily engaged in distribution and marketing (the D&M firm). This is within the context of a differential game. The differential game is represented in continuous time over a finite planning horizon. Analytical solutions to differential games are for the most part restricted to linear quadratic objective functions. For this reason, a numerical solution is obtained by randomly generating parameters for second degree polynomials for the decision variables of the R&D and D&M firms and from this derives an estimate of the state equation. Specifically, second degree polynomials to represent the debt function of the R&D firm and the investment function of the D&M firm. Investment here refers to the purchase by the D&M firm of the intellectual property of the R&D firm. Given the debt function and the investment function, the path of the state variable, the level of R&D capital, is determined. It is the interaction between the path of R&D debt, D&M investment, and R&D capital that determines the capital structure of the R&D firm over time.

The results suggest that optimal capital structure for the R&D firm does not come from converging to a target debt-to-equity ratio, as in tradeoff theory, but evolves over time as it responds strategically to decisions made by the D&M firm; and that even though capital formation is higher when financial distress is lower, the debt-to-equity ratio can be higher in the high financial distress case. Further, this paper presents a solution method that can be extended to a number of similar differential games. A feature of this method makes use of the iterative power and mathematical functions of modern spreadsheets many use on personal computers and laptops.

2.0 Capital Structure and R&D

In their now classic paper Modigliani and Miller (1958) argue that, given a number of assumptions, financial decisions are independent of operational decisions and free cash flow. Firm value does not depend on whether the firm is financed with debt or equity. This result quickly breaks down in the presence of taxes because of the so-called tax shield and the tax savings that result from deducting interest expenses from taxable income (Mogdigliani and Miller, 1963). This suggests that firm value is maximized when it is mostly funded with debt. Since this is seldom observed in the capital structure of actual firms, Stiglitz (1972) and later Castanias (1983) extended this with the tradeoff theory which includes the possibility of financial distress. The tradeoff is between the full utilization of the tax shield and the increase in the probability of default as a firm takes on more debt. Since then a number of studies have addressed the issue of whether firms revert to an optimal target debt-to-equity ratio or not. An alternative to the tradeoff theory is the pecking order theory. As developed by Myers (1984) and Myers and Majluf (1984), it argues that firms prefer internal financing whenever possible since outside investors require a premium based on asymmetric information as to the actual financial and operational health of the firm. Closely related are theories that approach capital structure from the agency cost and corporate governance perspective. Galai and Masulis (1976), Jensen and Meckling (1976), and Stultz (1990) argue that debt encourages decision makers to make riskier investments since the downside is borne disproportionately by bondholders. Grossman and Hart (1982) argue that debt is beneficial in that it acts as a constraint on managers and provides incentives to work harder to meet shareholder expectations. Along the same lines Jensen (1986) argues that debt reduces the amount of free cash flow available for managers to waste on risky ventures.

There is some evidence supporting each of the theories of capital structure discussed above. This evidence is not conclusive. For instance, Rajan and Zingales (1995) argue that although their results support some of the implications of the tradeoff and pecking order theories there are also other very feasible explanations for these results. As an example, both the tradeoff and pecking order theories imply that firms with higher market-to-book ratios have less leverage. The logic is firms with higher leverage are less likely to take advantage of investment opportunities and it has been suggested that the market-to-book ratio is a good proxy for this. Their research found a significant negative relationship for this variable. However, as they point out, there are other reasonable explanations for this negative relationship such as the tendency of firms to issue additional equity when its stock price is relatively high. Despite this, researchers continue to test models on the assumption that the tradeoff theory holds and that firms have a long run optimal capital structure that they revert to when possible. As a recent example, Mukherjee and Wang (2013) show that firms that deviate from a target capital structure have a greater speed of adjustment the bigger the deviation, and that the speed of adjustment is greater for overlevered firms when compared to underlevered firms.

This paper considers capital structure and formation within the context of a duopoly market characterized by an upstream firm that is primarily a research and development firm and whose value comes from the market valuation of these activities, and a larger downstream firm primarily engaged in distribution and marketing. An actual market that fits this model is that of R&D in the pharmaceutical industry. These R&D firms provide a good example because they seldom have positive free cash flow and can go years without generating any revenue at all. Their value comes from the potential of their R&D program and the possibility of a breakthrough resulting in a license agreement, an IPO, or the sale of the firm to a larger, usually downstream firm. Higgins and Rodrigues (2006) show that pharmaceutical companies experiencing pipeline deterioration, due mainly to projected drug treatments failing critical clinical tests, are more likely to acquire other pharmaceutical firms with complimentary pipelines. As an example, in 2002 Merck had 11 drug treatments in its pipeline. All of which were highly touted by management. Of the 11 drug treatments: two were launched in 2003, one was in the process of final FDA approval, two were delayed until 2006, and the remaining six were cancelled or indefinitely delayed. As a result, to restore their pipeline Merck acquired Aton Pharmaceuticals in 2004. In a similar vein, Graham and Higgins (2008) argue it can be more efficient for a large, mature pharmaceutical firm to specialize in downstream activities such as distribution and marketing and to use smaller firms engaged in research to develop its drug pipeline. This allows the larger firm to minimize the risk of failed clinical tests by choosing among the more promising R&D firms.

3.0 Differential Game Model

The differential game developed in this paper is based on a market relationship much like the one discussed in Graham and Higgins (2008). There are two players: an R&D firm whose principal value comes from its R&D pipeline and license fees from the sale of its research to the D&M firm; and a D&M firm whose value comes from investing in R&D and producing a marketable product from this. The R&D firm has an initial endowment of capital that comes from an IPO or some other source. Then it must choose the level of debt, D_t, that maximizes firm value over a finite planning horizon. The D&M firm chooses the level of R&D investment to purchase, I_t, that maximizes firm value over the same finite planning horizon. Since these are continuous variables, the levels of R&D debt and D&M investment are continuous functions that span the T-period planning horizon. A Markovian Nash equilibrium occurs when neither the R&D firm nor the D&M firm has an incentive to change its strategy. The strategies are Markovian since they are time dependent and respond to the intertemporal decisions of the other player.

The R&D firm's optimal control problem is to maximize the following:

$$J^{RD} = \int_0^T e^{-\rho t}\{AK_t^{\alpha} + (g - hK_t)I_t - rD_t - cD_t^{\delta}\}dt\,, \qquad (1)$$

where $J^{RD}$is the value of the R&D firm; ρ is the weighted average cost of capital (WACC) and is the firm's discount rate; $K_t$represents capital assets devoted to R&D at time t; A is a scale coefficient and α is the exponent for the R&D production function which represents the value of the firm's R&D; $I_t$is the D&M firm's investment in R&D projects; $D_t$is new debt; r is the cost of acquiring debt; and the term $cD_t{}^{\delta}$ is the cost of financial distress where the parameter c is positive and determines the level of financial distress and δ is the exponent of the financial distress function. For simplicity a downward sloping price function for R&D investment is assumed. This is captured by the expression: g - hK_t. The capital assets here refer specifically to those assets used in the production of R&D. Any other capital created from net earnings is assumed to be available to shareholders at some future point in time.

The D&M firm's optimal control problem is to maximize the following:

$$J^{DM} = \int_0^T e^{-\omega t}\{bI_t{}^{\beta} - (g - hK_t)I_t - zK_t{}^{-\gamma}\}dt\,, \qquad (2)$$

where J^{DM} represents the value of D&M firm; ω is the firm's WACC; b is a scale coefficient and β is the exponent of the D&M production function which represents the value of a marketable final product; and $zK_t{}^{-\gamma}$ is a term that allows for additional costs of intellectual property. All other variables are as defined above.

The state variable for the system given by Equations 1 and 2 defines the instantaneous growth of capital for the R&D firm. That is:

$$\dot{K}_t = D_t - I_t\,, \qquad (3)$$

with the condition:

$$K_0 = initial\ capital\,, \qquad (4)$$

where K_0represents the initial equity of the firm from, say, an IPO. Equation 3 states that R&D capital increases as the R&D firm takes on more debt and decreases as the D&M firm invests in R&D for the production of a marketable good by purchasing it from the R&D firm. The Hamiltonian equation and the first order conditions based on Pontryagin's maximum principle for the R&D firm for the system represented by Equations 1 through 4 are stated in Equations 5 through 7 below.

$$H^{RD} = \left\{AK_t^{\alpha} + (g - hK_t)I_t - rD_t - cD_t^{\delta}\right\} + \lambda^{RD}(D_t - I_t)\,, \qquad (5)$$

$$\frac{\partial H^{RD}}{\partial D_t} = -r - c\delta D_t^{-(1-\delta)} + \lambda^{RD} = 0\,, \qquad (6)$$

$$\frac{\partial H^{RD}}{\partial K_t} = A\alpha K_t^{\alpha-1} - hI_t = \rho\lambda^{RD} - \dot{\lambda}^{RD}\,. \qquad (7)$$

The Hamiltonian equation and the first order conditions for the D&M firm are stated in Equations 8 through 10.

$$H^{DM} = \left\{bI_t^{\beta} - (g - hK_t)I_t - zK_t^{2}\right\} + \lambda^{DM}(D_t - I_t)\,, \qquad (8)$$

$$\frac{\partial H^{DM}}{\partial I_t} = b\beta I_t^{\beta-1} - (g - hK_t) - \lambda^{DM} = 0\,, \qquad (9)$$

$$\frac{\partial H^{DM}}{\partial K_t} = hI_t + z\gamma K_t^{-(1+\gamma)} = \omega\lambda^{DM} - \dot{\lambda}^{DM}\,. \qquad (10)$$

With respect to the model parameters two key assumptions are made: (i) production functions for both the R&D firm and the D&M firm exhibit diminishing marginal productivity ($\alpha < 1$ and $\beta < 1$), a standard assumption in economic models; and (ii) the financial distress function exhibits increasing marginal costs ($\delta > 1$). For a study that supports this assumption see Van Bingsbergen, Graham, and Yang (2010). For the duopoly described by Equations 1 through 10 and given these key assumptions, the existence of a Nash equilibrium is proved. The proof relies on the continuity and convexity of the control variables, $D_t$and I_t; and the concavity of the Hamiltonians with respect to the control variables and the state variable K_t. This proposition is stated below and the proof is provided in the appendix:

Proposition (existence): Given the assumptions stated above, the duopoly game described by Equations 1 through 10 has a Nash equilibrium.

4.0 Numerical Solution Results

Since analytical solutions to differential games are for the most part restricted to linear quadratic objective functions, a numerical solution is sought. For a differential game like the one modeled in this paper, a numerical method can be developed based on the optimal control problem of each of the players. In this case, an optimal control problem in continuous time. In a Nash equilibrium each player's strategy is consistent in the sense that none of the player's have an incentive to change their best reply strategies. Thus any numerical method that can take one player's best reply and compute the best replies of other players would be a candidate for a numerical solution method. Here there are several numerical methods that can be applied. These can be categorized as either indirect or direct methods. Indirect methods utilize the necessary conditions of the Pontryagin maximum principle. Direct methods attempt to directly maximize the objective function subject to a state equation and control constraints. Both approaches can employ shooting or collocation methods. Shooting methods discretize the time space, make an initial guess on the control variables, state variables, or costate variables; then calculate the implied path over time for the state variable. This continues until convergence to the target occurs.

Collocation methods also discretize the time dimension. Polynomials are fitted across the subintervals such that continuity across the subintervals is met. There are also global collocation methods that search for polynomials that estimate optimal paths across the entire time interval. See Rao (2009) for a survey of numerical methods applied to optimal control problems. The method developed here is a type of collocation method in which, given the current optimal path of the control variable for one player, random guesses for a second degree polynomial are used to calculate the optimal path of the control variable for the other player. From this, the path of the state variable for this iteration can be calculated. This is an iterative process that continues until a convergence criteria based on the state space trajectories for all players is met. Appendix B explains in more detail the algorithm for the numerical solution and provides convergence charts for the case parameterized in this paper.

The solution represents a Markovian Nash equilibrium in which the R&D firm's optimal debt is consistent with the D&M firm's optimal choice of investment in R&D in the sense that neither firm has an incentive to move from the equilibrium. What drives the model is the relationship between R&D debt, D_t, D&M investment, I_t; and R&D capital, K_t. This relationship is captured in the state equation, Equation 3. The parameterization of the model defines a typical growth case, where R&D capital is expected to grow over the planning horizon. The initial issue of equity from, for instance, an IPO is set to $50 million. (Although arbitrary, all dollar amounts are in millions.) This scenario is roughly similar to a firm emerging from the startup stage and entering a rapid growth stage. Since the focus of this paper is on the impact of financial distress, two cases are considered: low financial distress (c = 0.1) and high financial distress (c = 0.4). The parameters must also meet the two key assumptions discussed above: (i) production functions for both the R&D firm and the D&M firm exhibit diminishing marginal productivity; and (ii) the financial distress function exhibits increasing marginal costs.

Each firm's weighted average cost of capital (WACC) is used as its discount rate. The assumption is the R&D firm is younger and smaller and faces a higher WACC (set at 14%) than the D&M firm which is assumed to be larger and with deeper pockets (set at 7%). All parameters, the assigned values, and descriptions are listed in Table 1. The following variables are tracked: firm value for the R&D and D&M firms; the path of the D&M investment function; the path of the R&D debt function; changes in the debt-to-equity ratio, and R&D capital formation. For each of these a comparison between low financial distress and high financial distress is made. These results are summarized in Table 2 and Figures 1 through 4. Table 2 displays the terminal values achieved at the end of the planning horizon for R&D and D&M firm value, the debt-to-equity ratio, total R&D debt, and total D&M investment. Figures 1 through 4 plot the D&M investment function, the R&D debt function, the path of the debt-to-equity ratio based on book value, and the path of R&D capital. In these plots, points on low financial distress curves are represented by triangles and points on high financial distress curves are represented by diamonds.

Table 1: Parameter Values

This table lists the parameters, their values, and a description.

Parameter	Value	Description
A	12.0	Scaling factor for R&D production
α	0.75	Exponent for R&D production
c	0.1/0.4	Cost of financial distress (low/high)
δ	1.9	Exponent for financial distress
r	10%	Interest on debt
b	35	Scaling factor for D&M production
β	0.85	Exponent for D&M production
K_0	50	Initial equity from an IPO
T	7	Planning horizon
ρ	14%	R&D WACC
ω	7%	D&M WACC

Firm Value. From Table 2, comparing low financial distress to high financial distress, R&D firm value decreases, going from $3663.40 million in the low financial distress case to $3234.26 million in the high financial distress case, a decrease of 13.27%. The loss of R&D firm value comes from the reduced level of capital formation across the planning horizon as exhibited in the Capital Formation chart of Figure 4.Firm value for the D&M firm goes from $2368.97 million to $1821.53 million, a decrease of 23.11%.

This decrease is due in large part to the increase in the price of R&D that results from the lower level of capital formation in the high financial distress case. (Recall, to simplify the model, a downward sloping demand curve for R&D is assumed.)

Table 2: Terminal Values

This table compares terminal values for low financial distress (c = 0.1) to high financial distress (c = 0.4) given T = 7, K_0 = $50, Dollars in millions.

	C=0.1	C=0.4	% Change
R&D Firm Value	$3663.40	$3234.26	-13.27%
D&M Firm Value	$2368.97	$1821.53	-23.11%
R&D Debt	$143.73	$113,66	-20.92%
D&M Investment	$94.49	$88.17	-6.69%
D/E	0.98	1.11	13.27%

D&M Investment. Total D&M investment decreases somewhat going from $94.49 million for low financial distress to $88.17 million for high financial distress, a decrease of 6.69%. From the D&M Investment Function chart in Figure 1, it is clear that the optimal path for the D&M investment function under low financial distress is slightly higher across the planning horizon.

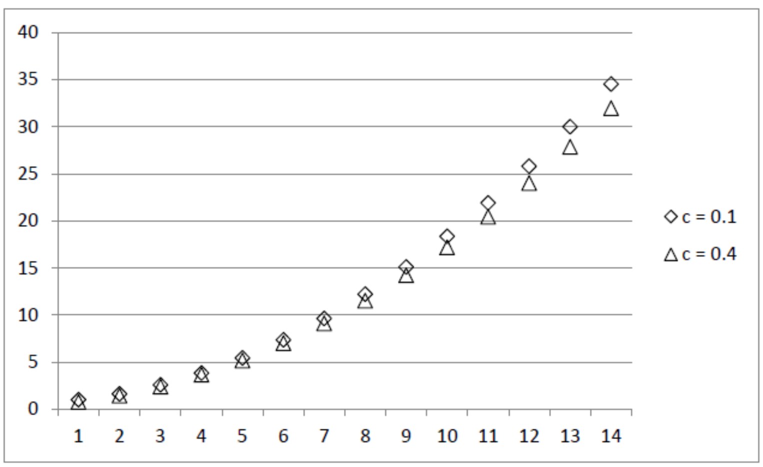


Figure 1: D&M investment function

This figure graphs the D&M investment functions for low financial distress (c = 0.1) and high financial distress (c = 0.4) given T = 7, K_0 = $50. Triangle indicates low financial distress; diamond indicates high financial distress. Dollars in millions.

R&D Debt Formation. Total debt is $143.73 million for low financial distress and $113.66 million for high financial distress, a decrease of 30.92%. From the R&D Debt Function chart of Figure 2 the path of the debt function for low financial distress is higher across the planning horizon.

Debt-to-Equity Ratio (D/E). D/E is calculated based on the book value of equity and the book value of debt. Considering the terminal D/E, it actually goes up from 0.98 for low financial distress to 1.10 for high financial distress case, an increase of 13.27%. This somewhat counterintuitive result is primarily due to the reduced equity in this case. This comes for the most part from a reduction in the creation of net earnings created by the increased cost of financial distress. This can be seen from the D/E Ratio chart in Figure 3.

Also note there is no single optimal debt-to-equity ratio. It varies over time as the R&D firm and the D&M firm make optimal strategic decisions with respect to debt formation and investment.

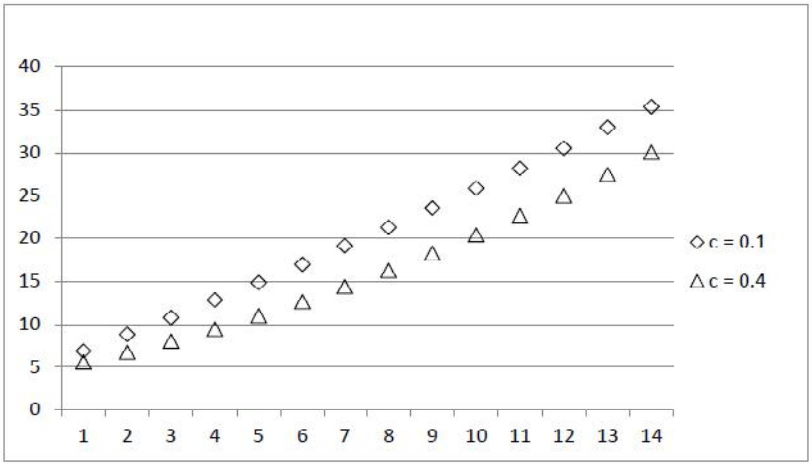


Figure 2: R&D Debt Function

This figure graphs the R&D debt functions for low financial distress (c = 0.1) and high financial distress (c = 0.4) given T = 7, K_0 = $50. Triangle indicates low financial distress; diamond indicates high financial distress. Dollars in millions.

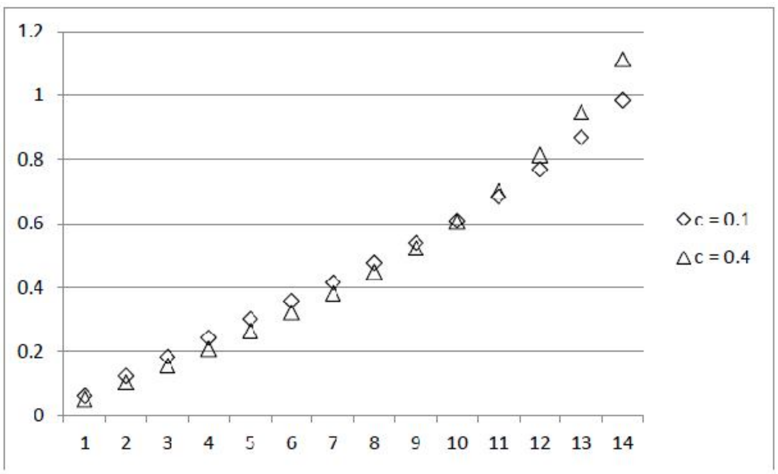


Figure 3: D/E Ratio Based on Book Value

This figure graphs the debt-to-equity ratio based on book value of equity for low financial distress (c = 0.1) and high financial distress (c = 0.4) given T = 7, K_0= $50. Triangle indicates low financial distress; diamond indicates high financial distress.

R&D Capital. From Figure 4, given initial equity of $50 million, R&D capital formation in the low financial distress case increases over the planning horizon to $146 million. For high financial distress it rises to a peak of $104 million and tails off to $102 million by the end of the planning horizon. That R&D capital formation is less in the high financial distress case is directly related to the path of the debt function. Under high financial distress debt formation is less across the planning horizon as described above.

Summary. Comparing the results for low financial distress to those of high financial distress: the path of D&M investment is slightly higher for low financial distress across the planning horizon; the path of R&D debt is significantly higher than for low financial distress across the planning horizon. The debt-to-equity ratio, somewhat counter intuitively, is higher under high financial distress due to lower equity creation; and R&D capital formation under low financial distress is higher across the planning horizon.

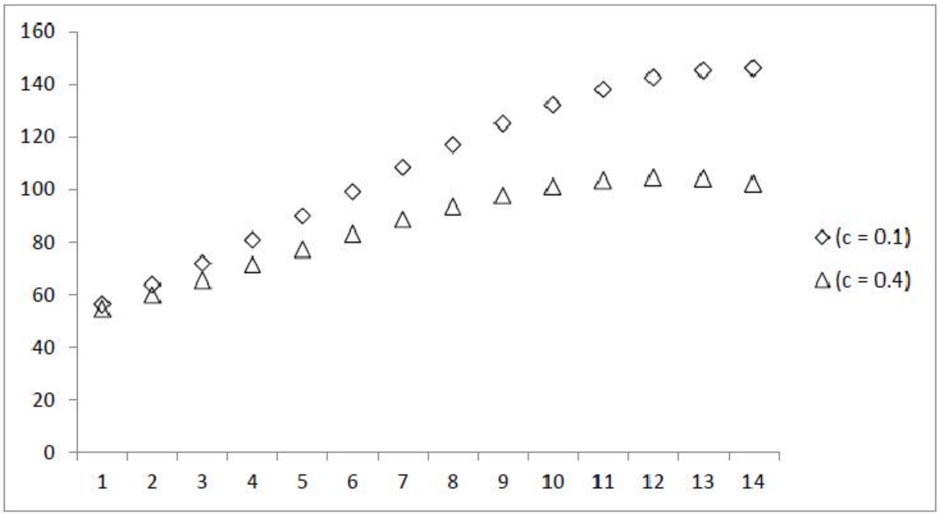


Figure 4: R&D Capital Formation

This figure graphs R&D capital formation for low financial distress (c = 0.1) and high financial distress (c = 0.4) given T = 7, K_0= \$50. Triangle indicates low financial distress; diamond indicates high financial distress. Dollars in millions.

5.0 Conclusion

The results of the numerical solution are in contrast to tradeoff models pioneered by Stiglitz (1972) and Castanias (1983). These are still the most widely accepted and widely studied models of capital structure. In tradeoff models, firms have a target debt-to-equity ratio and over time converge to this optimal capital structure. Here, it is argued that for some firms a reasonable strategy is to grow the firm over a finite planning horizon. This embodies a strategy with respect to the growth of the business and a possible exit strategy. Taking this approach, numerical solutions to the differential game suggest that the resulting capital structure can cover a broad range of outcomes. Decisions of both firms must take into account the best response of the other. It is clear that in this case capital structure is not driven by a reversion to a target debt-to-equity ratio, but by maximizing firm value under strategic considerations. This paper also provides a solution method for differential games similar to the one here in which there is a small number of players and each player has a single decision or control variable. It is a type of collocation method that finds the best polynomials that maximize the players' objective function. The primary innovation here is the parameters for the polynomial are randomly generated for each player's decision variable. An important advantage of this method is that it relies on functionality available for most spreadsheet software.

Appendix A: Proof of the Proposition

Proposition (existence of the differential game): To prove the existence of a Nash Markowitz equilibrium for the duopoly differential game represented by Equations 1-10 one must show: (i) the convexity and continuity of the control variables for the D&M and R&D firms, (ii) the convexity and continuity of the state variable, and (iii) the concavity of the Hamiltonians for the D&M and R&D firms. See Dockner, et al (2000, Theorems 4.1 and 4.2).
Convexity (i and ii): Convexity of the control variables, $D_t$and I_t, follows from the non-negativity of these variables. Convexity of the state variable, K_t, also follows from this condition. That is the following is assumed to hold:

$$D_t \geq 0;$$

$$I_t \geq 0;$$

$$K_t \geq 0.$$

Concavity of the Hamiltonians (iii): Concavity is based on the negative definiteness of the Hessian matrices associated with the Hamiltonian functions for the R&D firm and the D&M firm. For a given function $H(x_1, x_2)$, the Hessian is defined as follows:

$$Hessian = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix},$$

where H_{ij}represents the second order partial derivatives for the i and j variables. It should be noted by the Cauchy's Theorem that H_{ij}must equal H_{ji}.

For the R&D firm the Hessian is derived by taking the second order derivatives of the Hamiltonian in Equation 5:

$$Hessian^{RD} = \begin{bmatrix} A\alpha(\alpha-1)K_t^{\alpha-2} & 0 \\ 0 & -c\delta(\delta-1)D_t^{\delta-2} \end{bmatrix}. \qquad (A1)$$

For the Hessian matrix to be negative definite and the Hamiltonian strictly concave, the principal minors must alternate in sign. For a 2X2 matrix, this reduces to the following conditions:

$$A\alpha(\alpha-1)K_t^{\alpha-2} < 0, \qquad (A2)$$

$$(A\alpha(\alpha-1)K_t^{\alpha-2})(-c\delta(1-\delta)D_t^{\delta-2}) > 0, \qquad (A3)$$

which given our assumptions that α is less than one and δ is greater than one must hold.

For the D&M firm the Hessian is derived by taking the second order derivatives of the Hamiltonian in Equation 8:

$$Hessian^{DM} = \begin{bmatrix} -z\gamma(\gamma-1)K_t^{-(2+\gamma)} & h \\ h & b\beta(\beta-1)I_t^{\beta-2} \end{bmatrix} \qquad (A4)$$

The conditions that must hold are the following:

$$-z\gamma(\gamma-1)K_t^{-(2+\gamma)} < 0, \qquad (A5)$$

$$(-\gamma z K_t^{\gamma-1})(b\beta(\beta-1)I_t^{\beta-2}) - h^2 > 0. \qquad (A6)$$

Under a typical parameterization these would be expected to hold since the first condition (Equation A5) is obviously negative and the second condition (Equation A6) would be expected to hold since the product of the first two terms should be greater than the product of the second two terms. For instance, using the parameter values in this paper, Equation A6 equals 0.74. □

Appendix B: Numerical Solution Algorithm and Convergence Charts

The steps for the numerical solution algorithm are:

1. Discretize the space for the decision variables, $D_t$and I_t. Here there are seven years discretized into 14 periods.
2. Set the initial value of R&D capital, K_0.
3. Set initial guesses for $I_t$over the discretized space.

Loop through steps 4 and 5 a large number of times:

4. Randomly generate guesses for parameters of a second order polynomial for R&D debt:

$$D_t - a_1 + a_2 t + a_3 t^2 \qquad (B1)$$

5. Compare R&D firm value for each guess to the previous best guess. In the process calculate the path of the state variable given the current $D_t$and I_t; that is, K_t.

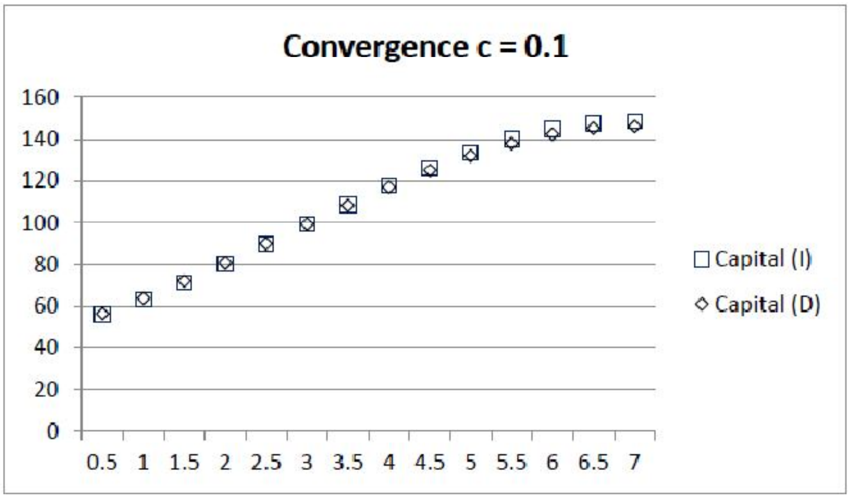
Given the best result from steps 4 and 5, loop through steps 6 and 7 a large number of times:

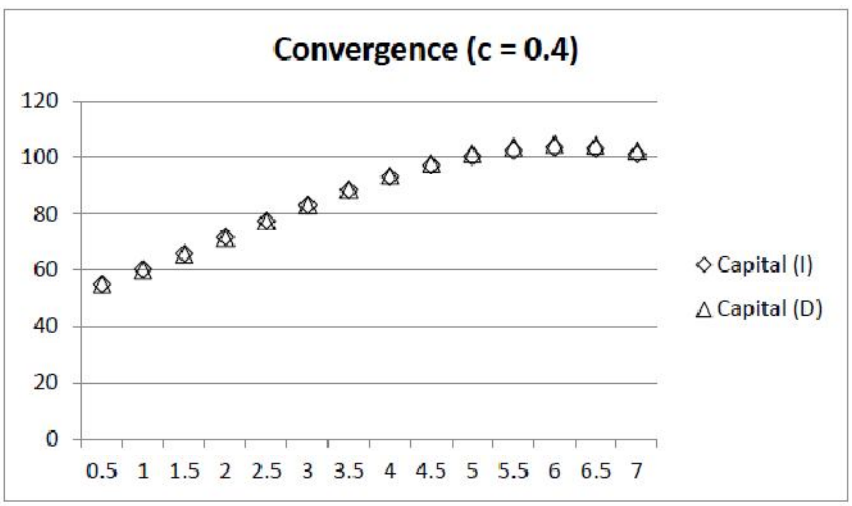
6. Randomly generate guesses for parameters of a second order polynomial for D&M investment:

$$I_t - b_1 + b_2 t + b_3 t^2 \qquad (B2)$$

7. Compare D&M firm value for each guess to the previous best guess. In the process calculate the path of the state variable given the current D_t and I_t; that is, K_t.
8. Repeat steps 4 through 7 until the estimated curves for D_t and I_t meet some convergence criteria. The criteria used here is that the path of R&D capital (K_t) for the two solutions (I_t and D_t) should also converge.

The charts below represent the convergence of R&D capital in the solution for the decision variable for the R&D firm (D_t) and the D&M firm (I_t) in the low financial distress case (c = 0.1) and high financial distress case (c=0.4).





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