

Productivity, Economic Equilibrium and von Neumann's Model of General Economic Equilibrium

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Abstract

This study examines the reliability of the von Neumann Model of General Economic Equilibrium to determine economic equilibrium in economic systems. The findings show that the model is of considerable interest to economists as well as mathematicians, because it deals with many areas of economics such as productive processes and scales of production. Most importantly the model demonstrates the mechanism which determines the interest rate and the rate of expansion of the whole economy at the equilibrium state. This will allow for the application and calculation of the process to determine economic equilibrium in economic systems. This will be presented as a graphic presentation and solution.

Keywords: Foreign Exchange Rates, Labor Productivity and Economic Equilibrium

Introduction

The work of Harrod (1933), Balassa (1964), Samuelson (1964) and Olson (2012) argue that productivity growth will lead to a real exchange rate appreciation only if it is concentrated in the traded goods sector of an economy. Productivity growth that has been equally strong in the traded and non-traded sectors will have no effect on the real exchange rate. Bailey and Wells (2001), argue that an increase in US productivity increases the rate of return on capital and increased capital flows in the United States, which might explain in part the appreciation of the US dollar during the early part of the 2000's. Tille and Stoffels (2001) argue that developments in relative labor productivity account for part of the change in the external value of the US dollar over the last 30 years. Alquist and Chinn (2002) argue that there is a robust correlation between the euro area United States labor productivity differential and the dollar/euro exchange rate. This study will employ the von Neumann Model which demonstrates the use of a mechanism to determine the optimum rate of interest and rate of expansion of the whole economy. This is shown in Figure 1.

According to the Balassa-Samuelson model, the distribution of productivity gains is important for assessing the impact of productivity advances on the real exchange rate. Increases in productivity can lead to an increase in exchange rates and growth of the economy as shown in Figure 1. With this change the growth rate of the economy increases from A to B and the interest rate decreases from A to B. The increase in the exchange rate is shown as point B to point D. The optimum growth and interest rate is at point D. The growth rate can be increased to point D but any further increase in the growth of the national output beyond D will result in a less than optimum rate of interest and maximum growth rate.

Olson (2014) argues that there is evidence of stable long-run relationships between the real dollar/euro exchange rate, the productivity measure, and the other variables. The model specification was estimated for the productivity measure. The sample covers the period from 1985 to 2007. This study includes the following tests shown below from the paper of Olson (2013).

Fuller (1976) and Dickey & Fuller (1979) proposed the augmented Dickey-Fuller (ADF) test for the null hypothesis of a unit root. It is based on the t -statistic of the coefficient ϕ from an OLS estimation (see table 1). Schmidt & Phillips (1992) propose another group of tests for the null hypothesis of a unit root when a deterministic linear trend is present.

The empirical analysis employed cointegration tests as developed by Johansen (1995). The presence of the cointegration relationships is tested in a multivariate setting. Table 2 and 3 show the results of the cointegration tests. Over all, the results suggest that it is reasonable to assume a single cointegration relationship between the variables and suggest being an order of I(1).

Data for Variables for the Referenced Study Olson (2013)

For the period 1985-2007, the real dollar/euro exchange rate was computed as a weighted geometric average of the exchange rates of the euro currencies against the dollar. In addition, the model was estimated with several other variables, which included US productivity, M2, oil prices, government spending and US GDP

Olson (2013) argues that the decline of the euro against the US dollar during the period 1995-2007 can be attributed to relative changes in productivity in the United States and the euro area. Figure 2 shows the impact of a change in relative productivity changes over these periods on the equilibrium real exchange rate. The impact is significant. Figure 3 shows the stable relationship of changes in the Dollar/Euro Real Exchange rate and the US Dollar.

Refer to the figure 4 for the US and Euro productivity differentials which show the long-run impact of productivity shocks on the dollar/euro real exchange rate. Figure 4 shows the significance of large gaps in the euro and US productivity differentials especially around the years 2000-2001 when the dollar started to depreciate against the euro.

The findings of the Olson (2013) and Samuelson (1964) show that the US/Euro area productivity differentials have a significant impact on real exchange rates and economic growth. These factors play a role in the determination of the of the productivity processes necessary in achieving economic equilibrium and economic growth. These factors play a role in the determination of the production processes and intensity levels necessary in achieving the velocity of quantity of goods produced.

Von Neumann's Model of General Economic Equilibrium

The following section describes von Neumann's Model of General Economic Equilibrium and its application to this study.

The supreme merit of John von Neumann's Model of General Economic Equilibrium lies in the elegance of the mathematical solution of a highly generalized problem in theoretical economics. However, the paper is of considerable interest to economists as well as to mathematicians, because it deals simultaneously with questions on several fields of economics. For example, in this paper von Neumann considers which goods will be free goods, and the determination of the prices of goods which are not free. He examines which productive processes and scales of production will be optimum and which will be unprofitable. He also examines the degree in which each optimum process will be used and the relative amounts of different goods that will be produced. The model demonstrates the mechanism which determines the rate of interest and the rate of expansion of the whole economy.

The conclusions can be expressed as follows:

An economic system in economic equilibrium will produce the following:

1. The greatest factor of expansion of the whole economy.
2. The lowest interest factor at which a profitless system of prices is possible.

Summary

When applying the findings of Olson (2013) to the van Neumann Model (1937) the results show a strong evidence of a long-run relationship between the real dollar/euro exchange rate and productivity measures. This relationship allows for assumptions in the von Neumann model which include the following:

1. Assume constant returns (to scale).
2. Assume natural factors of production, including labor, can be expanded in unlimited quantities.
3. Assume it is necessary to determine which production processes will be used.
4. Assume it is necessary to determine the relative velocity which the total quantity of goods increases.
5. Assume it is necessary to determine what level prices will obtain.
6. Assume it is necessary to determine what the rate of interest will be.

It is suggested that with these assumptions the production processes can be determined necessary to achieve economic equilibrium in the economic system.

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Appendix

The data for Olson (2013) was collected from the following sources:

Economic Data Base (FRED) of the Economic Research Department of the *Federal Reserve Bank of St. Louis*.

The PPI and CPI are used as proxies for tradable and nontradable goods.

Data Bases and Tables of the *Bureau of Labor Statistics*.

The source of all of the graphs, figures and charts was the Software JMulTi. available from Lutkepohl, Helmut. Applied Time Series Econometrics.2004, Cambridge University Press.

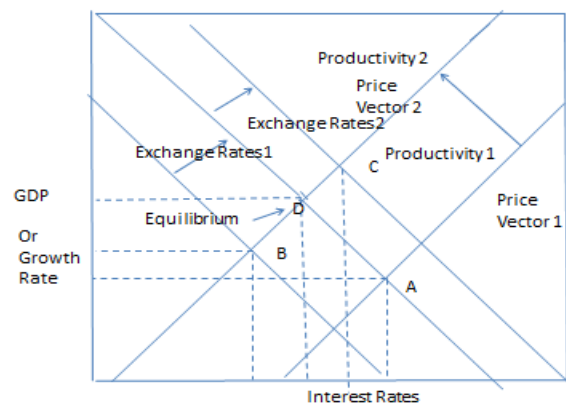


Figure 1

Table 1

ADF Unit Root Tests				Schmidt & Phillips		
	Sample Range	Lagged Difference	Critical Values	Test Values	Critical Values	Test Values
US Prod	1985-2008	2	-3.2535	3.13*	-9.9532	18.1**
Euro Prod	1985-2008	2	-4.1978	3.96	-17.3112	18.1**
US GDP	1985-2008	2	-5.4389	3.41	-11.5869	18.1**
Euro GDP	1985-2008	2	-3.2786	3.96***	-11.4467	25.2**
US CPI	1985-2008	2	-5.4851	3.13	-18.5775	25.2**
Euro CPI	1985-2008	2	-3.7792	3.41**	-12.1413	18.1**
US PPI	1985-2008	2	-2.013	2.56***	-5.4734	18.1**
Euro Govt % of GDP	1985-2008	2	-1.0952	1.94**	-15.0563	18.1**
Oil Prices	1985-2008	2	-2.7965	3.96***	-2.5623	25.2**

Significance at the 99%, 95% and 90% levels are noted by ***, ** and * respectively. The S and L critical values are taken from tables computed by Saikkonen and Lutkepohl

Table 2

Cointegration Without Oil	Period	Specification	LR Ratios	Critical Ratios & Test Results
US Prod	1985-2008	2 lags	3.72	16.22***
Euro Prod	1985-2008	2 lags	2.7	12.45**
US GDP	1985-2008	2 lags	2.23	12.53**
Euro GDP	1985-2008	2 lags	3.32	9.14**
US CPI	1985-2008	2 lags	10.59	12.45**
Euro CPI	1985-2008	2 lags	2.48	12.45**

Significance at the 99%, 95% and 90% levels are noted by ***, ** and * respectively. The S and L critical values are taken from tables computed by Saikkonen and Lutkepohl.

Table 3

Cointegration With Oil	Period	Specification	LR Ratios	Critical Ratios & Test Results
US Prod	1985-2008	2 lags	15.34	25.73**
Euro Prod	1985-2008	2 lags	31.68	42.77**
US GDP	1985-2008	2 lags	13.61	16.22***
Euro GDP	1985-2008	2 lags	26.07	30.67***
US CPI	1985-2008	2 lags	17.82	25.73**
Euro CPI	1985-2008	2 lags	16.62	30.67**

Significance at the 99%, 95% and 90% levels are noted by ***, ** and * respectively. The S and L critical values are taken from tables computed by Saikkonen and Lutkepohl

Figure 2: US Prod > Dollar/Euro Exchange Rate

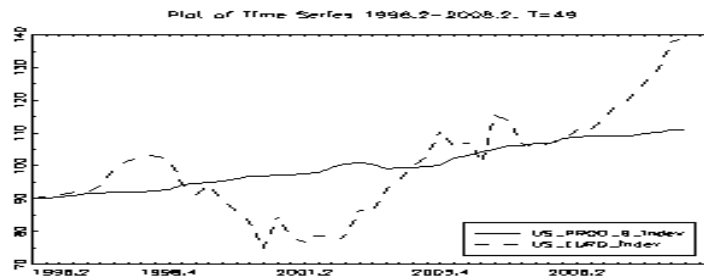


Figure 3: US GDP > USD/EURO Exchange Rate

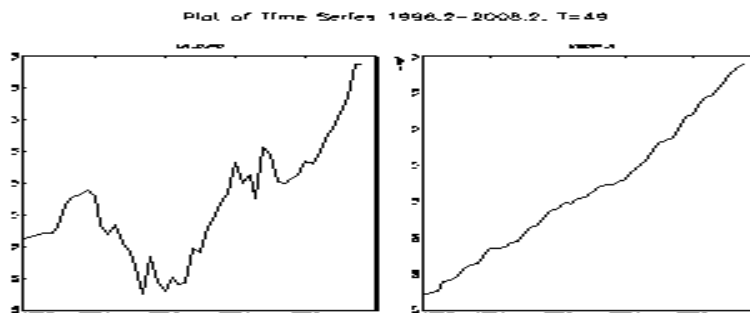


Figure 4

Time Series
Euro Productivity and US Productivity
Dollar/Euro Real Exchange Rate

