Application of Continued Fractions in Controlling Bank fraud

Fadipe-Joseph O.A.

Titiloye E. O.

Department of Mathematics
University of Ilorin
P.M.B. 1515, Ilorin
Nigeria

Abstract

A finite continued fraction is a finite expression obtained through an iterative process of representing a number. We investigate the use of continued fraction in controlling bank staff fraud.

Key words: Continued fraction, fraud, bank, internal control.

1 Introduction

Fraud is defined as any behavior by which one person intends to gain a dishonest advantage over another. In other words fraud is an act of commission which is intended to cause wrongful gain to one person and wrongful loss to the other, either by way of concealment of facts or otherwise.

Fraud has been the precipating factor in the distress of banks, and as much as various measures have been taken to minimize the incidence of fraud, it still rises by the day because fraudsters always device tactical ways of committing fraud. This has become a point of great attention in the banking sector.

Fraud is one of the numerous enemies of the business world. The high incidence of fraud within the banking industry has become a problem to which solution must be provided in view of the large sums of money involved and its adverse implications on the economy.

Fraud in its effects reduces the assets and increases the liability of any company. In the case of banks, this may result in the loss of potential customers or crisis of confidence of banking by the public and in the long run end up in another failed bank situation. Special organizations have been formed to combat it and International Police (Interpol) tries to deal with it at the international level, but it has not been eradicated (Nwankwo, 1991). An analysis made of cases brings out broadly the following major elements responsible for the commission of frauds in banks.

a. Active involvement of the staff-both supervisor and clerical either independent of external elements or in connivance with outsiders
b. Failure on the part of the bank staff to follow meticulously laid down instructions and guidelines
c. External elements perpetuating fraud on banks by forgeries or manipulations of cheques, drafts and other instruments.
d. There has been a growing collusion between business, top banks executives, civil servants and politicians in power to defraud the banks, by getting the rules bent, regulations flouted and banking norms thrown to the winds.

In 2008, Ogidefa [10] reported that the problem of fraud in our banking system may have some attachments which are:

a. Bank malpractices.
b. Failure to appoint trusted and honest official as the representative in the clearing house.
c. Failure to change representative on regular basis.
d. Failure to provide locked boxes or bags for carrying cheques to and from the central banks.
e. Inadequate training facilities for clearing staff both in the offices and central bank.
f. Negligence in checking clearing cheques from the banks to avoid a case of possible short change of cheque

It was also reported that fraud is an anti economic process and must properly be dealt with. He made it clear that for any banking institution to stand there must be proper management and trusted worker that are psychologically fit before employment. Fraud was also referred to as a virus which spreads from the banking sector to other economic activities and organization even the government and that for any economy to be balanced; an antifraud virus should be injected to the banking industry to completely eradicate fraud from the system.

In view of the gravity of fraud in banks, the management of various banks had employed different measures, such as establishment of internal control unit, fraud alerts, security measures etc., yet fraud has continued in an upward trend, and this has called the effectiveness of these measures into question (Okubena, 1998). It was suggested by Nwankwo (1991) that on the discussion of the anatomy of frauds, management should evolve positive attitudes towards safeguarding the banks assets and ensuring that staffs do not exploit the weakness in internal control. He further said that the policies should stress the cardinal principles of separation of duties to ensure that one person does not originate and complete an assignment or entry.

In 2009, Abiola [1] investigated some practical means of minimizing the incidence of fraud in banks. It was revealed that so many factors contributed to the incidence of fraud in the banks amongst which are poor management of policies and procedures, inadequate working conditions, bank's staff staying longer on a particular job, and staff felling frustrated as a result of poor recommendations.

In this work, we introduce an internal control method since fraud is caused majorly by the bank staff. Job rotation involves shifting a person from one job to another, so that he is able to understand and learn what each job involves. In an organization like bank, job rotation should be undertaken to prevent employees from doing any kind of fraud, that is, if a person is handling a particular job for a very long time he will be able to find loopholes in the system and use them for his benefit and indulge in fraudulent practices. Job rotation avoids this.

2 Preliminaries

Mathematics as a science, studies past discoveries in calculations. Those who wish a particular field of mathematics, whether it is statistics, abstract algebra, or continued fractions, will first need to study their fields past. The origin of continued fraction is traditionally placed at the time of the creation of Euclid's algorithm. However, Euclid's algorithm is used to find the greatest common denominator (GCD) of two numbers or integers a and b. for example, if we are given two integers a and b, one can find the (GCD) of (3, 5) = 1 and (GCD) of (12, 60) = 12 and so on. However, algebraically manipulating the algorithm, one can derive the simple continued fraction of the rational $\frac{p}{q}$ as opposed to the GCD of p and q .

A finite continued fraction is a finite expression obtained through an iterative process of representing a number as the sum of its integer part and the reciprocal of another number, then writing this other number as the sum of its integer part and another reciprocal, and so on. The iterations terminated after finitely many steps. All integers in the sequence must be positive. Continued fractions are extremely important in the theory of rational approximation. Continued fraction representation for a number is finite if and only if the number is rational. The study of continued fractions is motivated by a desire to have "mathematically pure" representation for the real numbers. A continued fraction is an expression of the form

$$a_{0+}\cfrac{1}{a_{1+}\cfrac{1}{a_{2+}\cfrac{1}{a_3 \quad + \ddots}}}$$

where $a_0, a_{1,} a_{2,} \ldots$ are real numbers with $a_{1,} a_{2,} \ldots$ positive. If the expression contains finitely many terms, then it is called a *Finite Continued fraction*; otherwise it is called an *Infinite Continued fraction*.

The real numbers $a_0, a_{1,} a_{2,} \ldots$ are called the *partial quotients* of the continued fraction. The continued fraction above is said to be *Simple*, if the real numbers $a_0, a_{1,} a_{2,} \ldots$ are all integers.

Every finite continued fraction represents a rational number, and every rational number can be expressed as a finite continued fraction, c.f. [3, 4]

Continued fraction has a number of useful applications in solving physical problems such as some diophantine equations and chaotic problems. In this work, we used continued fraction to solve bank staff fraud.

3 Main Results

Here, we state the lemma that would be used in the work.

3.1 Lemma A. [7]

Every rational number can be expressed as a finite continued fraction.
To calculate the continued fraction of $r = \frac{p}{q}; q \neq 0$, we write the integer part which is called the *floor of r*. Then we subtract this integer part from *r*, hence, we proceed to find the reciprocal of the difference and repeat the process over and over again. The process will only terminate if *r* is rational. Thus, the process terminates since *r* is a rational number.

I. We find the continued fraction representation of $\frac{137}{1600}$
We follow the algorithm

1. $b = \frac{137}{1600}$ = 0.085625 and $i = 0$
2. $a_0 = \lfloor b \rfloor = 0$
3. $b = \frac{1}{(b-a_0)}$ = 11.67883211 and $i = 1$
4. $a_1 = \lfloor b \rfloor = 11$
5. $b = \frac{1}{(b-a_1)}$ = 1.47311828 and $i = 2$
6. $a_2 = \lfloor b \rfloor = 1$
7. $b = \frac{1}{(b-a_2)}$ = 2.1136 and $i = 3$
8. $a_3 = \lfloor b \rfloor = 2$
9. $b = \frac{1}{(b-a_3)}$ = 8.8 and $i = 4$
10. $a_4 = \lfloor b \rfloor = 8$
11. $b = \frac{1}{(b-a_4)}$ = 1.26 and $i = 5$
12. $a_5 = \lfloor b \rfloor = 1$
13. $b = \frac{1}{(b-a_5)}$ = 4 and $i = 6$
14. $a_6 = \lfloor b \rfloor = 4$

Since $b - a_6 = 0$; we stop

The required continued fraction is [0; 11, 1, 2, 8, 1, 4]
$\Rightarrow \frac{137}{1600}$ = [0; 11, 1, 2, 8, 1, 4]

II. We find the continued fraction representation of $\frac{1}{3}$
We follow the algorithm

1. $b = \frac{1}{3}$ = 0.33333. . . and $i = 0$
2. $a_0 = \lfloor b \rfloor = 0$
3. $b = \frac{1}{(b-a_0)}$ = 3 and $i = 1$
4. $a_1 = \lfloor b \rfloor = 3$

Since $b - a_1 = 0$; we stop.

The required continued fraction representation is [0; 3]
$\Rightarrow$ ⅓ = [0; 3].

III. We show the continued fraction representation of 7/11 .

1. b = 7/11 = 0.63636… and $i = 0$.
2. $a_0 = \lfloor b \rfloor = 0$.
3. $b = \frac{1}{(b-a_o)}$ = 1.57143… and $i = 1$.

4. $a_1 = \lfloor b \rfloor = 1$.
5. $b = \frac{1}{(b-a_1)} = 1.75\ldots$ and $i = 2$.
6. $a_2 = \lfloor b \rfloor = 1$.
7. $b = \frac{1}{(b-a_2)} = 1.33333\ldots$ and $i = 3$.
8. $a_3 = \lfloor b \rfloor = 1$.
9. $b = \frac{1}{(b-a_3)} = 3$ and $i = 4$
10. $a_4 = \lfloor b \rfloor = 3$

Since $b - a_4 = 0$; we stop

$\Rightarrow 7/11 = [0; 1, 1, 1, 3] = [0; 1, 1, 1, 2, 1]$

Now, $7/11 = [0; 1, 1, 1, 3] = [0; 1, 1, 1, 2, 1]$

4 Application of Results

In this section, we discuss the application of the results in controlling bank fraud. Usually, as the account is being balanced at the end of the month, the algorithm in the section above will run automatically. If the steps terminate, the bank staff A will be rotated to another assignment. If it does not, bank staff A will maintain the position until another month when the algorithm will run. Once the algorithm terminates at the end of the month, the staff is rotated. Suppose we have $r = \frac{137}{1600}$ it implies that 137 customers out of 1600 customers that visited a particular bank were attended to by a bank staff, A. The process terminates at step 14. This means for staff A to have attended to 137 customers that month out of total of 1600 customers that were attended to in the bank, staff A must be rotated. Also, for r = ⅓ the process terminates at step 4. For $r = \frac{7}{11}$ it stops at step 10. We found out that if we have $r = \frac{139}{1600}$, it terminates at stage 19, for $r = \frac{155}{1600}$ it terminates at stage 36.

5 Conclusion

a. The algorithm provided in this work will enable the management of the bank staff to rotate their staff.
b. The rotation of job cannot be influenced by the bank officials.
c. The bank staff cannot pre-determine when he will be rotated.
d. Even if the bank staff attends to few or many customers, this does not determine the rotation.

Acknowledgements

Continued fraction was introduced to the first author when she was a visiting researcher at African Institute for Mathematics Sciences, South Africa where she enjoyed numerous stimulating discussions with Professor Alan Beardon formerly at Cambridge University.

References

Abiola, Idowu(2009): An Assessment of Fraud and its Management in Nigeria Commercial Banks. European Journal of Social Sciences, Vol.10(4), 628-640.

Arthur T. Benjamin, F. E. Su, Quinn J. J. (2000): Counting on Continued Fractions, Maths Mag. Vol.73, pages 98-104.

Beardon A. F. (2001): Continued Factions, Discrete Groups and Complex Dynamics, Computational Methods and Function Theory, Vol. 1, No 2, 535-594

Beardon A. F. (1991): Iteration of Rational Fractions, Springer-Verlag.

Burn, R. P. (1982): A Pathway into Number Theory. Cambridge University press, New York , London.

Davison J. L. (1977): A series and its Associated Continued Fraction, Proc. Amer. Math. Soc. Vol. 63,pages 29-32.

Kenneth R. H. (1993): Elementary Number Theory and its applications. Published by Addison – Wesley publishing Company, United States of America

Nwankwo, G. O. (1991): Bank Management Principles and Practices, Malthouse Press Ltd. Lagos.

Okubena O. O. (1998): An investigation in frauds in banks, An Unpublished Thesis of University of Lagos.

Ogidefa Ivor (2008): Fraud in Banking Systems in Nigeria [Online] available via http://www Socyberty.com/law/fraud –in-banking-system-in-Nigeria.