Upgrading Corporate Equipment as an Asian Real Option with Constant Business Volatility

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Abstract

A method for handling the problem of financial mission statement has been suggested to evaluate effects of projected upgrading equipment of a manufacturing company. For this end, such project is analyzed as an Asian real option with constant business volatility. The problem is solved using the Black-Scholes model, a refined and modified binomial model and a modified trinomial model. It has been demonstrated that the most accurate valuation of the option and the entire project in general is provided by the trinomial model.

Keywords: upgrading equipment, real option valuation, Asian option, constant volatility, binomial model, trinomial model

1. Introduction

At present time, the technological progress to a large extent determines improvement of living standards of the community. But at the same time, it is important to understand processes of implementing technical and technological innovations into human social activities, primarily in the area of social economic development. In such a manner, economic implementation of technological innovations on a first-priority basis at leading and developing companies in many respects predetermines successful economic performance of the country, which has a direct say in improvement of living standards of the population.

In this connection, creating corresponding economic and financial techniques for successful implementation of technological innovations is one of the top-priority goals of manufacturing companies. One of such trends includes a theory and practice of real options that have been already used in business for a long period since the time when stock market option technologies were for the first time adapted to manufacturing requirements. The term 'real option' itself was introduced into financial science by Stewart Myers (1977). From then onward, the concept "real option" has been seriously progressing having developed both into a separate global scientific field and into quite a broad sphere of practical application in business.

Despite a broad coverage of different business lines with real option techniques, this method of financial analysis and strategic planning already boomed as far back as 1990s. At present, many web-sites dedicated to real options, such as www.real-options.com, look like frankly languorous, and only some of them, such as www.realoptions.org, continue conducting serious surveys in this area, but already in a fully scientific field using for this purpose the stochastic financial mathematics instrument with increasing frequency.

In its issue dated August 14, 1999, *The Economist* journal delivered the following viewpoint on its traditional page *Economics Focus*: real options will be able to obtain a wide circulation in practice unless and until most managers hold a doctorate in applied mathematics. However, due to exactly real options, many leading global companies managed to be greatly in advance of their competitors in business significantly increasing their market capitalization. Perhaps, the most shining example of this includes *Amazon.com* that was in due time even called 'cold table of real options' (Roche, 2005).

To our opinion, a reasonable understanding of this problem should imply a progressive perception of true requirements and missions of economics in general and business in particular. Thus, for instance, already for a long time, instruments used by businessmen and financial analysts in their work include computer resources support, which greatly accelerates processes of taking managerial decisions. For example, building a simple linear regression for predicting any economic indicators can be now done almost in any software program, including in *MS-Excel*. Naturally, no one will ever try to do this manually if there is a computer at hand. Another example of this includes the use in financial calculations of linear and integer programming that is necessary for certain investment missions. Nevertheless, the theory of these methods itself implies deep studies in applied mathematics. For real options, there are also appropriate software packages enabling quite easily to enter basic data into a program and quickly obtain a final result in the form of an eventual figure meaning, for example, the real option value, which then may be, for instance, added to NPV of an investment project. Such procedure already makes no businessman or analyst feel uneasy since it is elementary.

However, the use of real options in practice of doing business should not be satisfied by this. To our opinion, there are two reasons for that.

1. Many scientists, such as Roche (2005), fairly believe that real options are associated with many purely technical problems of financial nature, which should be primarily attributed to the fact that a considerable number of companies prefer to have real options at their disposal, but not to exercise them at the same time. This leads to unjustified overvaluation of investment and innovation projects that may in reality turn out to be even unprofitable. This adversely affects future marketable value of such a company.

2. The principle of real option building and analysis itself should focus its attention primarily on placing financial tasks, because incorrectly formulated investor's objectives will clearly lead to erroneous and, therefore, ineffective management decisions. A correct understanding by an investor of what it wants to get out of business is much more important than the mathematical methods themselves for the solution of standard tasks in many ways. Simply put, a correct statement of a problem is already a half-solved problem.

Taking into account the above reasons will contribute to moving the primary focus onto a more adequate building of the real option in order to solve the task of upgrading the company equipment. And only after that, it will be possible to select the most optimal method of valuating the option.

2. Statement of a ROV Task for Upgrading Company Equipment

A real option for equipment upgrading is a classical 'option for future development' (Limitovsky, 2008). While analyzing future development prospects, the value of an option is usually added to the business or project value determined according to the traditional DCF technology. Capital investments in development (expansion, experience replication) are used as the strike price K. The present value of basic asset S_0 is a current valuation of cash flows that are generated by business (quite often, it is less than the strike price). The time t in models as applied to real options is a period during which it is possible to take a decision on business expansion. A real option for equipment upgrading should be Asian, i.e. it should have a variable strike price, for instance, depending on inflation since cash assets, including investments, have different values at various times.

A classical Asian equity option is an option style in which the strike price is determined on the basis of an average value of the basic asset for a particular period of time. An Asian option is also called an average-price option or an average-rate option. Such options are usually entered into for commodities, stock indices, currency rates and interest rates. Asian options are popular in markets with a high volatility of basic assets (oil, non-ferrous metals, etc.) (Chance, 2001). The differential characteristic of such options consists in the fact that the strike price is unknown as of the contract date. One of the basic variants of Asian options implies a variable strike price (or a variable rate) of the Asian option (Chance, 2001). In this case, the price of a call option will be:

$$C(T) = \max\{S(T) - kA(0,T), 0\}, \quad (1)$$

where $A(0,T)$ is an average value of basic asset;
k - weighing (1 is usually excluded from descriptions).

Usually, $A(0,T)$ means an arithmetic average value. With continuous monitoring in view, it is calculated as follows:

$$A(0,T) = \frac{1}{T}\int_0^T S(t)dt. \quad (2)$$

With discrete monitoring at the time moments $t_1, t_2, ..., t_n$:

$$A(0,T) = \frac{1}{N}\sum_{i=1}^{N} S(t_i). \quad (3)$$

There are also Asian options where the average value is calculated as a geometrical mean. With continuous monitoring in view, it is calculated according to the formula:

$$A(0,T) = \exp\left\{\frac{1}{T}\int_0^T \ln S(t)dt\right\}. \quad (4)$$

Asian options are investment instruments having a moderate risk level. Since the price of an option is based on basic asset pricing data for a particular period, an investor has the opportunity to make a reasonable judgment on investment practicability. As an illustration of a reasonable task for estimating the value of a real option (a ROV task), we will consider an equipment replacement project at a hydrogeological well-drilling plant (Limitovsky, 2008). We will consider the same example in future to compare different methods of solving the ROV task.

Thus, LLC Vodyanoi provides services to gardeners' partnerships in the Moscow Region for drilling water wells. All in all, LLC Vodyanoi has on the books ten drilling rigs operating at different sites and in different areas of the region. The company director is considering a possibility of substantial upgrading of the drilling rigs, which would contribute to reducing operating expenses, increasing the equipment productivity and, accordingly, procuring more orders from potential customers. In order to handle the designated mission, the company management decided to carry out a feasibility study of the upgrading project.

Let us introduce basic data for calculations according to the most likely case of developments per one drilling rig (Table 1).

Table 1: Basic Economic Data for Calculations per Drilling Rig

Indicator Name	Indicator Value	
	Base Case	New Equipment
Productivity, m/machine-shift	8.1	12.2
Equipment utilization ratio by time	0.5	0.5
Average number of shifts per year	304	304
Average price of one drilled meter, USD	22	22
Average operating expenses per machine-shift, USD	123.4	96.1
Net capital costs, including procurement of new rigs less net salvage value of old rigs, USD	-	20,000

The project includes no additional costs and benefits associated with growth of working capital. The equipment rate of depreciation is 20%; at the end of a five-year period, the net value from retirement of equipment is equal to zero.

All calculations were carried out on a real basis – in a fixed scale of prices. The basic financial data for calculations is given in Table 2.

Table 2: Basic Financial Data for Calculations per Drilling Rig

Indicator Name	Indicator Value (% per year)
The company WACC in real terms	12
Risk-free interest rate	4
Income tax rate in the Russian Federation	20

Calculations carried out by the financial director according to the conventional technology show inexpedience of upgrading any drilling rig, not to mention ten drilling rigs (Table 3).

Table 3: Calculation of Cash Flow for Replacing One Drilling Rig

Indicator Name	Indicator Value by Year					
	0	1	2	3	4	5
1. Productivity, m/machine-shift: 1.1. New equipment 1.2. Basis of comparison 2. Equipment utilization ratio 3. Average number of shifts per year 4. Additional scope, m/year $((1.1-1.2)\times 2\times 3)$ 5. Average price of one meter, USD		12.2 8.1 0.5 304 623.2 22	12.2 8.1 0.5 304 623.2 22	12.2 8.1 0.5 304 623.2 22	12.2 8.1 0.5 304 623.2 22	12.2 8.1 0.5 304 623.2 22
6. Additional revenue per year, USD (4×5)		13,710.4	13,710.4	13,710.4	13,710.4	13,710.4
7. Average operating expenses per machine-shift, USD: 7.1. New equipment 7.2. Basis of comparison 8. Average number of shifts per year		123.4 96.1 304	123.4 96.1 304	123.4 96.1 304	123.4 96.1 304	123.4 96.1 304
9. Additional operating expenses per year, USD $((7.1-7.2)\times 8)$		8,299.2	8,299.2	8,299.2	8,299.2	8,299.2
10. Additional capital costs, USD	-20,000					
11. Rate of depreciation, % 12. Depreciation of additional capital investments, USD		20 4,000	20 4,000	20 4,000	20 4,000	20 4,000
13. Additional income per year, USD $(6-9-12)$		1,411.2	1,411.2	1,411.2	1,411.2	1,411.2
14. Income tax (20%), USD		282.24	282.24	282.24	282.24	282.24
15. Project net cash flow (CF), USD $(13-14+12)$	-20,000	5,128.96	5,128.96	5,128.96	5,128.96	5,128.96
16. WACC, %		12	12	12	12	12
17. Project NPV, USD	-1,511.25	CF discounting result at the rate of 12%				

Each of the projects reduces the wealth of owners by 1,511.25 USD, which is a considerable amount for this company.

At the same time, the director has great doubts about the calculation results connected with the accuracy of predicting cash flows. The issue is about that uncertainty, which is borne by the basic assumptions regarding:

a) the number of orders and related operating expenses per one drilled meter (saving on semi-constant expenses is possible) and the equipment utilization ratio;
b) faultless performance of new equipment and repair frequency;
c) average depth of drilled wells (payment is made not for meterage, but for the result of drilling, i.e. the number of productive wells) and others.

As a result, the efficiency calculation accuracy has the mean-square deviation $\sigma = 40\%$ (mean-static σ (%) in USD for the machine-building industry).

In order not to lay down the entire business at stake in general and to obtain more accurate information on the project results, the director of LLC Vodyanoi decides to conduct an experiment: despite the negative calculation results, to carry out upgrading of one of the drilling rigs. If the result turns out to be successful (which will be clear within a year), this experience may be repeated for the other nine rigs.

This approach produced a moderately negative impression upon the financial director who construed this decision as a distrust in the quality of his calculations. In order to stroke him down, the director had to issue a bonus to him and provide him with a special leave.

There remained, however, an open question: whose position was more reasonable in such situation, the director's or that of his deputy in charge of finance?

Thus, the pilot project provides us with information on what may happen to the following nine projects and reveals the uncertainty. As a matter of fact, it confers entitlement to investing money in the nine similar projects within a year under favorable circumstances (in case of a positive result of the pilot project). This entitlement represents a call option for 9 projects (or 9 options, each for 1 project).

On top of everything else, it should be noted that cash assets depreciate with the lapse of time even for a period of one year. Such problem is particularly topical for developing markets, including Russia. However, since the financial calculations are made in US dollars, it is required to consider in future calculations the inflation rate of exactly US dollar, which has been averaged to 3% per annum for the last years. With this factor in mind, the strike price will be $K = 20600$ USD in a year. Consequently, we come to an Asian option model, i.e. an option with a variable strike price (in this case, based on the inflation rate).

It is also important to note that during calculating the option value, we will use a risk-neutral approach as it implies no calculation of WACC for each respective year and requires no estimation of any actual transition probabilities in the event space.

3. Solving a ROV Task for Equipment Upgrading Using the Black-Scholes Model

The underlying problems relating to the use of the Black-Scholes model (OPM) to valuate real options include as follows (Trifonov, Yashin, Koshelev, 2012):

1. OPM includes σ of contract profitability, which is not possible to predict accurately.
2. If σ is predicted by experts, there appears a problem of reliability of the prediction.
3. OPM is only applicable to European options.
4. OPM was created for conditions and restrictions of a stock market.

The first problem is particularly topical for developing markets, including Russia. We are going to solve it switching to financial calculation in US dollars. Therefore, we can use, as we mentioned previously, the mean static σ (%) in USD for the machine-building industry. With a view to even greater specification of calculations, we can also adjust it for the project implementation conditions existing in Russia. But such adjustment itself also bears an uncertainty that is again very difficult to evaluate accurately.

In this case, there is one of the basic principles of evaluating volatility, which is used in stochastic financial mathematics, namely, the principle 'volatility is volatile in itself' (Shiriayev, 1998).

The second problem is also associated with the project implementation conditions in Russia. Here, expert evaluations are also notably volatile.

The third problem makes even more serious impact on the reliability of estimating the value of a real option since in reality, we understand that we can exercise it when we need it (within an option period under review). Therefore, it is more reasonable to analyze an American option. However, as is pointed out by many authors, such as Limitovsky (2008), in this case, OPM may be applied for conservative estimate of an American real option, i.e. the price of a European option is a lower limit for the price of an American option having the same terms of issue.

The fourth problem is perhaps the most serious one, but it may be approximately solved using the same method that was used for the third problem.

The formal OPM formula developed for valuation of a premium under a European call option looks like this:

$$C_0 = S_0 N(d_1) - Ke^{-rT} N(d_2); \quad (5)$$

$$d_1 = \frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}; \quad (6)$$

$$d_2 = d_1 - \sigma\sqrt{T}, \quad (7)$$

where C_0 – current price of call option;
S_0 – current price of basic asset (it is expected that the asset brings no current income, i.e. dividend or coupon);
K – strike price;
r – continuous yearly rate of risk-free return (growth power);
T – time to exercise of option (in years);
σ - mean-square deviation of basic asset price per year;
$N(d)$ – cumulative normal distribution function.

Please note that σ in the example under review does not change due to a short period of the real option – one year. Consequently, we will valuate an Asian real option with constant business volatility. Let us do this in Table 4 according to formulas (5)-(7).

Table 4: Valuation of an Asian Real Option with Constant Business Volatility Using OPM

Parameters and Indicators	Parameter and Indicator Values
Number of options in project	9
S_0 for each option, USD	18,488.75 (PV of project cash inflows)
K for each option, USD	20,600 (investments)
r	0.04 (continuous risk-free rate)
T	1 (option period – 1 year)
σ	0.4
d_1	0.103575
d_2	-0.296425
$N(d_1)$	0.54123
$N(d_2)$	0.38346
C_0, USD	2,417.15
Option project NPV, USD	20,243.14 ($2{,}417.15 \times 9 - 1{,}511.25$)

Consequently, the director of LLC Vodyanoi was right: despite the apparent inexpedience of upgrading, the experiment is fully justified.

4. Solving a ROV Task for Equipment Upgrading Using a Binomial Model

With a view to specifying ROV calculations in our example, it is required to solve the remaining two problems that we had in the Black-Scholes model (OPM):

1. OPM is only applicable to European options.
2. OPM was created for conditions and restrictions of a stock market.

They may be solved using in calculations, for instance, a binomial model slightly modified by us (Trifonov, Yashin, Koshelev, 2012). The first modification consists in changing the strike price of a real option in a certain period of time, depending on the inflation rate for the corresponding number of elapsed periods. The second modification consists in a practical opportunity to trace time moments beneficial for early exercise of the real option, i.e. in identifying nodes of a binomial tree where the price of a 'dead' (exercised) option is higher than that of a 'live' (non-exercised) option.

Let us apply this modified binomial model to the solution of the ROV task for equipment upgrading. The characteristic feature of its application includes an adequate transition from a continuous process to a discrete one using for this purpose the already known constant volatility σ.

The classical approach of Cox, Ross and Rubinstein (1979) (CRR model) implies a transition according to the equations:

$$u = e^{\sigma\sqrt{\Delta t}}; \qquad (8)$$

$$d = e^{-\sigma\sqrt{\Delta t}}; \qquad (9)$$

$$p = \frac{e^{r\Delta t} - d}{u - d}, \qquad (10)$$

where u – appreciation rate of basic asset;
d - depreciation rate of basic asset;
p – pseudoprobability of the event u;
$1-p$ - pseudoprobability of the event d;
σ - mean-square deviation of basic asset price per year;
Δt - time interval between nodes of binomial grid (in years);
r – continuous yearly rate of risk-free return (growth power).

The largest disadvantage of the CRR model includes the fact that it loses stability if $\Delta t > \sigma^2 / r^2$, and, as a consequence, calculations may contain negative pseudoprobabilities (Trigeorgis, 1991).

With a view to more accurate simulation with a longer time interval Δt, a binomial tree may, according to the viewpoint of Jabbour, Kramin, Young (2001) and Hull (2006), be derived in accordance with the following equations:

$$u = e^{\sqrt{e^{\sigma^2\Delta t} - 1} + r\Delta t}; \qquad (11)$$

$$d = e^{-\sqrt{e^{\sigma^2\Delta t} - 1} + r\Delta t}; \qquad (12)$$

$$p = \frac{e^{r\Delta t} - d}{u - d}. \qquad (13)$$

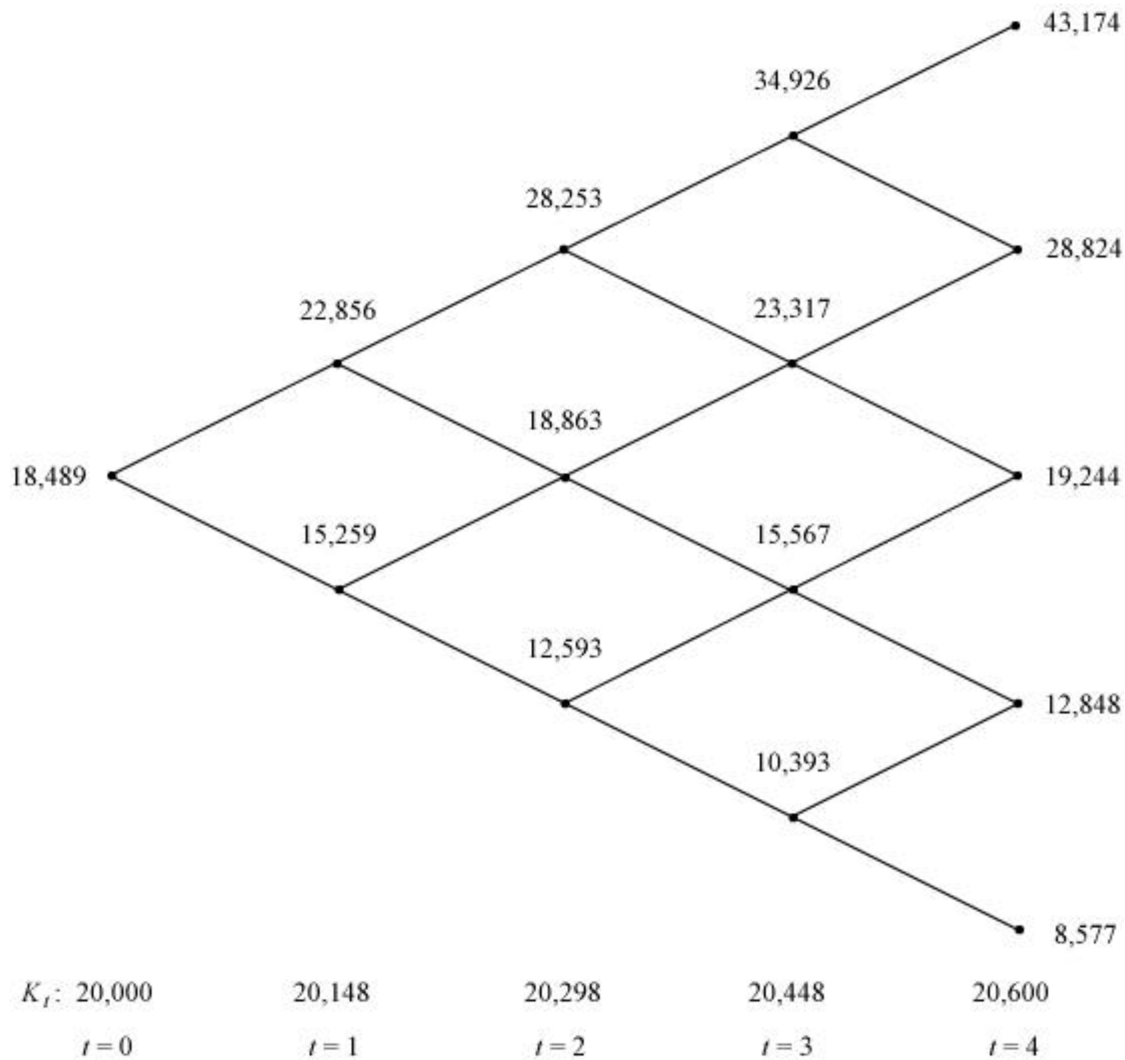
This tree may be regarded as a supplement to CRR parameterization. In this case, the both leaps, upward (u) and downward (d), slightly change. As a result, the central line of the tree follows a risk-free rate. Another advantage includes the fact that this parameterization is also always stable regardless of time interval length Δt (Haahtela, 2010).

Using model (11)-(13) for $\Delta t = 0.25$ year, we obtain the following parameterization in the example under our review:

$u = 1.236169$; $d = 0.825293$; $p = 0.449666$; $1 - p = 0.550334$.

The result is that based on the values *u* and *d*, we obtain a binomial tree for modifying the value S_t of the basic asset (PV of pilot project cash inflows) in US dollars (Figure 1). In the same figure, let us show changes in the strike price (K_t) as per quarterly inflation rate $i = \sqrt[4]{1.03} - 1 = 0.007417$.

Figure 1: Binomial Tree of Basic Asset Price Variance (USD)



In the binomial CRR model, the price of a 'live' option may be calculated according to the formula

$$C_t^N = \frac{pC_{t+1,u} + (1-p)C_{t+1,d}}{e^{r\Delta t}}. \tag{14}$$

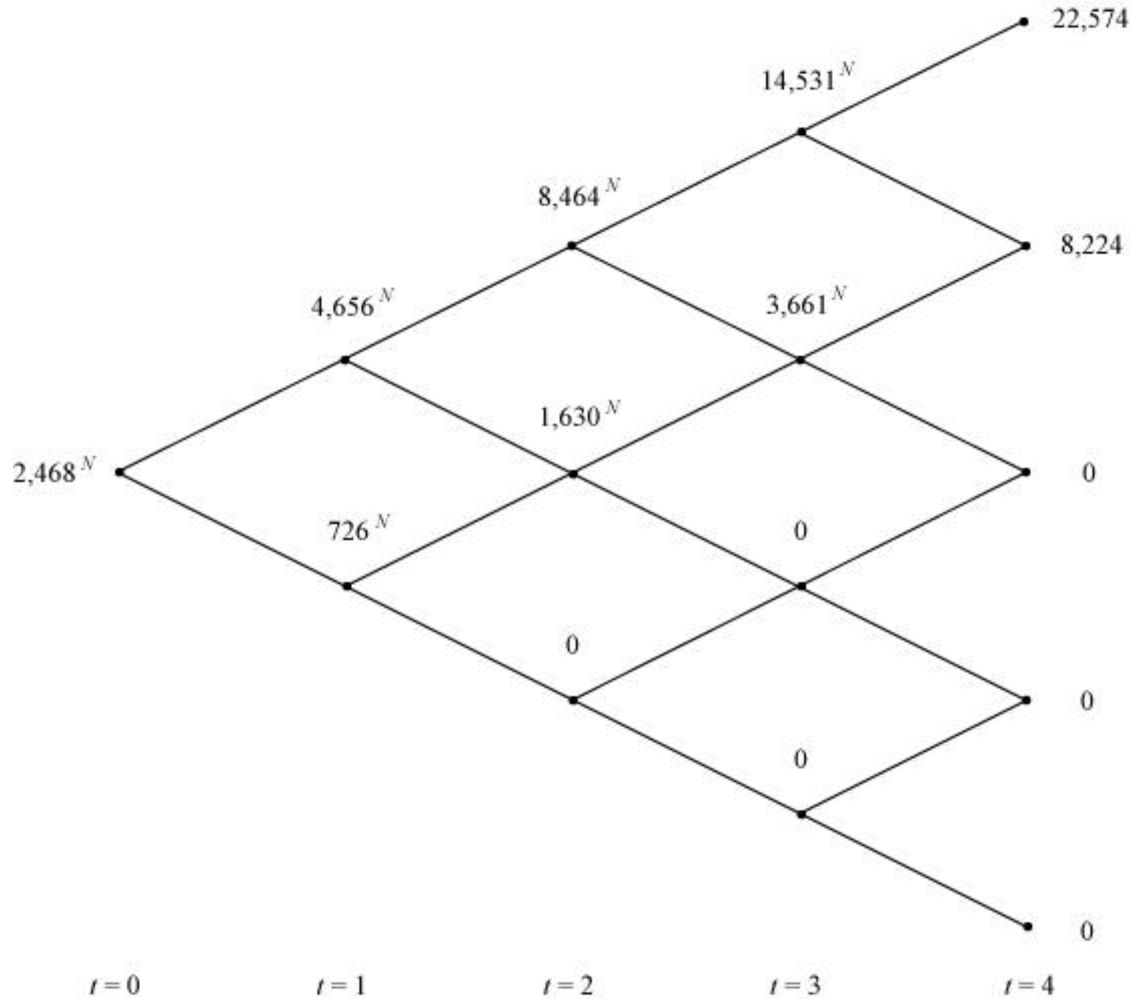
Consequently, it is possible to estimate the option value in any period *t* if $C_{t+1,u}$ and $C_{t+1,d}$ are known in the next period $t + 1$.

Since we are considering a call option, then in each period *t*, the price of a 'dead' option shall be calculated according to the formula

$$C_t^A = \max\{S_t - K_t, 0\}. \tag{15}$$

Using formulas (14) and (15), it is possible to sequentially calculate the option prices beginning with Quarter 4 and ending with the present moment of time (Figure 2). In this connection, in each node of the binomial tree, the maximum price is selected out of the prices C_t^N and C_t^A for the purposes of sequential calculation.

Figure 2: Binomial Tree of Real Option Price Variance (USD)



For instance, in Quarter 3 ($t = 3$) in the uppermost node, the option price shall be calculated as follows. At first, the price of a 'live' option is calculated according to formula (14):

$$C_{3,u^3}^N = \frac{0.449666 \cdot (43{,}174 - 20{,}600) + 0.550334 \cdot (28{,}824 - 20{,}600)}{e^{0.04 \cdot 0.25}} = 14{,}531 \text{ (USD)}.$$

Then the price of a 'dead' option is calculated according to formula (15):

$$C_{3,u^3}^A = 34{,}926 - 20{,}448 = 14{,}478 \text{ (USD)}.$$

The 'live' option turns out to be more expensive, that is why its price is selected in order to calculate the option earlier price.

The entire binomial tree shown in Figure 2 is derived on this principle. In our example, it turned out that a 'live' option was in any case more profitable for an investor. However, the other situation may arise as a matter of practice, i.e. a 'dead' option may be more expensive in some nodes of the tree, which is indicative of the necessity to early exercise it in this node. This may be affected by the inflation rate i, which would modify the strike prices K_t. Also, a similar effect may be provided by modifying the parameters u and d.

As a result, working in the tree from its end to the beginning, we may obtain the price of this pilot project option in zero. It will be $C_0 = 2{,}468$ USD. Then NPV of the equipment upgrading project with 9 options will be

$$\text{NPV} = 2{,}468 \cdot 9 - 1{,}511.25 = 20{,}700.75 \text{ (USD)},$$

which is somewhat greater than the calculation result according to OPM. This is an amended estimate of the project effect.

5. Solving a ROV Task for Equipment Upgrading Using a Trinomial Model

Using a binomial CRR model, though refined by means of equations (11)-(13), involves a particular set of weaknesses primarily associated with a situation of business volatility behavior in the time domain (Haahtela, 2010). But there is also a material weakness consisting in the fact that in case of very low or even insignificant volatility during a certain period of time, any upward or downward deviation of the basic asset price from the expected value in future, i.e. an increase according to the risk-free rate ($S_{t+1,m} = S_{t,m} e^{r\Delta t}$), will make a binomial tree derivation impossible (Haahtela, 2010).

A trinomial tree is derived with simultaneous selection of such parameters that establish a reasonable problem space, having in mind reasonable probabilities of transition between the tree nodes. Moreover, the tree recombination is set so that $ud = du = m^2 = e^{2r\Delta t}$ because otherwise, any rise of the system discreteness would not lead to low or even zero volatility. A trinomial tree is always stable regardless of time interval length. The equations describing upward and downward movements of the stochastic processes are more accurate, even with longer time intervals. This is necessary as the time intervals used to valuate a real option are selected on the basis of managerial practicability, so that they be longer than it usually occurs in financial options.

Trinomial trees are another discrete representation of the basic asset price behavior, similar to binomial trees. Trinomial grids have three leap parameters u, m and d and three corresponding probabilities p_u, p_m and p_d. During this time, the asset price increment may pass on to one of the three nodes: with the probability p_u to upper node as far as the value S_u, with the probability p_m to the node middle as far as the value S_m and to lower node as far as the value S_d with the probability p_d. We presume that the sum of probabilities is equal to one, that is why we set $p_m = 1 - p_u - p_d$. At the end of each time interval, there are five unknown parameters: two probabilities p_u and p_d, and three price nodes S_u, S_m and S_d.

In this respect, a minor modification suggested by Jabbour, Kramin, Young (2001) and Hull (2006) consists in the use of more accurately estimated deviation in accordance with the equation

$$\sqrt{e^{\sigma^2 \Delta t} - 1} \approx \sigma\sqrt{\Delta t} \tag{16}$$

instead of $\sigma\sqrt{\Delta t}$. According to the viewpoint (Haahtela, 2010), following such changes, a trinomial grid parameter derivation results in an improved general parameterization form for all the probability transitions and leap sizes u, m and d in accordance with the following equations:

$$p_u = \frac{m^2 (V-1)}{u^2 + md - um - ud}; \tag{17}$$

$$p_d = p_u \frac{m-u}{d-m}; \tag{18}$$

$$p_m = 1 - p_u - p_d\,; \tag{19}$$

$$u = e^{r\Delta t + \sqrt{e^{(\lambda\sigma)^2 \Delta t} - 1}}\,; \tag{20}$$

$$d = e^{r\Delta t - \sqrt{e^{(\lambda\sigma)^2 \Delta t} - 1}}\,; \tag{21}$$

$$m = e^{r\Delta t}\,; \tag{22}$$

$$V = e^{\sigma^2 \Delta t}\,, \tag{23}$$

where the justified value of the dispersion parameter λ is 1.12 (Haahtela, 2010). This makes the problem space dense and ensures quite good probabilities of transition between the trinomial grid (tree) nodes.

The movements upward (20) and downward (21), which determine the problem space, are calculated according to the maximum volatility during investments so that $\sigma = \max \sigma_t$. These values *u* and *d* are used for the entire problem space within all periods of time, regardless of any change in volatility. Nevertheless, the transition probabilities calculated according to (17)-(19) are only justified for a period of time having the highest volatility. In our example, a time-constant business volatility is assumed, which greatly simplifies our calculations.

Using model (17)-(23) for $\Delta t = 0.25$ year, we obtain the following parameterization in the example under our review:

$$u = 1.236169;\quad d = 0.825293;\quad m = 1.01005;\quad V = 1.040811;$$
$$p_u = 0.350268;\quad p_d = 0.439457\,;\quad p_m = 0.210275\,.$$

The result is that based on the values *u*, *m* and *d*, we obtain a trinomial tree for modifying the value S_t of the basic asset (PV of pilot project cash inflows) in US dollars (Table 5). In the same table, let us show changes in the strike price (K_t) as per quarterly inflation rate $i = \sqrt[4]{1.03} - 1 = 0.007417$.

Table 5: Trinomial Grid of Basic Asset Price Variance (USD)

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$
S_t					47,681
				37,626	28,004
			29,691	29,989	30,291
		23,430	23,665	23,903	24,144
	18,489	18,675	18,862	19,052	19,244
		14,885	15,035	15,186	15,338
			11,983	12,103	12,225
				9,647	9,744
					7,766
K_t	20,000	20,148	20,298	20,448	20,600

In a trinomial model, the price of a 'live' option may be calculated according to the formula

$$C_t^N = \frac{p_u C_{t+1,u} + p_m C_{t+1,m} + p_d C_{t+1,d}}{e^{r\Delta t}}\,. \tag{24}$$

Consequently, it is possible to estimate the option value in any period *t* if $C_{t+1,u}$, $C_{t+1,m}$ and $C_{t+1,d}$ are known in the next period *t* + 1.

Since we are considering a call option, then in each period *t*, the price of a 'dead' option (C_t^A) shall be calculated in the same way as in the binomial model case, i.e. according to formula (15).

Using formulas (24) and (15), it is possible to sequentially calculate the option prices beginning with Quarter 4 and ending with the present moment of time (Table 6). In this connection, in each node of the trinomial grid, as with the binomial one, the maximum price is selected out of the prices C_t^N and C_t^A for the purposes of sequential calculation.

Table 6: Trinomial Grid of Real Option Price Variance (USD)

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$
					27,081
				$17{,}231^N$	17,404
			$9{,}756^N$	$9{,}595^N$	9,691
		$5{,}094^N$	$4{,}715^N$	$4{,}098^N$	3,544
C_t	$2{,}510^N$	$2{,}170^N$	$1{,}677^N$	$1{,}229^N$	0
		670^N	426^N	0	0
			0	0	0
				0	0
					0

For instance, in Quarter 3 ($t=3$) in the uppermost node, the option price shall be calculated as follows. At first, the price of a 'live' option is calculated according to formula (24):

$$C_{3,u^3}^N = \frac{0.350268\cdot(47{,}681-20{,}600)+0.210275\cdot(38{,}004-20{,}600)+0.439457(30{,}291-20{,}600)}{e^{0.04\cdot 0.25}} =$$

$= 17{,}231$ (USD).

Then the price of a 'dead' option is calculated according to formula (15):

$$C_{3,u^3}^A = 37{,}626-20{,}448 = 17{,}178 \text{ (USD)}.$$

The 'live' option turns out to be more expensive, that is why its price is selected in order to calculate the option earlier price.

The entire trinomial tree shown in Table 6 is derived on this principle. In our example, it turned out again that a 'live' option was in any case more profitable for an investor. The alternative of occurrence of the other situation was described by us above in respect to the binomial tree.

As a result, working in the tree from its end to the beginning, we may obtain the price of this pilot project option in zero. It will be $C_0 = 2510$ USD. Then NPV of the equipment upgrading project with 9 options will be

$$\text{NPV} = 2{,}510\cdot 9 - 1{,}511.25 = 21{,}078.75 \text{ (USD)},$$

which is even greater than the calculation result according to the binomial model. This is even more accurate estimate of the project effect.

6. Conclusion

In conclusion, let us compare the results of the three described models for valuating an Asian real option for upgrading equipment of a company having constant business volatility. It is a reminder that for the purposes of analysis, three models have been used:

1. The Black-Scholes model (OPM).
2. A refined binomial model (BTM).
3. A trinomial model (TTM).

In the example under review, the option price comparison yields the following results:

$$C_0 = \left\{ \underbrace{2{,}417}_{\text{OPM}} < \underbrace{2{,}468}_{\text{BTM}} < \underbrace{2{,}510}_{\text{TTM}} \right\}.$$

Then NPV of the company equipment upgrading project with 9 options will be:

$$\text{NPV} = \left\{ \underbrace{20{,}243}_{\text{OPM}} < \underbrace{20{,}701}_{\text{BTM}} < \underbrace{21{,}079}_{\text{TTM}} \right\}.$$

These results make it possible to come to the following conclusions:

1. As a matter of fact, the Black-Scholes model is the lower limit for the price of an American option having the same terms of issue as a European option.
2. With the constant business volatility σ, which enables us to make the option existence condition throughout one year, the valuation variance for different models is insignificant.
3. A real option for equipment upgrading should be Asian, i.e. it should have a variable strike price, for instance, depending on inflation since cash assets, including investments, have different values at various times.
4. In interim calculations, it is always required to compare the price of a 'live' and 'dead' option in the tree nodes and select that option, which is more expensive. On top of everything else, this contributes to tracing a possibility of option early exercise.
5. The most important practical conclusion includes the fact that the trinomial model allows the most accurate valuation of an Asian real option with constant business volatility. In the example under review, a pecuniary advantage of this is insignificant, but in practice, there may appear situations, in which the advantage may be great to the extent that various models may lead both to positive and to negative NPV with options. And this, in turn, will greatly influence taking a managerial decision on investments.

The results obtained in this paper may contribute to upgrades of the software used to draw up and valuate real options. And the main thing is that they may be useful to businessmen, managers and financial analysts of primarily manufacturing companies with a view to developing and substantiating strategic decisions in innovative business development.

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