An Approach for Optimum Upgrades of Information Systems

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Abstract

The rapid pace of advancement in technologies has created possibilities for firms to invest in superior technologies to achieve higher utility. In such a technological evolving environment, it would be optimum for a firm to execute its decisions about technology purchases and "upgrades" to minimize the total system ownership cost. Upgrading and maintaining the current technologies in a system can be a time-consuming process and may involve training employees. Information Systems (IS) upgrades are particularly of business significance because they contribute a major portion to the total cost of system ownership in the long run. This paper considers a firm that utilizes an Information System, consisting of software and hardware technologies, and both of these technologies are expected to have a series of breakthroughs in the future. The dynamic programming model of the paper provides a theoretical basis for the firm to decide on its optimal upgrade decisions. The model is analyzed by a simulation study, and the ideas to expand this work have been discussed in the conclusion.

Keywords: System upgrades, multiple technologies, technology adoption

1. Introduction

In today's rapidly evolving technological environment, firms are finding it tougher by the day to keep up with the latest technologies. For instance, collaborative email system, Microsoft Exchange Server (Wikipedia 2011), which is utilized by several medium and large size organizations, have seen seven major releases (upgrades) in just the last 14 years. This system allows a firm to perform important productivity related tasks such as emailing, calendaring, support for mobile and web-based access to information, and support for data storage. In this system, the hardware is the server which runs the software application (Microsoft Exchange Server).

In the context of Information Systems (ISs), such as the one discussed above, upgrade decisions are of great importance, as they typically contribute highly to the total cost of system ownership. According to the Gartner Group and Forrester Research, a typical Management Information System (MIS) department spends 50 percent of its time installing software and performing upgrades, which accounts for a significant portion of the total cost of IT ownership (Ferris, 2004). Upgrading and maintaining technologies in a system can be a time-consuming process and may involve training employees. Figure 1 shows the breakup of the cost associated with upgrading from Microsoft Exchange Server system version 5.5 to version 2003 (Ferris, 2004). The acquisition cost of the upgrade only constitutes 53% of the total cost. This constitutes 40% in the purchase of the new hardware and 13% in the purchase of the new software. Almost half (47%) of the total upgrade cost is related to the implementation. Labor is a part of the implementation cost, which includes learning of the new technology and the related training. Also, it is known that the cost of training of a new technology is lesser if the firm has some experience with its previous versions (Ferris, 2004).

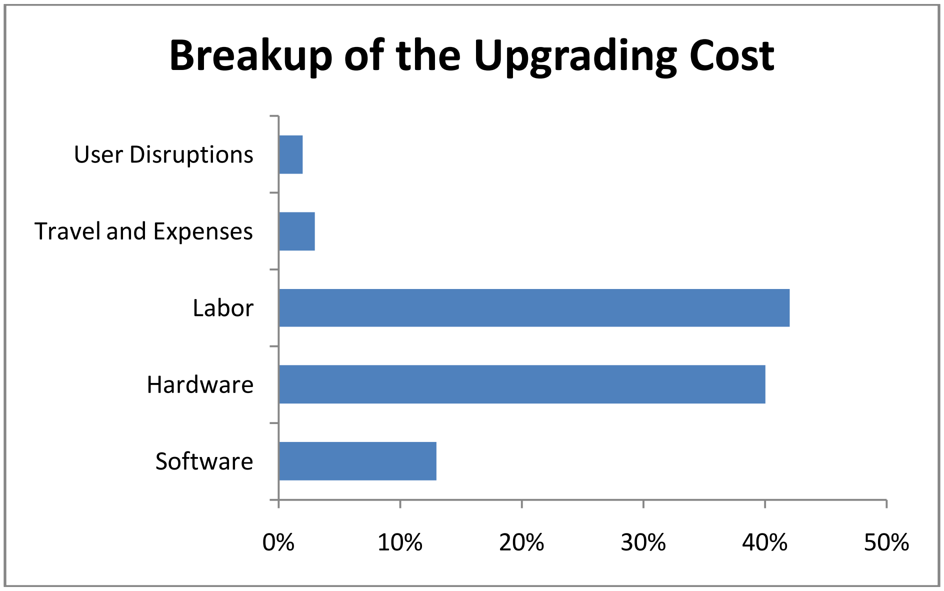


Figure 1: Exchange Upgrade Cost (Ferris, 2004).

Even if a system with only one technology is considered, as is the case in some of the papers of the relevant literature (Rajagopalan et al. 1998) a firm in a technologically evolving environment faces several challenges. Many of these challenges have been analyzed in the related literature and will be discussed in the next section. For example, a machine in manufacturing sector can be considered as a system with only one technology, because the machine typically can only be replaced (upgrade) as a whole. The issues discussed in this case include: whether to replace the older technology now or wait for a better and perhaps cheaper version in the future? If a new version hits the market, should the firm adopt the technology immediately or wait for the prices to drop? Is it always optimal to upgrade the older technology to its latest version? All of these questions stem from a more fundamental question; that is, how can a firm ensure optimal decisions, as far as running a system is concerned?

A system with multiple technologies (hardware and software, for instance) has not been considered in this setting. Some of the factors that would be relevant in this setting include the following:

(1) Each combination of technologies in the system would satisfy a set of functions and would provide a level of utility (productivity/efficiency). A newer technology is more "productive" than the older ones.

(2) Some combinations of technologies may be incompatible. This is likely if there is "substantial" difference between the versions of the technologies of the system. Technologies which are "generations apart" may not be compatible together in a system. So, a particular software upgrade may require a minimum version of hardware in the system.

(3) Replacing an older technology with a newer one also has a variety of costs. First, there is the cost in purchasing the new technology. In this paper this cost is called the "acquisition cost." The acquisition cost of a technology may also decline in time, as has been the observed trend in both software and hardware technologies in the recent past. Second, there is the cost related to the implementation of the new technology. This includes cost related to learning and implementation. In this paper this cost is called the "implementation cost." This cost is of a particular interest in the present setting as synchronizing the implementation of the technologies can potentially be a cost saving decision. Third, upgrading a technology is also likely to disrupt the system performance for a period of time. This cost is called the "disruption cost" in this paper. This cost can also be minimized by synchronizing the upgrade decisions of both of the technologies.

(4) (Chand et al. 1993) considers a model of single machine replacement problem in learning and training environment, and shows that machines are replaced more frequently in the presence of higher learning cost. However, in other situations, especially where training and labor costs are not substantial, the firm is likely to skip some generations of technologies in the upgrade decisions. In situations where the cost of training of technology is low and the cost of disruption is high, to avoid expensive disruptions, the optimum upgrade decisions are likely to have less number of upgrades.

(5) Finally, a technology may also have a salvage value. As and when the older technologies are replaced by the newer versions, the old versions may still fetch some salvage value in the market. This value structure allows the model of the paper to be applicable in evaluating buying and leasing options.

This paper presents a dynamic programming model that can be utilized to analyze upgrade decision of a firm, which uses a system that has two technologies. Therefore, differences between the technologies can also be considered while evaluating the optimum "upgrade" decisions. The rest of this paper is organized as follows. In the next section, the relevant literature is reviewed. Section 3 contains the definition and the mathematical formulation of the problem. In section 4, the dynamic programming model is implemented in some numeric examples. In section 5, the conclusion of the research and some future directions are discussed.

2. Review of the Relevant Literature

This work is closely related to the research in the areas of machine replacement, and capacity expansion of Operations Management. Early machine replacement models consider the optimal timing of replacement of a machine with a new machine of the same or better technology (Pierskalla and Voelker. 1976;Chand and Sethi. 1982). considers the issue of replacement of machines where the firm knows the timing of the origination of the new technologies with certainty. (Jones et al. 1991) models a deterministic machine replacement problem, where both fixed acquisition and variable operating costs are considered. (Cohen and Halperin. 1986) considers the problem where a known choices of technologies of a machine are available over time, but their output capacities can be technologically improved by a fixed investment. Some other papers consider the stochastic aspects of the machine replacement problem. (Goldstein et al. 1988) models a single machine replacement problem with one anticipated technological breakthrough in the future. (Nair and Hopp, 1992).discusses a version of the problem where several technological breakthroughs are anticipated. (Balser and Lippman, 1984) models the problem that considers both the uncertainty in the timing and the magnitude of the breakthroughs.

In all these machine replacement problems the firm has only two options: either to replace all the existing capacity or replace none of it. A more general problem that considers incremental acquisition of capacity; and thus, can also utilize the scale of economy in the analysis, is known as capacity expansion problem. (Monahan, 1989).considers one such problem where a labor-intensive process can be transformed into a better automated process in steps. There are also many papers, which consider the capacity expansion problem in isolation (Freidenfields, 1981;Luss, 1982). (Rajagopalan et al. 1998) provides a model which considers detailed capacities in technology acquisition and replacement problems in a technologically evolving environment. Another stream of research considers the software upgrade release problem of the software industry. This work focuses on the economic aspects of the timings of the software upgrade decisions (Swan, 1970a;Swan, 1970b). (Wilhelm and Xu, 2002) focuses on the issue of the timing of the content upgrade decisions in the domain of high-tech products.

The abovementioned papers consider a system of only one technology. This is the first research where the issue of optimally upgrading of two independent technologies in a system is considered. This makes the analysis applicable to the issues that relate to the interactions of the technologies in the system. The model of this article is in a way an extension of the model considered by (Nair, 1995). The model theoretically relates to the problem of Dynamic Assignment problem of the Operations Research. The problem deals with optimally assigning resources to tasks over time. The Dynamic Assignment Problem is also a fundamental problem in routing and scheduling (Spivey and Powell, 2004). The problem considered in this paper can also be viewed as variation of a more general dynamic assignment model, where one technology is being assigned to another set of technologies over time to produce the optimal system in long run.

3. Problem Definition and the Model

The two technologies of the system are denoted by **S** and **H**. In the problem horizon, n technological breakthroughs in **S** and m breakthroughs in **H** are expected. These technological breakthroughs are represented in the set notation as:

$\mathbf{H} = \{0,1,\ldots, n+1\}$

$\mathbf{S} = \{0,1,\ldots,m+1\}$

The index 0 represents the current version both the technologies of the system and 1 is a new version that is already available in the market.

There are n generations of technologies of type **H**, the new technologies are indexed by: 2,…,n+1. Similarly, there are m technologies of type **S**, and the new technologies are indexed by: 2, …,m + 1. For analysis purposes, the problem horizon is broken down into discrete time periods. In a period, k and l are the index of the latest available technology of type **S** and H respectively. The firm's current technological state of the system is represented by (i; j: k; l).

The actions available to the firm in the state (i; j :k; l) is either to "keep the system unchanged," or to "upgrade the system" to technologies indexed up to s and h. The first decision is represented by K_{ij} and the second by K_{ij}^{sh}. Note that by definition $s \leq k$ and $h \leq l$. p_{t+1}^{k+1} and q_{t+1}^{l+1} represent the probabilities of appearance of the next generation technology of type **S** and **H** respectively in time period t + 1. Also, (1 - p_{t+1}^{k+1}) and (1 - q_{t+1}^{l+1}) represents probabilities that new technologies will not appear in the next period. These probabilities can be estimated using technological forecasting methods such as Delphi method as utilized by (Cho et al. 1991). Table 1 presents the transition probabilities of all the four possible events possible in a time period:

Events	Transition probabilities
Only new generation of S appears in period t + 1	$p_{t+1}^{1} = p_{t+1}^{k+1} \times (1 - q_{t+1}^{l+1})$
Only new generation of H appears in period t + 1	$p_{t+1}^{2} = (1 - p_{t+1}^{k+1}) \times q_{t+1}^{l+1}$
New generations of both S and H appears in period t + 1	$p_{t+1}^{3} = p_{t+1}^{k+1} \times q_{t+1}^{l+1}$
No new technology appears in period t + 1	$p_{t+1}^{4} = (1 - p_{t+1}^{k+1}) \times (1 - q_{t+1}^{l+1})$

Table 1: Transition Probability Matrix

Note that $p_{t+1}^{1} + p_{t+1}^{2} + p_{t+1}^{3} + p_{t+1}^{4} = 1 \quad \forall t$.

3.1 Cost and Benefit Related Parameters Associated with an Upgrade Decision

In time, the firm would expect to have several opportunities to upgrade the technologies of its system. Let the cost of purchase of i level of technologies of type **S** and **H** be represented by x_i^t and y_i^t, respectively. Similarly, the implementation costs of these technologies are represented by $C_1(i,s)$ and $C_2(j,h)$. Table 3 below shows a sample cost of implementation of three versions of **S** and **H** technologies.

Initial Technology	S			H		
	Upgraded Technology Level			Upgraded Technology Level		
	0	1	2	0	1	2
0	0	100	150	0	100	250
1	-	0	100	-	0	100
2	-	-	0	-	-	0

Table 3: Technology Implementation Cost

As the firm typically have fixed resources (labor etc.) for technology implementation, it would be efficient to upgrade both the technologies in a synchronized (simultaneously) manner. D represents the discount percentage that the firm receives if both technologies are upgraded simultaneously. $C((i,j),(s,h))$ represents the total cost of system implementation, where **S** and **H** technologies are upgraded from the version (i, j) to the version (s, h). This cost is calculated using the following expression:

$$C((i,j),(s,h)) = \begin{cases} D \times (C_1(i,s) + C_2(j,h)), & \text{if the technologies are upgraded simultaneously} \\ C_1(i,s) + C_2(j,h), & \text{elsewhere} \end{cases}$$

Table 4 shows the total implementation costs of the system in D = 50% scenario.

	Upgraded System								
	(0,0)	**(0,1)**	**(1,0)**	**(1,1)**	**(0,2)**	**(2,0)**	**(1,2)**	**(2,1)**	**(2,2)**
(0,0)	0.0	100.0	100.0	100.0	250.0	150.0	175.0	125.0	200.0
(0,1)	-	0.0	124.25	100.0	100.0	149.25	100.0	150.0	125.0
(1,0)	-	-	0.0	100.0	-	100.0	250.0	100.0	175.0
(1,1)	-	-	-	0.0	-	-	100.0	100.0	100.0
(0,2)	-	-	-	-	0.0	-	100	-	150
(2,0)	-	-	-	-	-	0.0	-	100.0	250.0
(1,2)	-	-	-	-	-	-	0.0	-	100.0
(2,1)	-	-	-	-	-	-	-	0.0	100.0
(2,2)	-	-	-	-	-	-	-	-	0.0

Table 4: Cost of Implementation

r_{sh}^{t} represents one time revenue that the firm receives in time period t if it decides to have a system with technology combination of (s; h). β_t. is a discount factor that is used to transform the future revenues to their present values. By definition the discount factors are less than 1 in all future time periods. As discussed earlier, not all available versions of technologies may be compatible. Therefore, a "compatibility matrix," which summarizes the combinations of technologies that is feasible in the model, is used in the model. This matrix is also used to efficiently run the model, as infeasible options may not need to be explored. Table 5, shows the compatibility matrix for the running example of the paper.

Technology H	Technology S		
	0	**1**	**2**
0	1	1	1
1	1	1	1
3	0	0	1

Table 5: Compatibility Matrix

The profit maximization problem of the firm over the planning horizon T is formulated as a non-stationary Markov decision process. $f_t^T(i, j:k,l)$ returns the maximum revenue value from time period t to the end of planning horizon. The optimum decision at the time period t is given by $\pi_t^T(i, j:k,l)$. As $T \to \infty$, these functions are represented by $f_t\ (i, j:k,l)$ and $\pi_t\ (i, j:k,l)$, respectively.

3.2 Model Formulation

The following notation is used for brevity:

$(a,b) > (y,z)$ if $a \geq y, b \geq z$ and $a+b > y+z$, which means both a and b are greater than or equal to y and z respectively, and at least one of them is strictly greater. Similarly, $(a,b) \gg (y,z)$ if $a \geq y$ and $b \geq z$.

$$g(x,y) = \begin{cases} 0, & \text{if } y \leq x \\ 1, & \text{if } y > x \end{cases}$$

$T((i,j),(s,h))$: represents the total cost of upgrading the system from the state (i,j) to the state (s,h). The function can be calculated as:

$$T((i,j),(s,h)) = x_s^t g(s,i) + y_h^t(h,j) + C((i,j),(s,h))$$

The technology combinations (s,h) that can be chosen are defined by: $(i,j) < (s,h) \ll (k,l)$

3.3 Assumptions

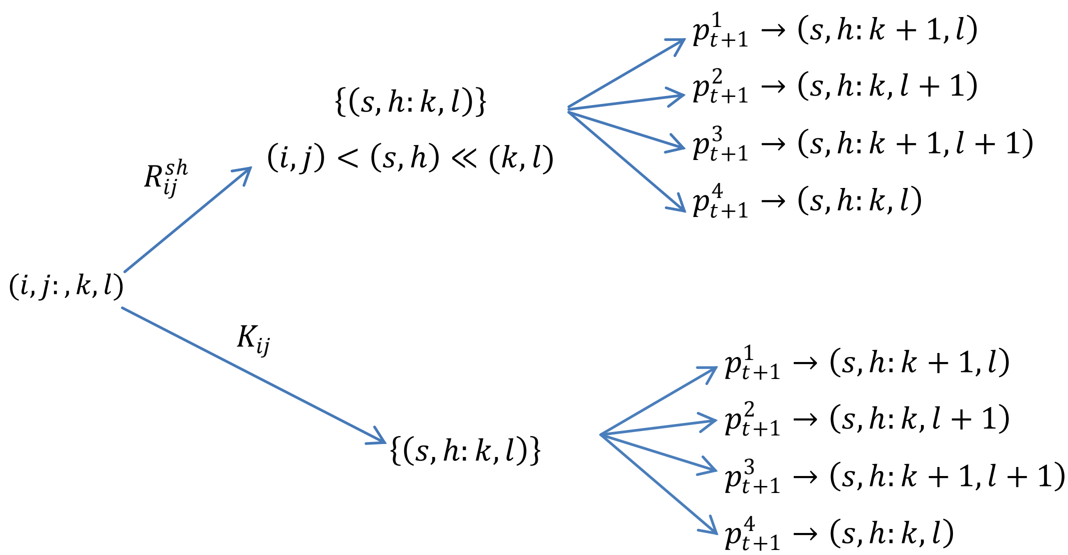
For efficiency purposes, the model is built around the following two assumptions:

1. REVENUE FUNCTION ASSUMPTION: $r_{sh}^t \geq r_{ij}^t$ when $(s,h) > (i,j)\ \forall t$. Any system with better technology generates greater revenue.

2. INCOMPATIBILITY ASSUMPTION: $\pi_t^T(i,j:k,l) \neq R_{ij}^{ab}$ when $(a-i)\times(b-j) \leq 0 \forall t$.

It would never be optimal to degrade one of the technologies while upgrading the other. This assumption is also expected to be true, because otherwise the upgrade would likely to create a system with incompatible technologies.

3.4 The Model and its Pictorial Representation



State at *t* Actions available at *t* Transition Probabilities State at *t*+1

Figure 2: The Model

The objective function value is defined below:

$$f_t^T(0,0:1,1) = Max \begin{Bmatrix} R_{00}^{11}: \begin{Bmatrix} -x_1^t - y_1^t - C\big((0,0),(1,1)\big) \\ +r_{11}^t + \beta_t[(p_{t+1}^1) f_{t+1}^T(1,1:2,1) \\ +(p_{t+1}^2) f_{t+1}^T(1,1:1,2) \\ +(p_{t+1}^3) f_{t+1}^T(1,1:2,2) \\ +(p_{t+1}^4) f_{t+1}^T(1,1:1,1)] \end{Bmatrix} \\ R_{00}^{01}: \begin{Bmatrix} -y_1^t - C\big((0,0),(0,1)\big) \\ +r_{01}^t + \beta_t[(p_{t+1}^1) f_{t+1}^T(0,1:2,1) \\ +(p_{t+1}^2) f_{t+1}^T(0,1:1,2) \\ +(p_{t+1}^3) f_{t+1}^T(0,1:2,2) \\ +(p_{t+1}^4) f_{t+1}^T(0,1:1,1)] \end{Bmatrix} \\ R_{00}^{10}: \begin{Bmatrix} -x_1^t - C\big((0,0),(0,1)\big) \\ +r_{10}^t + \beta_t[(p_{t+1}^1) f_{t+1}^T(1,0:2,1) \\ +(p_{t+1}^2) f_{t+1}^T(1,0:1,2) \\ +(p_{t+1}^3) f_{t+1}^T(1,0:2,2) \\ +(p_{t+1}^4) f_{t+1}^T(1,0:1,1)] \end{Bmatrix} \\ K_{00}: \begin{Bmatrix} -x_1^t - C\big((0,0),(0,1)\big) \\ r_{00}^t + \beta_t[(p_{t+1}^1) f_{t+1}^T(0,0:2,1) \\ +(p_{t+1}^2) f_{t+1}^T(0,0:1,2) \\ +(p_{t+1}^3) f_{t+1}^T(0,0:2,2) \\ +(p_{t+1}^4) f_{t+1}^T(0,0:1,1)] \end{Bmatrix} \end{Bmatrix}$$

The first row corresponds to the decision of replacing level zero technologies of both **H** and **S** typeswith level one technologies. This costs $-y_1^t - x_1^t - C\big((0,0),(1,1)\big)$ (including both acquisition and implementation costs). This also generates discounted revenue of r_{11}^t in the current period and the expected future revenue from the period $t+1$ to the end of the problem horizon. For the sake of simplicity, the salvage values of the technologies are not considered in the model explicitly. These cost, can, however, be considered as part of the "implementation cost."

Lemma 1: If $f_T^T(a,b:k,l) \geq f_T^T(i,j:k,l)$ where $(a,b) > (i,j)$then

(a) The optimal actions in state $(i,j:k,l)$ are

$$\pi_t^T(i,j:k,l) = \begin{Bmatrix} R_{ij}^{ab}: \text{upgrade } (i,j) \text{ to } (a,b), (i,j) < (a,b) \ll (k,l) \\ K_{ij}: \text{keep } (i,j). \end{Bmatrix}$$

(b) If $\pi_t^T(a,b:k,l) = R_{ab}^{sh} \Longrightarrow \pi_t^T(i,j:k,l) = R_{ij}^{sh} \;\; \forall (i,j) < (a,b) < (s,h) \ll (k,l)$
PROOF: To prove for $(k,l) \gg (s,h) > (i,j) > (p,q)$ is

$$f_t^T(i,j,:k,l) = Max\left\{\begin{matrix} R_{ij}^{sh}: \begin{Bmatrix} -T\big((i,j),(s,h)\big) \\ +r_{sh}^t + \beta_t[(p_{t+1}^1)f_{t+1}^T(s,h:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(s,h:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(s,h:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(s,h:k,l)] \end{Bmatrix} \\ K_{ij}: \begin{Bmatrix} r_{ij}^t \\ +\beta_t[(p_{t+1}^1)f_{t+1}^T(i,j:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(i,j:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(i,j:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(i,j:k,l)] \end{Bmatrix} \\ R_{ij}^{sh}: \begin{Bmatrix} -T\big((i,j),(p,q)\big) \\ +r_{pq}^t + \beta_t[(p_{t+1}^1)f_{t+1}^T(p,q:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(p,q:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(p,q:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(p,q:k,l)] \end{Bmatrix} \end{matrix}\right\} \quad (1)$$

For$(k,l) \gg (s,h) > (i,j) > (p,q)$, at $t = T-1$, we have,

$$f_{T-1}^T(i,j,:k,l) = Max\left\{\begin{matrix} R_{ij}^{sh}: \begin{Bmatrix} -T\big((i,j),(s,h)\big) \\ +r_{sh}^{T-1} + \beta_{T-1}[(p_T^1)f_T^T(s,h:k+1,l) \\ +(p_T^2)f_T^T(s,h:k,l+1) \\ +(p_T^3)f_T^T(s,h:k+1,l+1) \\ +(p_T^4)f_T^T(s,h:k,l)] \end{Bmatrix} \\ K_{ij}: \begin{Bmatrix} r_{ij}^{T-1} \\ +\beta_{T-1}[(p_T^1)f_T^T(i,j:k+1,l) \\ +(p_T^2)f_T^T(i,j:k,l+1) \\ +(p_T^3)f_T^T(i,j:k+1,l+1) \\ +(p_T^4)f_T^T(i,j:k,l)] \end{Bmatrix} \\ R_{ij}^{sh}: \begin{Bmatrix} -T\big((i,j),(p,q)\big) \\ +r_{pq}^t + \beta_{T-1}[(p_T^1)f_T^T(p,q:k+1,l) \\ +(p_T^2)f_T^T(p,q:k,l+1) \\ +(p_T^3)f_T^T(p,q:k+1,l+1) \\ +(p_T^4)f_T^T(p,q:k,l)] \end{Bmatrix} \end{matrix}\right\} \quad (2)$$

As $f_T^T(i,j:k,l) \geq f_T^T(p,q:k,l)$ according to the lemma, and $r_{ij}^t \geq r_{pq}^t, (i,j) > (p,q) \ \forall t$,

$$f_{T-1}^T(i,j:k,l) = Max\left\{\begin{array}{c} R_{ij}^{sh}:\left\{\begin{array}{c} -T\big((i,j),(s,h)\big) \\ +r_{sh}^{T-1} + \beta_{T-1}[(p_T^1)f_T^T(s,h:k+1,l) \\ +(p_T^2)f_T^T(s,h:k,l+1) \\ +(p_T^3)f_T^T(s,h:k+1,l+1) \\ +(p_T^4)f_T^T(s,h:k,l)] \end{array}\right\} \\ K_{ij}:\left\{\begin{array}{c} r_{ij}^{T-1} \\ +\beta_{T-1}[(p_T^1)f_T^T(i,j:k+1,l) \\ +(p_T^2)f_T^T(i,j:k,l+1) \\ +(p_T^3)f_T^T(i,j:k+1,l+1) \\ +(p_T^4)f_T^T(i,j:k,l)] \end{array}\right\} \end{array}\right\} \quad (3)$$

Hence, Lemma 1(a) is true at $T-1$. For$(k,l) \gg (s,h) > (i,j) > (p,q)$, the following will also be true:

$$f_{T-1}^T(a,b:k,l) = Max\left\{\begin{array}{c} R_{ab}^{sh}:\left\{\begin{array}{c} -T\big((a,b),(s,h)\big) \\ +r_{sh}^{T-1} + \beta_{T-1}[(p_T^1)f_T^T(s,h:k+1,l) \\ +(p_T^2)f_T^T(s,h:k,l+1) \\ +(p_T^3)f_T^T(s,h:k+1,l+1) \\ +(p_T^4)f_T^T(s,h:k,l)] \end{array}\right\} \\ K_{ab}:\left\{\begin{array}{c} r_{ab}^{T-1} \\ +\beta_{T-1}[(p_T^1)f_T^T(a,b:k+1,l) \\ +(p_T^2)f_T^T(a,b:k,l+1) \\ +(p_T^3)f_T^T(a,b:k+1,l+1) \\ +(p_T^4)f_T^T(a,b:k,l)] \end{array}\right\} \end{array}\right\} \quad (4)$$

From the equation (3), (4), and the revenue assumption, we conclude that at $t = T-1, f_{T-1}^T(a,b:k,l) \geq f_{T-1}^T(i,j:k,l)$. Suppose Lemma 1(a) was true at $t+1$ and $f_{t+1}^T(a,b:k,l) \geq f_{t+1}^T(i,j:k,l)$.Hence $f_t^T(a,b:k,l) \geq f_t^T(i,j:k,l)$. The result follows by induction.

To prove Lemma 1(b), we use the above result to obtain

$$f_t^T(a,b:k,l) = Max\left\{\begin{array}{c} R_{ab}^{sh}:\left\{\begin{array}{c} -T\big((a,b),(s,h)\big) \\ +r_{sh}^{t} + \beta_{t}[(p_{t+1}^1)f_{t+1}^T(s,h:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(s,h:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(s,h:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(s,h:k,l)], (i,j) < (s,h) \ll (a,b) \end{array}\right\} \\ K_{ab}:\left\{\begin{array}{c} r_{ab}^{t} \\ +\beta_{t}[(p_{t+1}^1)f_{t+1}^T(a,b:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(a,b:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(a,b:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(a,b:k,l)] \end{array}\right\} \end{array}\right\} \quad (5)$$

And

$$f_t^T(i,j:k,l) = Max\begin{cases} R_{ij}^{sh}:\begin{Bmatrix} -T\big((i,j),(s,h)\big) \\ +r_{sh}^t + \beta_t[(p_{t+1}^1)f_{t+1}^T(s,h:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(s,h:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(s,h:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(s,h:k,l)],(i,j)<(s,h)\ll(k,l) \end{Bmatrix} \\ K_{ij}:\begin{Bmatrix} r_{ij}^t \\ +\beta_t[(p_{t+1}^1)f_{t+1}^T(i,j:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(i,j:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(i,j:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(i,j:k,l)] \end{Bmatrix} \end{cases} \quad (6)$$

Hence, $f_t^T(a,b:k,l) \geq f_t^T(i,j:k,l)$ when $(a,b) \gg (i,j)$, then Lemma 1(b) is true at $t = T-1$. Therefore, by induction Lemma 1(b) is also true at t. ∎

Using Lemma 1, we can define the recursive dynamic programming expression for all the states $(i,j) < (s,h) \ll (k,l)$,

$$f_t^T(i,j:k,l) = Max\begin{cases} R_{ij}^{sh}:\begin{Bmatrix} -T\big((i,j),(s,h)\big) \\ +r_{sh}^t + \beta_t[(p_{t+1}^1)f_{t+1}^T(s,h:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(s,h:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(s,h:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(s,h:k,l) \end{Bmatrix} \\ K_{ij}:\begin{Bmatrix} r_{ij}^t \\ +\beta_t[(p_{t+1}^1)f_{t+1}^T(i,j:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(i,j:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(i,j:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(i,j:k,l)] \end{Bmatrix} \end{cases} \quad (7)$$

Lemma 2. If $f_T^T(a,b:k,l) \geq f_T^T(i,j:k,l)$ where $(a,b) > (i,j)$ then

(a) For any time period t for which, $\exists\ (s,h)$ such that $(r_{sh}^t - r_{ij}^t - T\big((i,j),(s,h)\big) > 0$, where current state is given by $(i,j:k,l)$. The only optimal actions are given by:
$\pi_t^T(i,j:k,l) = R_{ij}^{sh}$. That is, upgrade (i,j) to $(s,h), (i,j) < (s,h) \ll (k,l)$.

(b) If $(r_{kl}^t - r_{ij}^t - T\big((i,j),(k,l)\big) > 0$ and the maximum value of the function $r_{sh}^t - T\big((i,j),(s,h)\big)$ exists at (k,l) then only the optimal actions in state $(i,j:k,l)$ is given by:

$\pi_t^T(a,b:k,l) = R_{ab}^{kl}$. That is, it is optimal to upgrade both the technologies to the highest available technologies.

PROOF: From Lemma 1, we have the following DP formulation:

$$f_t^T(i,j:k,l) = Max\begin{cases} R_{ij}^{sh}:\begin{Bmatrix} -T\big((i,j),(s,h)\big) \\ +r_{sh}^t + \beta_t[(p_{t+1}^1)f_{t+1}^T(s,h:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(s,h:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(s,h:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(s,h:k,l) \end{Bmatrix} \\ K_{ij}:\begin{Bmatrix} r_{ij}^t \\ +\beta_t[(p_{t+1}^1)f_{t+1}^T(i,j:k+1,l) \\ +(p_{t+1}^2)f_{t+1}^T(i,j:k,l+1) \\ +(p_{t+1}^3)f_{t+1}^T(i,j:k+1,l+1) \\ +(p_{t+1}^4)f_{t+1}^T(i,j:k,l)] \end{Bmatrix} \end{cases} \quad (7)$$

For $(i,j) < (s,h) \ll (k,l)$, in order to have only R_{ij}^{sh} as the optimal solution there must exist at least one (s,h) pair for which the objective function value for the decision R_{sh}^{sh} is more than the function value from the decision K_{ij} .

$$\text{Equivalently,}\left\{\begin{matrix} -T\big((i,j),(s,h)\big) + r_{sh}^{t} - r_{ij}^{t} + \beta_t[(p_{t+1}^{1})(f_{t+1}^{T}(s,h{:}\,k+1,l) - f_{t+1}^{T}(i,j{:}\,k+1,l)) \\ +(p_{t+1}^{2})(f_{t+1}^{T}(s,h{:}\,k,l+1) - f_{t+1}^{T}(i,j{:}\,k+1,l)) \\ +(p_{t+1}^{3})(f_{t+1}^{T}(s,h{:}\,k+1,l+1) - f_{t+1}^{T}(i,j{:}\,k+1,l)) \\ +(p_{t+1}^{4})(f_{t+1}^{T}(s,h{:}\,k,l) - f_{t+1}^{T}(i,j{:}\,k+1,l))] \end{matrix}\right\} > 0$$

$f_t^T(s,h{:}\,i,j) \geq f_t^T(a,b,i,j)$ if $(a,b) \ll (s,h)$ from Lemma 1. Using this we can conclude that the value inside the square bracket in the above expression is greater than or equal to zero. Hence required condition for which lemma 2(a) would hold is given by $-T\big((i,j),(s,h)\big) + r_{sh}^{t} - r_{ij}^{t} > 0$.

We also observe that the function inside the bracket would be maximized at (k,l) because the function is non-decreasing in technologies. If the remaining part of the R_{ij}^{sh} expression, i. e. $-T\big((i,j),(s,h)\big) + r_{sh}^{t}$ attains maximum value at (k,l) then it would be optimum to choose (k,l). Hence, lemma 2(b) is proved. ■

4. *Numerical Examples*

In order to derive practical insights from the abovementioned model, this section considers a variety of numeric examples. These examples will be utilized to explore the impact of different parameters of the model on the system upgrade decision. For simplicity, examples with only 10 time periods are considered. These include four scenarios, which are based on different transition probabilities of the technologies, which are given in Table 5. The first two scenarios capture a situation where both technologies have the same expected pattern in their technological evolution. In scenarios p1, the probabilities are chosen in such a way that the higher probabilities of technical breakthroughs are in the initial portion of the problem horizon. In p2, the higher probabilities of technical breakthroughs are in the later part of the problem horizon. The last two scenarios are designed using the probabilities from the previous two scenarios to capture opposite trends in the technologies.

Time Period	**Scenarios**							
	P1		**P2**		**P3**		**P4**	
	p_t^h	q_t^s	p_t^h	q_t^s	p_t^h	q_t^s	p_t^h	q_t^s
***t*=1**	0:93	0:93	0:22	0:22	0:22	0:93	0:93	0.22
***t*=2**	0.94	0.94	0.32	0.32	0.32	0.94	0.94	0.32
***t*=3**	0.95	0.95	0.42	0.42	0.42	0.42	0.95	0.42
***t*=4**	0.96	0.96	0.52	0.52	0.52	0.96	0.96	0.52
***t*=5**	0.97	0.97	0.62	0.62	0.62	0.97	0.97	0.62
***t*=6**	0.98	0.98	0.72	0.72	0.72	0.98	0.98	0.72
***t*=7**	0.99	0.99	0.82	0.82	0.82	0.99	0.99	0.82
***t*=8**	1	1	0.92	0.92	0.92	1	1	0.92
***t*=9**	1	1	1	1	1	1	1	1

Table 5: Transition Probabilities

Two implementation cost scenarios are considered below. In the first scenario (I1) it is relatively cheaper to upgrade in less number of steps. It captures situations where disruption is the dominant part of the implementation cost. The second scenario (I2) considers the situations where it is less costly to do upgrades in steps. That is, learning is the dominant factor in the cost of implementation. The table below provides the parameter values to capture these scenarios.

Initial Technology	Scenarios					
	I1			I2		
	Upgraded System			Upgraded System		
	0	1	2	0	1	2
0	0	100	150	0	100	250
1	-	0	100	-	0	100
2	-	-	0	-	-	0

Table 6: Implementation Costs Scenarios

Three scenarios where acquisition costs are kept constant for both the technologies, in all the periods, are provided in Table 7. In the first scenario (C1) both the technologies have the same acquisition cost. In the last two scenarios, S and H technologies are respectively made cheaper by one third of their acquisition costs in C1 scenario.

Time	Scenarios											
	C1				C2				C3			
	C_{h1}^t	C_{s1}^t	C_{h2}^t	C_{s2}^t	C_{h1}^t	C_{s1}^t	C_{h2}^t	C_{s2}^t	C_{h1}^t	C_{s1}^t	C_{h2}^t	C_{s2}^t
***t*=0**	42	42	45	45	42	14	45	15	14	42	15	45

Table 7: Constant Acquisition Costs Scenarios

Declining Acquisition Costs

The acquisition costs may decline with time. This is captured by the following four scenarios: v4, v5, v6 and v7. In scenarios v4 and v5, the cost of acquisition decreases by 50 and 40 percentage in every two periods, respectively. The last two scenarios have been constructed using rate of decrease of 20% and 10%, respectively. In all the scenarios the costs of acquisition of both the technologies are the same in all the periods. These costs are given in Table 8.

Time	Scenarios							
	V4		V5		V6		V7	
	C_{h1}^t,C_{s1}^t	C_{h2}^t,C_{s2}^t	C_{h1}^t,C_{s1}^t	C_{h2}^t,C_{s2}^t	C_{h1}^t,C_{s1}^t	C_{h2}^t,C_{s2}^t	C_{h1}^t,C_{s1}^t	C_{h2}^t,C_{s2}^t
***t*=0**	42	-	42	-	42	-	42	-
***t*=1**	42	-	42	-	42	-	42	-
***t*=2**	21	45	25.2	45	33.6	45	37.8	45
***t*=3**	21	45	25.2	45	33.6	45	37.8	45
***t*=4**	10.5	22.5	15.12	27	26.88	36	34.02	40.5
***t*=5**	10.5	22.5	15.12	27	26.88	36	34.02	40.5
***t*=6**	5.25	11.25	9.07	16.2	21.5	28.8	30.61	32.8
***t*=7**	5.25	11.25	9.07	16.2	21.5	28.8	30.61	32.8
***t*=8**	2.62	5.62	5.44	9.72	17.2	23.04	27.55	29.52
***t*=9**	2.62	5.62	5.44	9.72	17.2	23.04	27.55	29.52

Table 8: Declining Acquisition Costs Scenarios

Reward Matrix

For simplicity one time reward function (r_{sh}^t) is kept constant with time. The reward values in different versions of the technologies are given in Table 9.

Technology H	Technology S		
	0	**1**	**2**
0	20	24	64
1	24	60	70
2	64	70	80

Table 9: Reward Matrix

Different combinations of the transition probabilities, the implementation costs, and the acquisition costs given above generate a total of 64 scenarios. For each scenario, the model is run for 9 different values of R (reward multipliers) and 8 different values of D (discount percentage). The reward multipliers are used to generate 9 different reward matrixes. The reward multiplier is a value multiplied to each element of the reward matrix to create the reward matrixes. This allows the analysis to capture how upgrade decisions change with the potential gain from the adoption of new technologies. The reward multiplier is varied from 1 to 5 with increments of 0.5. Discount factor (D) is the percentage discount that the firm would get if it upgrades both of the technologies simultaneously. The discount factor is changed from 10% to 45% with an increment of 0.05. This is utilized to analyze the impact of firm's resources on its upgrade decision. Here high discount percentage would mean limited resources for implementation. Also, note that if the firm upgrades both the technologies simultaneously, it can use some shared resources for upgrading both the technologies.

4.1 Results

The model is implemented in the scenarios mentioned above. The algorithm for the dynamic program model was coded in the java programming language. The analysis is mostly concerned about analyzing the first period upgrade decision with the parameter of the model. For other periods the firm can estimate the parameters (probabilities, costs and reward) again and solve the model. Here the digit 0 corresponds to the decision of keeping the current technology, and 1 represents the decision to upgrade. In Table 10, the upgrade decisions are summarized across R in all the scenarios. Different scenarios are named using letters, which are used to denote its subscenarios. For example, P1I1C1 column name corresponds to the scenario where probabilities are chosen from the probability scenario P1, the implementation costs from the scenario I1 and the acquisition costs from the scenario C1. In Table 11, the output of the model is shown for all D values in different scenarios. All the 0 values are shaded in both the output tables.

R	p1I1C1		p1I1C2		p1I1C3		p1I1C4		p1I1C5		p1I1C6		p1I1C7		p3I1C1		p3I1C2		p3I1C3		P3I1C4		p3I1C5		p3I1C6		p3I1C7	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.5	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

R	p2I1C1		p2I1C2		p2I1C3		p2I1C4		p2I1C5		p2I1C6		p2I1C7		p4I1C1		p4I1C2		p4I1C3		p4I1C4		p4I1C5		p4I1C6		p4I1C7	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

R	p1I2C1		P1I2C2		p1I2C3		p1I2C4		p1I2C5		p1I2C6		p1I2C7		p3I2C1		P3I2C2		p3I2C3		p3I2C4		p3I2C5		p3I2C6		p3I2C7	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.5	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

R	p2I2C1		p2I2C2		p2I2C3		p2I2C4		p2I2C5		p2I2C6		p2I2C7		p4I2C1		p4I2C2		p4I2C3		p4I2C4		p4I2C5		p4I2C6		p4I2C7	
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.5	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 10: Reward Output Table

We notice that there are more zeros in the upper half of Table 10. That is, the cases where implementation cost is “dominated by disruption", it would be more common to, not upgrade existing technologies. Also as R increases, it would be more likely to upgrade, since reward would dominate cost of upgrade. The reverse is true in Table 11, where an increase in D would force the firm to not upgrade, since firm would be better-off by just waiting for newer technologies. An increase in the probabilities of breakthroughs would support the decision to “not upgrade", as in that case firm would likely to “skip” directly to the future versions of the technologies in order to minimize the implementation cost. Also we can see from the tables that it would be more likely when technologies have higher cost of implementation and disruption (scenarios I1, for instance). Lastly, high acquisition cost of the technology would also correlate with firm’s “not upgrade" decision.

D	p1I1C1		p1I1C2		p1I1C3		p1I1C4		p1I1C5		p1I1C6		p1I1C7		p3I1C1		p3I1C2		p3I1C3		p3I1C4		p3I1C5		p3I1C6		p3I1C7	
10%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40%	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D	p2I1C1		p2I1C2		p2I1C3		p2I1C4		p2I1C5		p2I1C6		p2I1C7		p4I1C1		p4I1C2		p4I1C3		p4I1C4		p4I1C5		p4I1C6		p4I1C7	
10%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D	p1I2C1		P1I2C2		p1I2C3		p1I2C4		p1I2C5		p1I2C6		p1I2C7		p3I2C1		P3I2C2		p3I2C3		p3I2C4		p3I2C5		p3I2C6		p3I2C7	
10%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D	p2I2C1		p2I2C2		p2I2C3		p2I2C4		p2I2C5		p2I2C6		p2I2C7		p4I2C1		P4I2C2		p4I2C3		p4I2C4		p4I2C5		p4I2C6		p4I2C7	
10%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 11: Discount Output Table

Table 12 summarizes the important trends that are observed in the parameters of the model and decision variable from the output of the model.

Parameters	Decision to Upgrade
Implementation cost: “Disruption dominant” ⟹”Learning dominant”	↑
Probability of breakthrough ↑	↓
Acquisition cost ↑	↓
Reward (R) ↑	↑
Discount (D) ↑	↓

Table 12: Output Summary Table

5. Conclusions and Future Research

This paper develops a model to help firms in system upgrade decisions. While making the system upgrade decisions, firms must not only consider the impact of the future technologies but also analyze the nature of technologies in the existing system, especially when the firm has the option to upgrade multiple technologies in the system.

In this paper, the system upgrade problem is analyzed for an Information System, which is composed of two technologies. Some useful insights have been drawn from the model regarding possible relationship between parameters involved in the decision making. This model can be used in many IS upgrade settings by decision maker for upgrade decisions.

The model can be extended to include situations where firms use multiple systems. Other possible research can study the impact of "technology obsolescence" on complex product life cycle (here a product is made up of many parts). A real world study can be executed to see the applicability of the model using a real world upgrade data. One such setting has been identified as a topic of future research, where the upgrade decision of an organization will be analyzed. Presently, the organization is using an ad hoc approach of committee to decide their upgrade decisions. The research in this field can act as a blueprint to be used to take system upgrade decisions. These decisions are also becoming more important because a huge amount of IT related spending can be optimized by a proper upgrade approach. The research in this field still has to establish a theoretical framework to assist firms to make their upgrade decisions.

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