Impact of Extended Storage Capabilities in IBIS under Stochastic Hydrologic Regime

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Abstract

A scenario based model is developed for Indus River System which addresses the problem related issues and applicability under a complex system network. We apply this model considering the current and future storage capabilities. We conduct an analysis under stochastic hydrologic regime (stochastic inflow and rainfall) if these capabilities considered effective. We note that wheat gain the major rise in its area i.e. from 5.428 million hectares to 6.227 million hectares. We also provide some analysis according to current storages. A cropping pattern is presented consistent with system infrastructure and land resources in multiple canal areas.

1. Introduction

There are three basic uses of water in the modern civilization agriculture, industry and human consumption. Using water wisely in these three uses is one way of developing a country economically and socially. Water is one of the most important natural resource and an eminent driving force for the economy of a country. Only a few decades ago, Pakistan was considered to have abundance of good quality water. Now, however, in many other area of the world, population growth, economic development, urbanization and industrialization are applying continuous pressure on the already limited water resources of Pakistan. Pakistan is now towards a serious shortage of water.

Indus Basin Irrigation System (IBIS) is the largest contiguous irrigation system in the world developed over the last 140 years. The Indus river basin stretches from the *Himalayan Mountains* in the north to the dry alluvial plains of Sind in the south. The area of Indus basin is 944*;* 574*km*2 *(Avionics Agro-Dev. International (Pvt) Ltd., 2000).* The vast irrigation system in Pakistan is comprised of three major storage reservoirs, 19 barrages (Canal Head works) and44 main canals with a conveyance length of 57,000 kilometers, and 89,000 water courses withal running length of more than 1.65 million kilometers. This vast irrigation system feeds more than 18 million Hectares of irrigated land in Pakistan; a country with the highest irrigated an drain-fed land ratio in the world, 4:1. About 180,000 *Km*2 (6.6% of the global irrigated area) is presently being irrigated in Pakistan *(FAO, 2001).* The contribution of rainwater to crops in the IBIS is estimated at about 16.5 billion cubic meter *(Ahmad, 1993).*

Pakistan depends on irrigation and water resources for 90 percent of its food and crop production*(World Bank, 1992).*Irrigation water management has a number of economic implications for those countries who have been benefited by reservoir system. Developing country like Pakistan where the agriculture is the back bone of her economics has a huge structural infrastructure for irrigation such as Rivers, Reservoirs, Canals, Sub Canals and Water Courses (Distributaries). This system has been developed at a huge financial investment. It demands a developed scientific water management policies. Development in the system sciences, operation research and mathematical modeling for decision making under uncertainty have been usefully exploited for water resource management in many advanced countries. Application of such mathematical techniques to specify irrigation water management policies and their implementation in the developing countries will yield huge economic benefits to these countries.

Decision making for reservoir release for irrigation when it is being operated under a power generation policy involves much subtle consideration such as nature and timing of the crops being irrigated. Determining the amount of release from the reservoir which is a part of complex network system like Indus is, must be supported by a comprehensive mathematical decision making mechanism.

It is therefore necessary to consider the crop water requirement along with the competition with other crops, when there is scarcity in water resources. The present model addresses the policy related issues considering the following features.

A two-stage Stochastic model: Uncertainty due to randomness of hydrologic variables; single decision-making mechanisms for reservoir operation and crop water allocation was ad-dressed *[Houghtalen and lo_tis 1988; Dudley 1988; Dudley and Scott 1993; Vedula and Mujum-dar 1992; Vedula and Nagesh Kumar 1996; Ravikumar and Venugopal 1998]* by using stochastic dynamic programming (SDP). A two-phase stochastic dynamic programming model was developed by [Umamahesh and Sreenivasulu, 1997] for optimal operation of irrigation reservoirs under a multi-crop environment. In the first phase they maximize the release from reservoir and in the second phase they try to minimize the deficit of water when different crops are competing for scarce water resources.

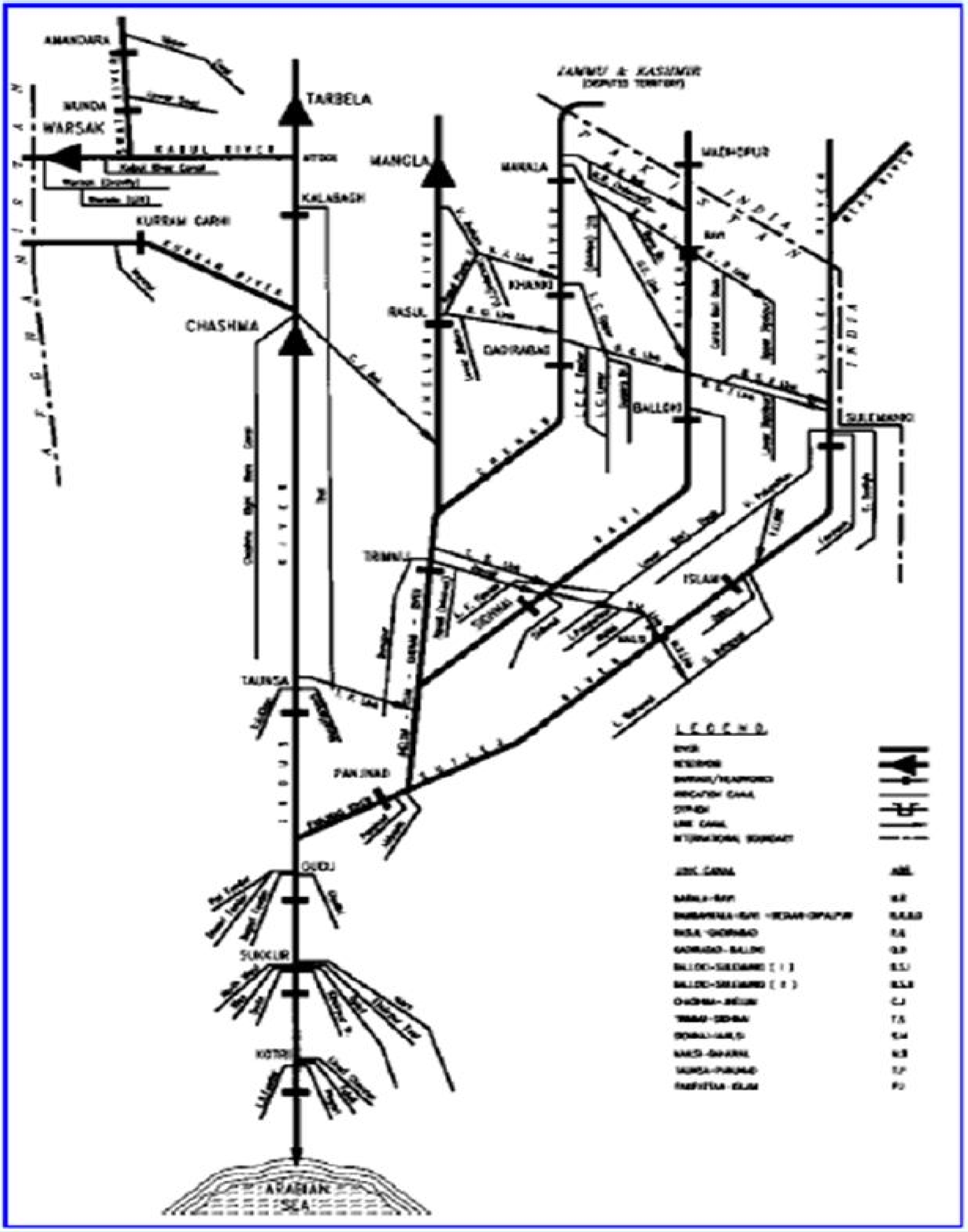
We developed a two-stage stochastic programming model for IBIS. First-stage decision variable (here and now) is the vector of crops area to be sown in different canal command. This decision is to be taken in the presence of uncertainty about future realization of scenario. The second stage decision variable (wait and see) is the vector of crops area cultivated with in different canal commands when actual scenario becomes known. First-stage decisions are, however, chosen by taking their future effects into account.

2. Stochastic Programming in Water Systems

The process of determining the best allocation and utilization of available scarce resources is as old as man himself. The uncertainty of the future water resources adds more complexity tithe problem of optimum allocation. This allocation problem is being studied by economists, engineers and mathematicians for centuries ago. But over last four decades, it is being studied under the preview of stochastic optimization.

In deterministic LP all components are considered as deterministic in nature. But in practice, this never happens. Consider the transition of reservoir storage from one volume in one period to some other volume in next period. The transition results from the partially release for various uses, which can be controlled, and partly from inflow to the reservoir and reservoir losses, such as evaporation, seepage etc., which cannot be controlled. So the First component can be made deterministic, but not the last two. They are random by their nature. Inclusion of such random components makes the LP formulation SLP.

Figure 1: Schematic Diagram of Indus River System: Network of Rivers, nodes, Links and Canals



Source: WAPDA, Pakistan

Due to uncertainty in randomness of hydrologic variables; single decision-making mechanisms for reservoir operation and crop water allocation was addressed *[Houghtalen and Lofitis1988; Dudley 1988; Dudley and Scott 1993; Vedula and Mujumdar 1992; Vedula and Nagesh Ku-mar 1996; Ravikumar and Venugopal 1998]* by using stochastic dynamic programming. Stochastic/Deterministic dynamic programming and linear programming used in seasonal and intraseasonal allocation of de_cit water, competing crops and crop yield optimization *[Paudyal and Das Gupta 1988; Rao et al. 1990; Azar et al. 1992; Mannocchi and Marcelli 1994; Sunantara and Ramirez 1997; Paul et al. 2000; Anwar and Clark 2001].* Dynamic programming; Linear programming and Simulation are effective tools in adaptive operation; real time forecasts of hydrologic variables *[Dariane and Hughes 1991; Rao et al. 1992; Mujamdar and Ramesh 1997;*

Wardlaw and Barnes 1999]. A state of the art review over stochastic dynamic programming (SDP)is presented by *[Labadie, J.W., 2004]* in which he enlist the researchers who implemented this technique in reservoir operations.

A Two-stage Stochastic Program for IBIS

A classical two-stage stochastic linear program with fixed recourse is

$$\min c^{T}x+E_{\xi}\left[\min q(\xi)^{T}y(\xi)\right]$$

$$\text{s.t } Ax=b$$

$$T(\xi)x+Wy(\xi)=h(\xi)$$

$$x\geq 0, y(\xi)\geq 0$$

Where c and b are known vectors, A and Ware matrices and W is assumed as fixed re-course matrix. $\xi$is a random variable representing the possible scenario and E_{ξ}represent the mathematical expectation with respect to ξ.

In IBIS scenario-based stochastic model, first-stage decision variable (here and now) *x* is the vector of crops area to be sown in every canal command. This decision is to be taken in the presence of uncertainty about future realization of scenario (ξ). The second stage decision variable (wait and see) y (ξ) is the vector of crops area cultivated within each canal command when actual scenario (ξ) becomes known. First-stage decisions are, however, chosen by taking their future effects into account.

The above formulation for IBIS scenario-based stochastic model may be presented as:

The objective is to maximize the net revenue from crops sale minus initial fixed cost and laborcost. The other expenditures are ignored i.e.

1- Water cost (A public property, farmers get their share proportional to their land holdings)
2- Farm rent (Farmers are owner of their land)
3- Initial cost (Includes: seed cost, fertilizer etc.)

$$\min c^{T}x+E_{\xi}\left[\min q(\xi)^{T}y(\xi)\right]$$

There are some constraints on first-stage decisions (no randomness involved)

$$\text{s.t } Ax=b$$

Randomness on the right hand side is involved in the constraints of second-stage decision variables only (i.e. no random technology used).

$$Wy(\xi)=h(\xi)$$

It is obvious that if we will not sow any, we cannot cultivate. And cultivated area is less than the sown area

$$x\geq 0, y(\xi)\geq 0$$

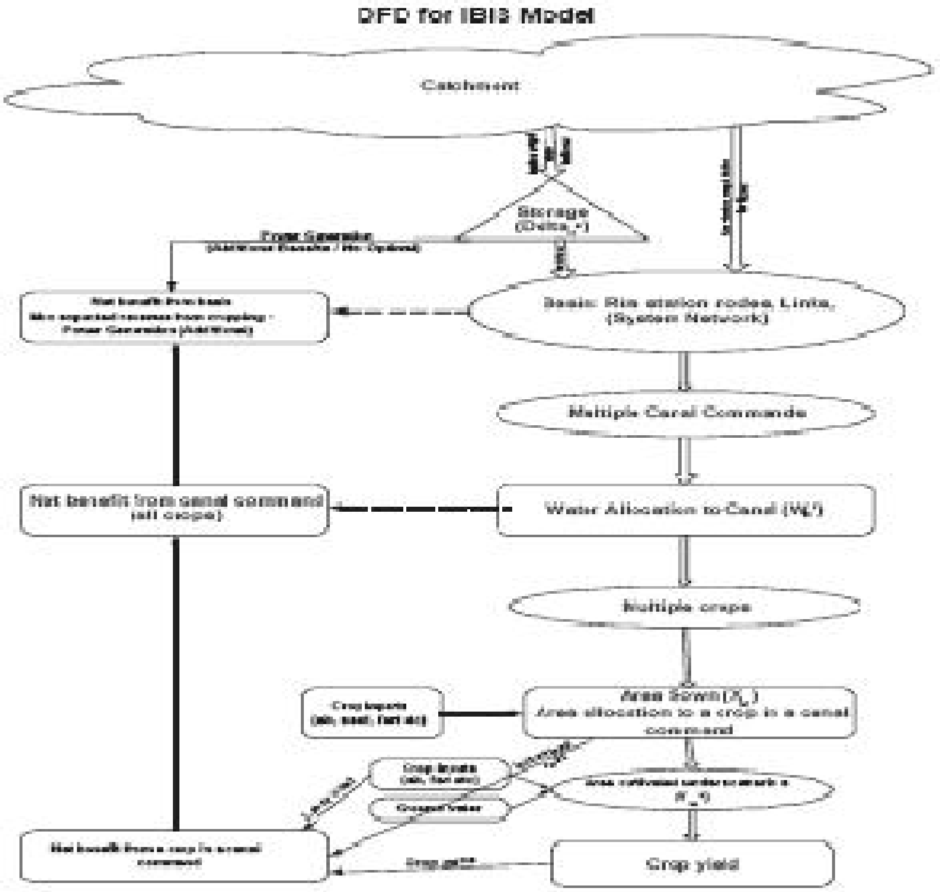


Figure 2: DFD for IBIS model

3. Model Descriptions

We are using our previously developed model for the system. The schematic representation is given in Fig.2. The model is developed for determining the optimal cropping pattern and irrigation scheduling for the said system. This model has the following distinguish features:

- Complex network involved
- Stochastic inflow and rainfall
- A two-stage decision problem.

In the formulation of the model, the following steps are considered.

- This is a 1-year planning model.
- Operating policy of hydrologic scheduling and cropping is ten-daily i.e. every month is divided into 3 ten-daily intervals and monthly.
- Every canal l belong to exactly one zone z(l), where z = z(l). Zone wise canal commands are given

Zone (z) Canals (l)

Zone1 2a, 03, 04, 05
Zone2 11, 12, 13, 14, 18, 22, 23, 24, 25, 26, 27
Zone3 01, 02, 06, 07, 08, 09, 10, 15, 16, 17, 19, 20, 21, 28, 29, 30 to 43

- Land occupation or cropping calendar is available.
- Considering historic inflow, whole year is further divided into two seasons' i.e. wet season(April to September) and dry season (October to March). Scenarios are generated within each season from their theoretical distribution of total inflow (see next section). Also for rainfall with \in each zone.

- Crop water requirement within each canal command are computed by using the results of a study in the basin (Kaleem Ullah et al., 2001).
- The other inputs on crops like labor, fertilizer, seed cost, crop yield etc. are used from available data from the documents of WAPDA Pakistan, Statistical Bureau of Pakistan and Economic Survey of Pakistan.
- Ground water, system inflow, evaporation from storage and canals, data about system network, area for each canal command etc. are available from documents of WAPDA Pakistan and IRSA Pakistan.
- Reservoirs are operated under a power generation policy. Power generation is not optimal. Water scheduling from reservoir are designed to get optimal cropping pattern in the basin.
- DFD for IBIS model is given in the Fig. 2.
- Objective function is to maximize the revenue from crops in the basin.
- Decision Variables are

$\mathbf{X_{lc}}$= area sown for crop c in zone z canal l (Hectares)
$\mathbf{Y_{lc}^{s}}$= area cultivated for crop c in zone z, canal l under scenario s (Hectares)
$\mathbf{W_{lc}^{s}}$= water released in canal l, zone z during month t (km^3)
$\mathbf{\Delta_{it}^{s}}$= storage level at storage i during month t (km^3)

A complete model is given in the APPENDIX. Further detail is available in Shad and Pflug (2014).

4. Results and Discussion

Solution Procedure

We solve this model under three different conditions.

- A monthly time horizon single stage deterministic model, setting now and rainfall at their average values.
- A monthly time horizon two-stage stochastic with 200 scenarios.
- A ten-daily time horizon two-stage stochastic with 200 scenarios.

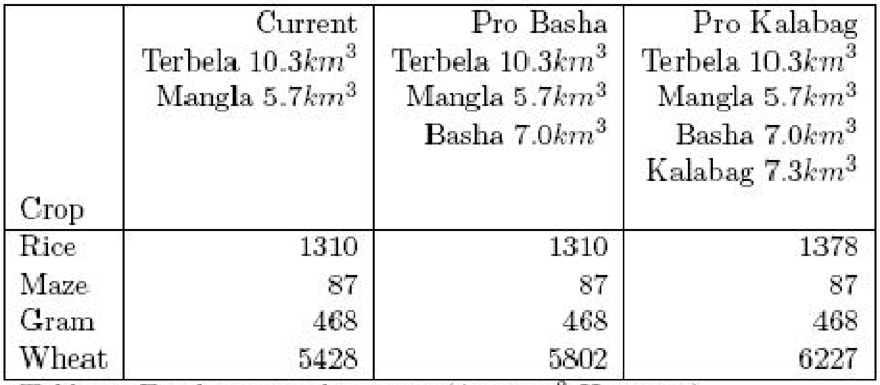
The (largest one) scenario based two-stage stochastic programming model comprises of750224 rows, 1,173,371 columns and 7,094,527 non zeros. This model uses 565 MB of memory space. We obtain the solution of this model after 531221 iterations. We run this model for 200 scenarios in both monthly and ten-daily time horizon. We compare the results of three kinds of model solutions.

1- Deterministic cropping pattern.
2- Stochastic cropping pattern with monthly time horizon.
4- Stochastic cropping pattern with ten-daily time horizon.

Cultivation Plan under Extended Storage Capabilities

Pakistan is planning to establish new storage capabilities in their river system. The two major storages which will be establish in near future are Basha and Kalabag-Diayamer dam. We conduct an analysis for the system under stochastic hydrologic regime (stochastic inflow and rainfall). We note that wheat gain the major rise in its area i.e. from 5.428 million hectares to 6.227 million hectares (see detail in the following table 2). This will help to fulfill the food requirements of the country whose present population is over 160 million which is growing at the rate of 2:13% (Pakistan Statistics Year Book, 2004-05). Wheat cultivated in the dry season.

These extended storage capabilities provide additional water resources in the dry season.

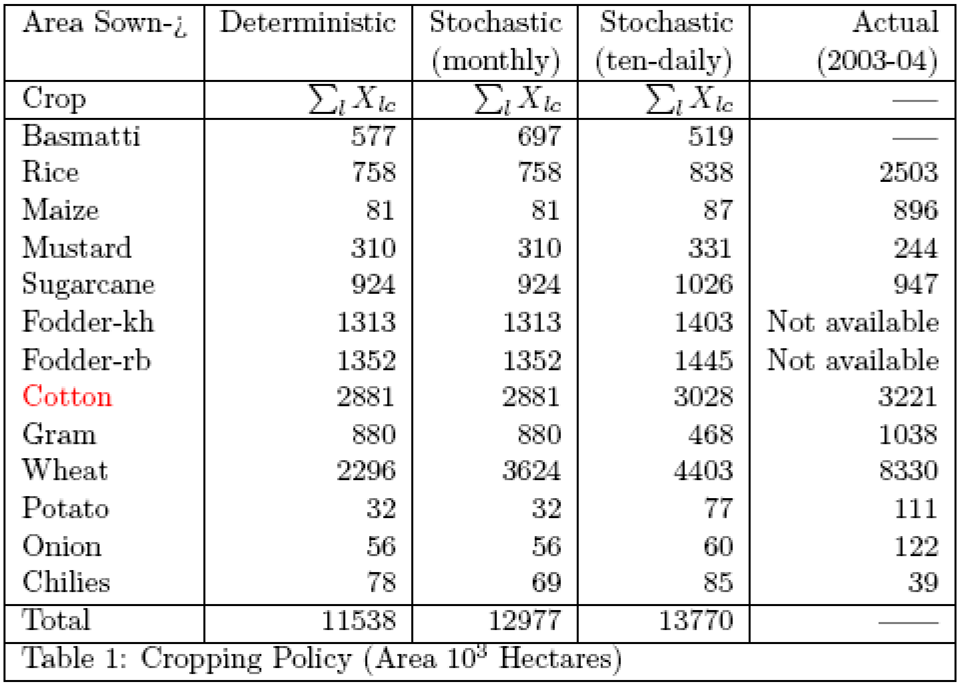


Crop	Current Terbela $10.3km^3$ Mangla $5.7km^3$	Pro Basha Terbela $10.3km^3$ Mangla $5.7km^3$ Basha $7.0km^3$	Pro Kalabag Terbela $10.3km^3$ Mangla $5.7km^3$ Basha $7.0km^3$ Kalabag $7.3km^3$
Rice	1310	1310	1378
Maze	87	87	87
Gram	468	468	468
Wheat	5428	5802	6227

Table 2: Food crops cultivation (Area 10^3 Hectares)

Cropping Pattern with Existing Storages

Total cropped area in the Indus Basin in 2003-04 was 22.94 million hectares where cultivation is possible on more than 32 million hectares *[Economic Survey, Pakistan 2004-05]*. Irrigated area is 18.78 million hectares. Among this irrigated area, 14.87 million hectares irrigated by a canal which is the input in the model. Remaining is irrigated by other conventional sources (Wells, Tub welletc.). 6.87 million hectares area is one that was used at least twice in 2003-04. Comparison in the actual area and results of the model in table 1 indicate the difference in rice, wheat, maize and gram areas. Rice represents the total of two varieties (Basmatti and Irri). Moreover it is also sown where irrigation is done with other sources. Maize is a major crop of rain fed areas along with wheat and gram. Pre mature maize is also used as fodder for cattle. It is also considered as fodder crop throughout the year. Farmers cultivate it two to three times in a year. All the rainfed areas are used for wheat cultivation. That is the sole reason which shows its area double with all simulated solutions. In the rain fed areas average yield of wheat reduces two to three times as compare with the irrigated areas. Gram is most suitable crop for rain fed areas due toits low requirement of water across the cultivation calendar. The results for area sown by three methods of solution and actual *[Economic Survey, Pakistan 2004-05]* are given in the table below.



Area Sown-¿	Deterministic	Stochastic (monthly)	Stochastic (ten-daily)	Actual (2003-04)
Crop	$\sum_l X_{lc}$	$\sum_l X_{lc}$	$\sum_l X_{lc}$	——
Basmatti	577	697	519	——
Rice	758	758	838	2503
Maize	81	81	87	896
Mustard	310	310	331	244
Sugarcane	924	924	1026	947
Fodder-kh	1313	1313	1403	Not available
Fodder-rb	1352	1352	1445	Not available
Cotton	2881	2881	3028	3221
Gram	880	880	468	1038
Wheat	2296	3624	4403	8330
Potato	32	32	77	111
Onion	56	56	60	122
Chilies	78	69	85	39
Total	11538	12977	13770	——

Table 1: Cropping Policy (Area 10^3 Hectares)

Storage and Power Generation

Pakistan consumed 57,491 GWH electric power in the year 2003-04. The average production for the last five years (1999-00 to 2003-04) from hydro power generation is 28,085 GWH i.e. 37% of the total consumption now [*Source*: *Economic Survey; Pakistan*2012-2013].

The power demand is projected to grow at an annual average rate of 7.9 percent during next five years. With the available hydro power production capability, this ratio of hydro power production will go on decreasing unless new storages are not built. With a ten-daily scenario based model, hydro power generation vary from 10,316 GWH to 29,352 GWH from a very low inflow scenario to very high inflow scenario. These results look very much consistent with the present power generation policy in the system. We run this model primarily for optimal cropping policy. Storage levels were not maintained to maximize power generation from the reservoirs. Even then the model results regarding power generation performed well.

5. Summary

The present study provides the comparison between deterministic programming and the application of Stochastic Linear Programming (SLP) in IBIS to find a suitable cropping pattern. In the stochastic model we consider randomness in hydrologic variables, inflow and rainfall in the basin. The operating policy is ten-daily and monthly. Depending on the actual canal commands requirement, target reservoir storage and target release for every time interval is set. This basin has a huge and complex network of nodes and canals. This is a potential river system to generate food due to highly variability in inflow and seasonality as well. We incorporate a constraint avoiding food over the maximum amount of surplus water in the system below a certain level during all time periods, which is according to system infrastructure. This river system is performing below to its potential because of political differences among the provinces of Pakistan. We tried to give reconciliation by incorporating a political constraint. This system produces more the 28,000 GHW electric power. This system has the8potential to produce more than 30,000 MW hydroelectric power. Due to differences among provinces and lack of consensus over storages construction, 45 km^3 water have to divert towards sea annually. All the decisions about hydrologic variables are implemented according to downstream need of agriculture requirements. Although, a storage policy is adapted to pro-duce hydroelectric power i.e. storages are maintain with in minimum maximum bounds. We calculate the amount of power generated from outflow when it takes place from the storage but storage levels are not maintained to produce optimal hydropower. All decisions are governed by irrigation related policies but not for power generation.

The length of Indus River is about 2880 km, where in Pakistan; it is about 2000 km. The downstream canal commands in low rainfall zone performing below to their potential. Considerable amount of evaporation takes place in this region. Moreover this is a low rainfall and high temperate zone which expedite in the evaporation. This zone is most suitable for cotton which is a major cash crop of Pakistan. Cultivation of cotton yields a huge loss in the form of evaporation from the system.

References

Ahmad, N.(1993). Water Resources of Pakistan, Lahore.

Anwar A. A. and D. Clarke.(2001). Irrigation Scheduling Using Mixed-Integer Linear

Programming. Journal of Irrigation and Drainage Engineering 127, No. 2: 63-69.

Asianics Agro-Dev. International (Pvt) Ltd. (2000).Tarbela Dam and related aspects of the Indus River Basin, Pakistan, A WCD case study prepared as an input to the World

Commission on Dams, Cape Town.

Azar A. H., V. V. N. Murty, and H. N. Phien.(1992). Modeling Irrigation Scheduling for Lowland Rice with Stochastic Rainfall. Journal of Irrigation and Drainage Engineering118, No. 1: 36-55.

Dudley, N. J. (1988). A Single Decision-maker Approach to Irrigation Reservoir and Farm Management Decision Making.Water Resources Research 24, No. 5: 633-640.

Dudley, N. J. and B. W. Scott. (1993). Integrating Irrigation Water Demand, Supply and Delivery Management in a Stochastic Environment. Water Resources Research 29, No.9: 3093-3101.

Economic Survey Pakistan (2013). Govt. of Pakistan. Ministry of Finance.

Houghtalen, R. J. and J. C. Loftis. (1988). Irrigation Water Delivery System Operationvia Aggregate State Dynamic Programming. Water Resources Bulletin 24, No. 2: 427-434.

KaleemUllah M., Habib Zaigham and Saim Muhammad. (2001). Spatial Distribution of Reference and Potential Evaporation Across the Indus Basin Irrigation System. International Water Management Institute, Lahore. Working Paper 24.

Labadie, J.W., (2004). Optimal Operation of Multireservoir Systems: State-of-the-Art Review. Journal of Water Resources Planning and Management, 130, 93- 111.

Mannocchi, F. and P. Mecarelli. (1994). Optimization Analysis of De_cit Irrigation
Systems.Journal of Irrigation and Drainage Vol. 120, No. 3: 484-503.9

Mujumdar, P. P. and T.S.V. Ramesh. (1997). Real-time Reservoir Operation for Irrigation. Water Resources Research 33, No. 5: 1157-1164.

Paudyal, G. N. and A. D. Das Gupta. (1990). Irrigation Planning by Multilevel Opti-
mization. Journal of Irrigation and Drainage Engineering 116, No. 2: 273-291.

Paul, S., S. N. Panda, and D. Nagesh Kumar. (2000). Optimal Irrigation Allocation: A Multilevel Approach. Journal of Irrigation and Drainage Engineering 126, No. 3:149-156.

Rao, N. H., P. B. S. Sarma, and S. Chander. (1990). Optimal Multicrop Allocationof Seasonal and Intraseasonal Irrigation Water.Water Resources Research 26, No. 4:551-559.

Rao, N.H., P.B.S. Sarma, and S. Chander. (1992). Real-time Adaptive Irrigation Scheduling under a Limited Water Supply.Agricultural Water Management 20: 267-279.

Ravikumar, V. and K. Venugopal. (1998). Optimal Operation of South Indian Irrigation Systems. Journal of Water Resources Planning and Management 124, No. 5: 264-271.

Rehman, Abdul. (2003). Presentation: Water Resources Data Base and Indus Basin
Model Revised. Workshop on Spatial Information Analysis for Sustainable Management of Indus Basin in Pakistan.

Shad M. Y. and G.C. Pug (2014). Stochastic vs. Deterministic Programming in Water Management: The value of exibility. Annals of Operation Research.DOI 10.1007/s10479-013-1455-8.

Sunantara J. D. and J. A. Ramirez. (1997). Optimal Stochastic Multicrop Seasonal and Intraseasonal Irrigation Control. Journal of Water Resources Planning and Management123,No. 1: 39-48.

N. V. UMAMAHESH and P. SREENIVASULU (1997).Two-Phase Stochastic Dynamic Programming Model for Optimal Operation of Irrigation Reservoir. Water Resources Management 11: 395406..

Vedula, S. and P. P. Mujumdar. (1992). Optimal Reservoir Operation for Irrigation of Multiple Crops. Water Resources Research 28, No. 1: 1-9.

Vedula, S. and D. Nagesh Kumar. (1996). An Integrated Model for Optimal Reservoir Operation for Irrigation of Multiple Crops. Water Resources Research. 32, No. 4: 1101-1108.

Wardlaw, R. and J. Barnes. (1999). Optimal Allocation of Irrigation Water Supplies in Real Time. Journal of Irrigation and Drainage Engineering, 125, No. 6: 345-353.

Appendix

Indices

c = crop
i = storage
k = states
l = canal commands
n = nodes
r = rivers
s = scenarios
t = time intervals (monthly, ten-daily)
z = rainfall zones

Non Stochastic Data/Parameters

$\Delta_{i,min}$ = minimum storage level at storage i (km^3)
$\Delta_{i,max}$ = minimum storage level at storage i (km^3)
h = risk free upper bound of system infrastructure (km^3)
p_c = price of crop c (Dollars/ 10^3 kg)
$Share_k$ = %age share of state k in aggregate surface water
$Labor_z$ = labor force available in zone z
$CropAreaz_c$= maximum area suitable for crop c in zone z (10^6hectares)
$Land_l$ = land resources of canal l in zone z (10^6hectors)
ε_{it} = evaporation from storage i during month t (km^3)

$\boldsymbol{\lambda}_{\mathbf{cl}}$ = average production of crop c in canal l (10^3 kg/ hectare)
$\boldsymbol{\upsilon}_{\mathbf{cl}}$ = initial cost of crop c in canal l (dollars/ hectare)
$\boldsymbol{\delta}_{\mathbf{l}}$ = carrying to field efficiency of canal l

$\boldsymbol{v}_{\mathbf{clt}}$ = water needed for crop c in zone z during time t (mm)
$\boldsymbol{\beta}_{\mathbf{clt}}$ = labor hours for crop c in zone z during time t (dollars/ hectare)
$\boldsymbol{\beta}_{\mathbf{clt}}$ = labor cost for crop c in zone z during time t (dollars/ hectare)
$\boldsymbol{a}_{\mathbf{clt}}$ = indicator for crop c in zone z during time t (present 1, absent 0)
$\boldsymbol{T}_{\mathbf{lt}}$ = ground water available in zone z canal l during time t (*km*3)

Stochastic Data

$\boldsymbol{\sigma}_{\mathbf{t}}^{\mathbf{s}}$= rainfall in zone z during month t under scenario s (mm)
$\boldsymbol{\alpha}_{\mathbf{t}}^{\mathbf{s}}$= inflow in river r during month t under scenario s (*km*^{3})

Objective function is to maximize the revenue from crops and energy generation from reservoir operations.

$$\max\left[\sum_{s\in S}\pi^{s}\left\{\sum_{l\in L}\sum_{c\in C}\left(p_c\lambda_{cl}-\theta_{cl}\right)Y^{s}{}_{lc}\right\}-\sum_{c\in C}\sum_{l\in L}\upsilon_{cl}X_{lc}\right]$$

Subject to

(Seconds)
$$\times\sum_{n:(i,n)\in N}\omega^{s}{}_{int}\le\Delta^{s}{}_{i(t-1)}+\sum_{r:(i\in N_s)}\tilde{\alpha}^{s}_{r(i)t}-\varepsilon_{it};\forall i\in N_s,t\in T,s\in S$$

$$\Delta^{s}{}_{i(t-1)}+\sum_{r:(i\in N_s)}\tilde{\alpha}^{s}_{r(i)t}-\Delta^{s}{}_{it}-\sum_{n:(i,n)\in N}\omega^{s}{}_{int}-\varepsilon_{it}=0;\forall i\in N_s,t\in T,s\in S$$

$$\Delta_{it;min}\le\Delta^{s}{}_{it}\le\Delta_{it;max}+h^{i}{}_{l};\forall i\in N_s,t\in T,s\in S$$

$$\sum_{r:n\in N_R)}\tilde{\alpha}^{s}_{r(n)t}+\sum_{i:(i,n)\in N}\sum_{i:(i,n)\in N}\omega^{s}{}_{int}-\sum_{l\in L}W^{s}{}_{lt}\le h;\forall,t\in T,s\in S$$

$$\sum_{c\in C}a_{clt}X_{lt}\le \text{Land}\ \ \forall l\in L,t\in T$$

$$X_{lc}-Y_{lc}\ge 0\ \forall,l\in L,c\in C$$

$$\sum_{l:z(l)=z}X_{lc}\le \text{CropsArea}\ \ \forall l\in L,z\in Z$$

$$\sum\tilde{\alpha}^{s}_{r(n)t}+\sum_{i:(i,n)\in N}\sum_{i:(i,n)\in N}\omega^{s}{}_{int}-\sum_{l\in L}W^{s}{}_{lt}\ge 0\forall,t\in T,s\in S$$

$$\sum_{l:k(l)=k}\sum_{t\in T}W^{s}{}_{lt}=\text{Share}\sum_{l\in L}\sum_{t\in T}W^{s}{}_{lt}\forall k,s\in S$$

$$\sum_{l:z(l)=z}\sum_{c\in C}\beta_{clt}Y^{s}{}_{lc}=\text{Labor}_z;\forall,t\in T,s\in S$$

$$X_{lc},Y^{s}{}_{lc},W^{s}{}_{lt},\Delta^{s}{}_{it},\omega^{s}{}_{int}\ge 0$$