Multi-Objective Compromise Allocation Stratified Sampling in the Presence of Non-Response Using Quadratic Cost Function

Sarhad Ullah Khan
Yousaf Shad Muhammad
Nouman Afgan

Quaid-I-Azam University
Islamabad

Abstract

In this paper we propose a quadratic cost function for allocating sample size in multivariate stratified random sampling in the presence of the non-response. We use the separate linear regression estimator. In this multi-objective non-linear integer programming problem, we use extended lexicographic goal programming for solution purpose. To illustrate the application, we apply this formulation on a real data set.

Keyword: Multivariate stratified sampling, Non-response, Compromise allocation, Multi-objective programming, Quadratic cost function, Lexicographic goal programming

1- Introduction

A good sampling plan plays a significant role to make the results useful, obtained from statistical studies and provides close approximation to the population estimates. A suitably selected sampling plan and samples, representing population, produce more reliable estimates.

The important consideration in stratified random sampling is the sample size allocation in each stratum with the criteria either to minimize variance of stratified sample mean for a fixed cost or to minimize cost for the specified variance.

Stratified random sampling is used to increase precision following some cost mechanism. Allocation of sample size $n_h$to individual stratum becomes more complicated in a study or survey in practical utilization of stratified random sampling scheme. The researcher meets problem to select a sample that maximizes precision of finite population mean under cost constraint.

Sampling efficiency depends largely on how the sample size is allocated. In univariate stratified sampling, individual optimum allocation can be used when the characteristics are correlated but in case when the characteristics are uncorrelated a suitable criterion is needed for allocation of sample size which is optimum for all the characteristics. Cochran [24] discussed that is difficult to work out an allocation which is optimum for all characteristics unless the characteristics are highly correlated and the variation between stratum variance in very small. Compromise allocation is based on such criteria. Holmberg [25] addressed the problem of compromise allocation in multivariate stratified sampling by taking into consideration the minimization of sum of variances or coefficients of variation of population parameters and minimization of sum of efficiency loss which may results due to increase in variance because of using the compromise allocation.

The solution of a problem needs some compromise allocation criteria which make the allocation optimum for all characteristics. For example an allocation which minimizes the trace of variance-covariance matrix of the estimator of population mean or that minimizes the weighted average of variances or that maximizes the total relative efficiency of the estimators as compared to corresponding individual optimum allocation Varshney et al. [21]. Many authors Haseen et al. [26], Khan et el. [18], Kokan [27], Folks and Antle [9], Dalenius [8], Ghosh [10], Ali [23], Khan et al. [20], Ansari el al. [1], Guddat et al. [11], Haimes et al. [12], Hiller and Lieberman [13], Khan et el. [17], Charnes et al. [6], Charnes et al. [4], Khan et al. [19], Bethel [2], Chromy [7] and Khowaja [28] used different compromise criterions to solve allocation problem in stratified sampling.

This paper is organized as follows: Section 2 explains the model for the cost function. In Section 3, we formulate the problem. Section 4 explains lexicographic programming. The results and discussions are given in Section 5.

2- Quadratic Cost Functions

The cost of survey is a major factor of sample allocation to various strata. Tschuprow [30] and Neyman [29] proposed an allocation procedure that minimize variance of sample mean under a linear cost function of sample size $n = \Sigma_{h=1}^{L} n_h$in stratified random sampling. Neyman [29] used Lagrange multiplier optimization technique to get optimum sample size for single variable under study. The linear cost function used in stratified random sampling in case of the non-response is given as;

$$C_h = C_{h0} n_h + C_{h1} n_{h1} + C_{h2} u_{h2} \quad (1)$$

Where**C** denotes total budget available for survey and C_{h0} be the per unit cost of selecting nh units. C_{h1} be the per unit enumerating cost of n_{h1} units. C_{h2}be the per unit enumerating cost of uh2 units from the non-respondents. The expected values of $n_{h1} = W_{h1} n_h$ **and** $u_{h2} = W_{h2} n_h / k_h$,

where (h = 1; 2; : : : ;L)represent measurement per unit cost in the h^{th}stratum, and n_h is number of sample units selected from the h^{th} stratum.

Considering a quadratic cost function, including measurement unit cost and traveling cost within strata as Beard wood et al. [3] proposed the shortest route among k randomly allocated sampling units in the region is asymptotically proportional to pSqrt(k) for a large k. Varshney et al. [21] used a quadratic cost function for large sample size given in (2).

$$C = c_0 + \sum_{h=1}^{L} c_h + \sum_{h=1}^{L} \tau_h \sqrt{n_h} (2)$$

where$\tau_h$is travel cost for a unit within the h^{th} stratum.

Equation (2) can further be extended to case of presence of the non-response given in (3).

$$\sum_{h=1}^{L} (t_{h0} + t_{h1} W_{h1}) \sqrt{n_h} + \sum_{h=1}^{L} t_{h2} \sqrt{u_h} \leq C_0 \quad (3)$$

Wheret_{h1} is travel cost for the respondents unit within the h^{th} stratum and t_{h2} is travel cost for the non-respondents unit within the h^{th} stratum. And u_{h2}isthe sub sample from non-respondents units.

3- Optimum Allocation Techniques

The different techniques to solve the multi objective programming problem of multivariate stratified sampling in case of non-response are explained below.

3.1- Individual Optimum Technique

It is an allocation technique that optimize coefficient of variation of one characteristic of population among Y_j(j = 1; 2; : : : ; p) characteristics and use that allocation for estimating other characteristics of the population.

Let V_j*be the optimum value of objective function V_j obtained by solving the following integer nonlinear mathematical programming problem (*INMPP*).

Minimize V_j

Subject to

$$\sum_{h=1}^{L} (C_{0h} + C_{h1} W_{h1}) n_h + \sum_{h=1}^{L} C_{h2} u_h \leq c_0 \qquad (4)$$

$$2 \leq n_h \leq N_h$$
$$2 \leq u_h \leq n_{\hat{h}2}$$

3.2 The Goal Programming Technique

Charnes et al. [4], Charnes et al. [6], Charnes and Cooper [5] and Ijiri [14] used the goal programming technique for multi-objective optimization problems.

We can use the goal programming technique when all information about the characteristics are given and the importance of each characteristics is known. Formulation under goal programming technique can be written as:

Minimize (V_1, V_2, ….,V_p)

Subject to

$$\sum_{h=1}^{L}(C_{0h} + C_{h1}W_{h1})n_h + \sum_{h=1}^{L} C_{h2}\, u_h \le c_0 \qquad (5)$$

$$2 \le n_h \le N_h$$

$$2 \le u_h \le n_{\hat{h}2}$$

$n_h$and u_h are integers, for all **h** and **j**.

3.3- Extended Lexicographic Goal Programming Technique

Romero [31] starts reviewing the satisfying philosophy of Goal Programming(GP) and interpreting their solutions from the point of view of the utility theory. This interpretation leads to a very general optimization structure called Extended Lexicographic Goal Programming (ELGP). It is then demonstrated that there are a significant number of Multiple Criteria Decision Making (MCDM) approaches that, from a logical point of view, can be reduced to the ELGP structure.

$$\mathbf{Minimize}V_j = f(n_{jh}, u_{jh})$$

Subjectto

$$\sum_{h=1}^{L}(C_{0h} + C_{h1}W_{h1})n_h + \sum_{h=1}^{L} C_{h2}\, u_h \le c_0 \qquad (6)$$

$$2 \le n_h \le N_h$$

$$2 \le u_h \le \hat{n}_{h2}$$

$$n_{jh}\mathbf{and}u_{jh}\mathbf{areintegersand}n_{jh} \in F; h = 1, 2, \dots, L$$

Note that in this generic form no assumptions have yet been made about the nature of thedecision variables of goals. The decision maker(s) sets a real target level for each goal whichis denoted by **Vj**(generally an individual optimal of the **j**thobjective). This then leads to the basic formulation of the **j**thgoal:

$$\hat{V}_j + d_j^{-} - d_j^{+} = V_j^{*}$$

Where d_j^- and, d_j^+ are –ve and +ve deviational variables.

The utility formulation of the Archimedean and MINMAX (Chebyshev) GP models undertaken in the preceding section suggests the following generalization:

$$\mathbf{Minimize}(1-\rho)D + \rho\sum_{j=1}^{p}\left[f_j\left(\underline{W_{1j}d_j^-}, \underline{W_{2j}d_j^+}\right)\right]$$

Subjectto

$$\left[\left(\underline{W_{1j}d_j^-}, \underline{W_{2j}d_j^+}\right)\right] \le D \qquad (7)$$

$$\hat{Z}_j + d_j^- - d_j^+ (\le \text{ or } \ge) Z_j^*$$

$$\underline{\hat{n}_j} \in F$$

$$n_{jh}\mathbf{and}u_{jh}\mathbf{areintegersand}n_{jh} \in F; h = 1, 2, \dots, L$$

Where parameter $W_1$and W_2 are the weights reflecting preferential and normalizing purposes attached to the negative and positive deviation variables of j^{th}goal, respectively. Parameter weights the importance attached to the minimization of the weighted sum of unwanted deviation variables and $0 \leq \sigma \leq 1$. Hare f_j(j = 1,2,…, q) are goals and F is the feasible space.

Integer nonlinear programming problems have a small feasible solution grid and we are already compromising on allocating sample size. This will help us to find feasible and optimal solution considering larger grid using this relaxation.

4- Application

In stratified random sampling every stratum is divided into two mutually exclusive groups of respondents and non-respondents, with N_{h1} size of respondents and $N_{h2}(N_{h2} = N - N_{h1})$ size of non-respondents in the h^{th} stratum. We select a sample of size **n** from the given population n_h of the respondents units and u_h from the non-respondents units are selected from N_h units in the h^{th} stratum such that $\Sigma_{h=1}^{L} n_h = n$. let $p \geq 2$ the characteristics are defined one of the population unit and the estimation of the **p** unknown population mean Y_j,j = 1,2,…, p is of interest.

Necessary formula:

$$\bar{Y}_j = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{Nh} y_{jhi}\,, \bar{x}_{jh} = \frac{1}{nh}\sum_{i=1}^{nh} x_{jhi}\, \bar{X}_j = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{Nh} x_{jhi}\, \bar{Y}_{jh} = \frac{1}{N_{h2}}\sum_{i=1}^{Nh} y_{jhi}$$

$$S^2{}_{xjh} == \frac{1}{n_h - 1}\sum_{i=1}^{nh}\left(x_{jhi} - \bar{X}_{jh}\right) S^2{}_{yjh} == \frac{1}{n_h - 1}\sum_{i=1}^{nh}\left(y_{jhi} - \bar{Y}_{jh}\right)$$

$$S_{xyjh} == \frac{1}{N_h - 1}\sum_{i=1}^{nh}\left(x_{jhi} - \bar{X}_{jh}\right)\left(y_{jhi} - \bar{Y}_{jh}\right)$$

Using Hansen [22] rule, as the non-response is in study variable Y_{jhi}.

Let$\bar{y}*_{jh} = (n_{h1}\bar{y}_{jh1} + n_{h2}\bar{y}_{jh2})/n_h$be an unbiased estimator of the population mean$\bar{Y}_{jh}$ where $\bar{y}_{h1}$the mean of respondents sampling units n_{h1} and$\bar{y}_{jh2}$is the meanof non-respondents sample units u_h.

The traditional regression estimator is $\bar{y}_{j,lrs} = \sum_{h=1}^{L} W_h \bar{y}_{lr,jh}$where
$\bar{y}_{j,lrs} = \bar{y}_{jh} + b_{jh}(\bar{X}_{hjh} + \bar{y}_{jh})$ **and** are b_{jh} is sample regression coefficient.

The MSE of $\bar{y}_{j,lrs}$is:

$$MSE(\bar{y}_{j.lrs}) = \sum_{h=1}^{L} W^2{}_h\left(\frac{1}{n_h} - \frac{1}{N}\right)\left(S_{yjh}{}^2 - 2\beta_{jh}S_{xyjh} \quad - 2\beta_{jh}{}^2 S^2{}_{xj}\right) + \sum_{h=1}^{L} W^2{}_h\left(\frac{W_{h2}}{\mu_h} - \frac{1}{nh}\right) W_{h2}S_{yjh}{}^2$$

Where

$\bar{y}_{jh}$= Sample Mean of j^{th} study characteristic in h^{th}stratum.
$\bar{x}_{jh}$= Sample Mean of j^{th} auxiliary characteristic in h^{th} stratum.
$\bar{X}_{jh}$= population Mean of j^{th} auxiliary characteristic in h^{th} stratum.
S^2_{yjh}= population Variance of j^{th} study characteristic in h^{th}stratum.
S^2_{xjh}= population Variance of j^{th} auxiliary characteristic in h^{th} stratum.
S^2_{yxjh}= population Covariance between the j^{th} study and the j^{th} auxiliary characteristic in h^{th} stratum, and$\beta_{jh} = S^2_{yxjh}/S^2_{xjh}$is population regression coefficient.

Now let

$$\left(S_{yjh}{}^2 - 2\beta_{jh}S_{xyjh} \quad - 2\beta_{jh}{}^2 S^2{}_{xj}\right) = \rho_{jh}$$

Then the above equation can be written as:

$$MSE(\bar{y}_{j.lrs}) = \sum_{h=1}^{L} \frac{W_h^2 p_{jh}}{n_h} - \sum_{h=1}^{L} \frac{W_h^2 p_{jh}}{N_h} + \sum_{h=1}^{L} W^2{}_h \left(\frac{W_{h2}}{\mu_h} - \frac{1}{nh}\right) W_{h2} S_{yjh}{}^2$$

As the units of all characteristics measured are not same, therefore it needs to use an estimate which is free from unit measurement. So coefficient of variation is used instead of *MSE*.

$$CV_{(\bar{y}_{j.lrs})} = \frac{\sqrt{MSE(\bar{y}_{j.lrs})}}{\bar{Y}}$$

$$CV_{(\bar{y}_{j.lrs})} = \sqrt{\sum_{h=1}^{L} \frac{W_h^2 p_{jh}}{n_h} - \sum_{h=1}^{L} \frac{W_h^2 p_{jh}}{N_h} + \sum_{h=1}^{L} W^2{}_h \left(\frac{W_{h2}}{\mu_h} - \frac{1}{nh}\right) W_{h2} S_{yjh}{}^2}$$

$$CV^2_{\bar{y}_{j.lrs}} = \sum_{h=1}^{L} \frac{W_h^2 p_{jh}}{n_h} - \sum_{h=1}^{L} \frac{W_h^2 p_{jh}}{N_h} + \sum_{h=1}^{L} W^2{}_h \left(\frac{W_{h2}}{\mu_h} - \frac{1}{nh}\right) W_{h2} S_{yjh}{}^2$$

A sample size $\mathbf{n=\Sigma_{h=1}^{L} n_h}$ for (h = 1, 2,…, L) is determined using proposed Quadratic cost function in Eq.3 that minimizes coefficients of variation of the estimator of population mean for each characteristics $\mathbf{Y_j}$(j = 1, 2, …, Q**)**. This problem is formulated in multi-objective integer nonlinear programming as:

Minimize (V_1, V_2, ….,V_p)

Subject to

$$\sum_{h=1}^{L} (C_{0h} + C_{h1} W_{h1}) n_h + \sum_{h=1}^{L} C_{h2}\, u_h \le c_0$$

$$2 \le n_h \le N_h$$
$$2 \le u_h \le n_{\hat{h}2}$$

$\mathbf{n_h}$and $\mathbf{u_h}$ are integers, for all **h** and **j**.

Allocation through Individual optimum technique

$$\text{Minimize}\, V_j \quad \textit{Subjectto}$$

$$\sum_{h=1}^{L} (C_{0h} + C_{h1} W_{h1}) n_h + \sum_{h=1}^{L} C_{h2}\, u_h \le c_0$$

$$2 \le n_h \le N_h$$
$$2 \le u_h \le \hat{n}_{h2}$$

$$n_{jh}\, \text{and}\, u_{jh}\, \text{areintegersand}\, n_{jh};\, h = 1, 2, \ldots, L$$

Allocation through Extended lexicographic goal programming

$$\text{Minimize}(1-\rho)D + \rho \sum_{j=1}^{p} \left[\left(\underline{d_j^-}, \underline{d_j^+}\right)\right]$$

$$\text{Subjectto}$$

$$\left[\left(\underline{d_j^-}, \underline{d_j^+}\right)\right] \le D$$

$$\hat{V}_j + d_j^- - d_j^+ \le V_j^*$$

$$\sum_{h=1}^{L}(C_{0h} + C_{h1}W_{h1})n_h + \sum_{h=1}^{L} C_{h2}\, u_h \leq c_0$$

$$2\leq n_h \leq N_h$$

$$2\leq u_h \leq n_{\hat{h}2}$$

n_{jh} and u_{jh} are integers and n_{jh}; $h = 1, 2, \ldots, L$

Where $d_j (j= 1, 2, \ldots, p)$ are the deviation variables.

Here *Vj*are the optimum values obtained from extended goal programming.

5- *Numerical Illustration*

The data are taken from the agricultural census 2007 conducted by National Agricultural Statistics Service, USDA, and Washington D.C. (Source: www.agcensus.usda.gov).

Y_1 = Corn harvested in 2007.
Y_2 = Soybean harvested in 2007.
X_1= Corn harvested in 2002.
X_2= Soybean harvested in 2002.

Where Y_1 , Y_2 are study variables and X_1 , X_2 are auxiliary information.

Here $\overline{Y}_1$ = 22698622:75 and $\overline{Y}_2$ = 4306561:045. It is assume that the total cost of the survey is 331 units.The last 27, 30, 27 and 20 percent values consider as non-response in each stratum respectively.

There are four strata in the population.The complete data are shown in APPENDIX.

5.1 Results and Discussion

In the three allocation techniques, extended lexicographic goal programming (*ELGP*) gives minimum values of CV than the other two techniques. Extended lexicographic goal programming set two additional constraints to bound Coefficients of variation maximum to their individual optimum values. Using an arbitrary weight **ϭ**= 0:4 for unwanted sum of deviations from individual optimum values and (**1 - ϭ**) = 0:6 for maximum deviation from utility, we minimize the goal objectives or achievements function under originally defined cost and decision variables constraints. By changing arbitrary weight **ϭ**, different results are expected (see Table 1)

Table 1: Compromise Allocations and Corresponding Values of the Objective Functions Obtained by Different Techniques

Allocation	Indiviual opt. technique	Goal prog.	Extended lexico.
(n_1, u_1)	(14, 3)	(17, 4)	(17, 4)
(n_2, u_2)	(35, 10)	(34, 10)	(36, 10)
(n_3, u_3)	(17, 4)	(13, 3)	(14, 3)
(n_4, u_4)	(8, 2)	(10, 2)	(10, 2)
CV_1	03100	0.03040	0.3020
CV_2	03010	0.02910	0.2900

References

Ansari, A. H., Najmussehar and Ahsan, M. J.: On multiple response strati_ed random sampling design. International Journal of Statistical Sciences, Kolkata, India. 1(1), 45-54 (2009)

Bethel, J.: An optimum allocation algorithm for multivariate surveys. Journal of the American Statistical Association. 209-212 (1985)

Beardwood, J., Halton, J. H. and Hammersley, J. M.: The shortest path through many points. Mathematical Proceedings of the Cambridge Philosophical Society. 55, 299-327 (1959)

Charnes, A., Cooper, W. W. and Ferguson, R. O.: Optimal estimation of executive compensation by linear programming. Management Science. 1, 138-151 (1955)

Charnes, A. and Cooper, W. W.: Management Models and Industrial Applications of Linear Programming. John Wiley and Sons, New York. (1961)

Charnes, A., Clower, R. W. and Kortanek, K. O.: Effective control through coherent decentralization with preemptive goals. Econometrical. 35, 294-320 (1967)

Chromy, J. R.: Design optimization with multiple objectives. Journal of the American Statistical Association 194-199 (1987)

Dalenius, T.: Sampling in Sweden: contributions to the methods and theories of sample survey practice. Almqvist and Wiksell, Stockholm.(1957)

Folks, J. L. and Antle, C. E.: Optimum allocation of sampling units to strata when there are R responses of interest. Journal of the American Statistical Association. 60, 225-233 (1965)

Ghosh, S. P.: A note on stratified random sampling with multiple characters. Calcutta Statistical Association Bulletin. 8, 81-89 (1958)

Guddat, J., Guerre, V. F., Tammer, K. and Wendler, K.: Multi-objective and stochastic optimization based on parametric optimization, Akademie-Verlag, Burlin. (1985)

Haimes, Y. Y., Lasdon, L. S. and Wismer, D. A.: On a bi-criterion formulation of the problems of integrated system identification and system optimization. IEEE Transactions on System, Man and Cybernetics. SMC-1, 296-297 (1971)

Hiller, F. S. and Lieberman, G. J.: Introduction to operation research. McGraw-Hill, Inc, New York. (1995)

Ijiri, Y.: Management Goals and Accounting for Control, Amsterdam North-Holland. (1965)

Jahan, N., Khan, M. G. M. and Ahsan, M. J.: A generalized compromise allocation. Joural of Indian Statistical Association. 32, 95-101, (1994)

Jahan, N., Khan, M. G. M. and Ahsan, M. J.: Optimum compromise allocation using dynamic programming. Journal of Indian Statistical Association. 49(2) 197-202 (2001)

Khan, M. G. M., Khan, E. A. and Ahsan, M. J.: An optimal multivariate stratified sampling design using dynamic programming. Australian and New Zealand Journal of Statistics. 45(1), 107-113 (2003)

Khan, M. G. M., Khan, E. A. and Ahsan, M. J.: Optimum allocationin multivariate stratified sampling in presence of non-response. Journal of the Indian Society of Agricultural Statistics. 62(1), 42-48 (2008)

Khan, E. A., Khan, M. G. M. and Ahsan, M. J.: Optimum stratification: a mathematical programming approach. Calcutta Statistical Association Bulletin. 52(Special volume.) 323-333 (2002)

Khan, M. G. M., Maiti,T. and Ahsan, M.J.: An optimal multivariate stratified sampling design using auxiliary information, an integer solution using goal programming approach. Journal of official Statistics.26(4), 695-708 (2010)

Varshney, R., Najmussehar and Ahsan, M. J.: Estimation of more than one parameters in strati_ed sampling with _xed budget. Mathematical Methods of Operations Research. 75(2), 185-197 (2012)

Hansen, Morris H and Hurwitz, William N.: The problem of non-response in sample surveys. Journal of the American Statistical Association. 41(236), 517-529(1946)

Ali, Irfan and Raghav, Yashpal Singh and Bari, Abdul: Compromise allocation in multivariate stratified surveys with stochastic quadratic cost function. Joural of Statistical Computation and Simulation. 83(5),962-976(2013)

Cochran and William G.: Sampling Techniques New York: John Wiley and Sons (1977)

holmberg and A.: A multiparameter perspective on the choice of sampling design in surveys. Joural of Statistics in Transition. 5(6),969-994(2002).

Haseen, Sanam and Iftekhar, SANA and Ahsan, MJ and Bari, Abdul: A fuzzy approach for solving double sampling design in presence of non-response. International Journal of Engineering Science and
Technology. 4(06), 2542-2551(2012)

Kokan, AR and Khan, Sanaullah.: Optimum allocation in multivariate surveys: An analytical solution. Journal of the Royal StatisticalSociety. Series B (Methodological). 29(1), 115-125(1967)

Khowaja, Saman and Ghufran, Shazia and Ahsan, MJ.: On the Problem of Compromise Allocation in Multi-Response Strati_ed Sample Surveys. Communications in Statistics-Simulation and Computation42(4), 790-799(2013)

Neyman and Jerzy: On the two di_erent aspects of the representative method: the method of stratified sampling and the method of purposive selection. Journal of the Royal Statistical Society. 97(4),558-625(1934)

Tschuprow and Al A.: On the mathematical expectation of the moments of frequency distributions in the case of correlated observations (Chapters 4-6). Metron. 2(1), 646-683(1923)

Romero and Carlos: Extended lexicographic goal programming: aunifying approach. Omega. 29(1), 63-71(2001)

Appendix

Table 2: Summary Statistics of Data

h	N_h	W_h	W_{h1}	W_{h2}	β_{1h}	β_{2h}	S^2_{y1h}	S^2_{y2h}	S^2_{y1h2}
1	22	0.222	0.73	0.27	9833710.37	2175050.54	5.76×10^{13}	1.67×10^{12}	7.80×10^{13}
2	40	0.404	0.70	0.30	17318227.95	2308431.77	1.21×10^{14}	2.50×10^{12}	1.22×10^{14}
3	24	0.242	0.73	0.27	11778330.92	2876092.92	5.57×10^{13}	3.58×10^{12}	2.67×10^{13}
4	13	0.131	0.80	0.20	11546442.92	2980656.00	7.08×10^{13}	4.44×10^{12}	4.01×10^{13}

S^2_{y2h2}	x_{1h}	x_{2h}	S^2_{x1h}	S^2_{x2h}	S_{yx1h}	S_{yx2h}
1.48×10^{12}	0.591	0.591	0.253	0.267	2622322.77	550824.49
2.807×10^{12}	0.575	0.475	0.255	0.242	4340658.42	590425.82
3.02×10^{12}	0.50	0.542	0.259	0.167	3072607.89	745074.54
6.28×10^{11}	0.308	0.461	0.269	0.333	2664563.75	3489303.00