Monthly Cluster of Hourly Solar Irradiation in Kumasi-Ghana

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Abstract

In some practical problems of statistical inference we may be required to take decision concerning the parameters of the population instead of finding estimates for them. Monthly cluster of hourly solar irradiation in Kumasi-Ghana is a study conduct to determine the various clusters in which the months of the year could be grouped. Classical hypothesis testing was used in the significance test and the various month of the year was tested among other to identify difference if they exist. The result in this paper indicate that, the month of the year could be put into two cluster and these clusters are made up of the month of January –October as the first cluster while the second cluster comprise of the month of November and December.

Keywords: Solar irradiation; Hypothesis testing; level of significance-value.

Introduction

Knowledge of the solar irradiation climate of an area is of paramount importance in assessing the potential use of a solar energy system, converted into either thermal or electrical energy, as a power source in that area. Such information is a prerequisite for the design of such solar energy conversion system The mobilization of adequate national financial resources for the planning and development of the local solar energy resource depends on the availability of solar radiation data which could be used to evaluate available resources and to assess the probable long term performance of systems and hence their economic viability.

The solar radiation received at the earth's surface is subject to daily, seasonal and annual variations and hence many years of observation (perhaps at least 20 years) must be acquired in order to obtain a fairly accurate estimate of long term availability and distribution. However many locations in the developing countries do not have the facilities for continuous and accurate measurements of solar radiation and it is then necessary to use empirical methods which are based on easily measured meteorological parameters such as temperature, relative humidity, rainfall, cloudiness and duration of bright sunshine.

Many such formulae have been documented in the literature (Knight et al, 1991) although the most widely used correlation and perhaps the simplest, is the Angstrom (B) linear regression equation as modified by Page (1964) and others. This correlation relates the monthly average, daily global irradiation on the horizontal to the relative duration of sunshine, and it has been applied to a variety of climates including tropical locations. Except for the recent work of Neba-Fabs et al., (1988) and of Exel(1978) nearly all the work done for locations in the West Africa sub-region and other tropical locations have sought to determine a single regression equation which could be used for all months and hence all seasons of the year. The results of Eze and Ododo (1988) and of other researchers however tend to indicate that the Angstom-Page correlation coefficients depend on both the local climate and the season. Furthermore, it is anticipated that more accurate estimates of monthly average global irradiation would be obtained from correlations for particular months.

Liu and Jordan (1963) as well as Bendt et al. (1981) conducted extensive statistical analysis of daily global irradiation on the horizontal particularly investigating possible variations of the frequency distribution with both location and season. Their results showed that frequency distribution of daily global irradiation on the horizontal for the monthly period corresponding to a specified value of a monthly mean clearness index, is almost independent of the location and the time of the year. Bendt*et al.* (1981), moreover, went ahead and showed that the generalized cumulative distribution function may be obtained from a probability density function which assumed among others random daily insolation sequences.

In this study, we shall not dwell on the regression methods used to estimate the monthly global averages which, in any case, has already comprehensively been dealt with by Jackson et al (1990) but rather, taking advantage of the currently abundant data on solar irradiation data for Kumasi, we undertake the determination of the pertinent probability density curves based on randomly selected samples in respect of monthly or seasonal variations.

Materials and Methods

Pyranometer is the device used to record the sunlight data.It is made up of solar cell modules which harvest energy from the sun. The output of the solar cell modules depends on the amount of sunlight (or solar radiation) falling on them and it is affected by seasonal and daily solar radiation changes. It also changes depending on how cloudy or dusty the site is. It records two types of radiations: global average and diffused average.

The global average radiation is the hourly average irradiance of the direct solar energy reaching the earth's surface and the diffused average radiation is as a result of the direct solar energy being blocked by a black ring, clouds or dust before reaching the solar cell modules. However, this research work makes use of the global average radiation which is useful in the production of solar energy.

Data on hourly solar irradiation in Kumasi was collected from the Solar Energy Laboratory of the Mechanical Engineering department of KNUST, Kumasi. The irradiation data which was measured in kilowatt hours per meter squared was collected by means of a pyranometer. The data consists of fourteen years of hourly solar irradiation data from 1995-2008. Analysis was conducted on the hourly solar irradiation data obtained in the past years to gain much insight of the data to constructively solve the problems as stated.

Hypothesis Tests

The mean and the variance the various standard distributions that were fitted to the observed solar irradiation data was computed for each month.These mean and variance values were used in hypothesis testing to help determine the existence of clustering in terms of similarity in patterns of solar irradiation for various months of the year. The hypothesis was conducted to test the difference between the population means of the various months of the year solar irradiation based on the sample selected. January being the first month of the year was used to test with the rest of the months until there was a rejection (existence of significant difference between the means corresponding to the months).

The hypothesis test used in the test about the difference between two populations means. Here the population corresponds to the month of the year. Testing:

$H_O : \mu_1 = \mu_2 \text{or} \mu_1 - \mu_2 = 0$ $H_a : \mu_1 \neq \mu_2$ With the test statistic:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_o}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

We reject H_O in favor of H_a at a level of significance α=0.05 if and only if the appropriate rejection point condition holds. I.e. if $Z > Z_{\alpha/2} or Z < -Z_{\alpha/2}$

The test between each month and subsequent months is performed until the rejection point condition holds. The test is repeated between the last month used in the latter test and subsequent months until the rejection point condition holds again. This procedure grouped the months into clusters, shown in the tables below.

Results

The month of January was used to test against the month of February, it was noticed that there is no significant difference between mean solar output for January and February. Thus January and February belong to the same cluster. The Table1 shows the results of the test.

The month of January was tested with March and it was found that there is no significant difference between the two months which indicate that January and March can also be in the same cluster. The table 2shows the result of the test.

The month of January was tested with April and it was found that there is no significant difference between the two months which indicate that January and April can also be in the same cluster. The table 3 shows the result of the test.

The month of January was tested with May and it was found that there is no significant difference between the two months which indicate that January and May can also be in the same cluster. The table 4 shows the result of the test.

The month of January was tested with June and it was found that there is no significant difference between the two months which indicate that January and June can also be in the same cluster. The table 6 shows the result of the test.

The month of January was tested with July and it was found that there is no significant difference between the two months which indicate that January and July can also be in the same cluster. The table 7 shows the result of the test.

The month of January was tested with August and it was found that there is no significant difference between the two months which indicate that January and August can also be in the same cluster. The table 8 shows the result of the test.

The month of January was tested with September and it was found that there is no significant difference between the two months which indicate that January and September can also be in the same cluster. The table9 shows the result of the test.

The month of January was tested with October and it was found that there is no significant difference between the two months which indicate that January and October can also be in the same cluster. The table 10 shows the result of the test.

The month of January was tested with November and it was found that there is a significant difference between the two months which indicate that January and November cannot be in the same the same cluster. The table 11 shows the result of the test.

Since the test for January and November shows a significant difference, we will now test November with December to check the result. It was found that the there is no significant difference between the solar irradiation for November and December and these two month can also form another cluster. Table 12 shows the test results.
The month of November was tested with January and it was found that there is a significant difference between the two months which confirms the fact that January and November cannot be in the same cluster. Table13 shows the result of the test.

The result of the series of tests indicate that the there are two clusters in the year in respect of similar patterns of solar irradiation, namely, one from January to October and the other from November to December.

Further hypothesis testing analysis was conducted on these two clusters of the year and it was also found that there is a significant difference between the mean outputs of solar irradiation for the two clusters of the year. The result of the cluster analysis is shown in Table 18.

Discussion

There are certain climatic factors that are accounting for the rise and fall of the hourly solar irradiation pattern in the year. The factors include cloudiness and the presence of dust particles in the atmosphere.

Fig 1 indicates that the maximum mean solar irradiation received occurred in February. This perhaps may be due to the fact that February marks the end of the harmattan and the beginning of the rainy season where both cloudiness and dust particles level in the atmosphere are low and hence the solar irradiation level reaching the earth is high. The months of March, April and May were found to be the months with the least mean solar irradiation. Perhaps this may be due to the fact by March, the rainy season would have begun for some time and the cloudiness level would be quite high resulting in a low level of solar irradiation. In the month of June there was a rise in solar irradiation even though the solar irradiation level falls in July, August, September and October which may perhaps be due to the fact that in June there is intermittent cloudiness and sunshine level. In some cases we have a cloudy day throughout and also sunshine through certain day's amount of rainfall is not as in march April and May. The cloudiness level reduces and so the solar irradiation output is higher. The month of July, August, September and October marks the end of the raining season which also results in higher amount of rainfall hence higher cloudiness level resulting in low solar irradiation output. The month of November and December which form the second cluster also had higher solar irradiation level above the solar irradiation level of July, August, September and October. This may be due to the fact that the month of November is the beginning of the harmattan season with low cloudiness and low dust particles level resulting in high solar irradiation.

Conclusion

Based on the theory of hypothesis testing the various months of the year with similar solar irradiation patterns were put into clusters. From the hypothesis testing carried out it reveals that the months of the year can be put into two such clusters and this cluster could be associated partially to the two major climatic conditions that pertain to our nation. The first cluster extends from the month of January to October and the second cluster is from November to December.

References

B.Y.U. Liu and R.C. Jordan, (1963) A rational procedure for predicting the long term average performance of flat plate energy collector. Solar Energy 7, (Page 53)

P.Bendt,M.Collares-Pereira and A.Rabl, (1981) The frequency distribution of daily insolation values Solar Energy 27, (Page 1)

G.Y. Saunier, T.A. Reddy and S. Kumar, (1987) Monthly probability distributions for both tropical and temperate locations Solar Energy 38, (Pages 325-330)K.G. Hollands and R.G. Huget (1983) Probability density function for the clearness index with application. Solar Energy 30, (Pages 195-209)

F.J.K. Ideriah and S.O. Suleman, (1989) Sky conditions at Ibadan during 1975-1980 Solar Energy 43, (Pages 325-330)

T. Feuillard, J.M. Abillon, and C. Martias, (1989) The probability density function of the clearness index Solar Energy 43, (Pages 363-372)

J.A. Olseth and A. Skartvieit, (1984) A probability density function for daily insolation within the temperate storm belts, Solar Energy 33, (Pages 533-542)

J.M. Gorden and T.A Reddy, (1988) Time series analysis of daily horizontal solar radiation Solar Energy 41, (Pages 215-226)

K.M. Knight, S.A. Klein and J .A. Duffie, (1991) A methodology for the synthesis of hourly weather data, Solar Energy 46, (Pages 109-120)

A.K.E. Ussher, (1986) Distribution of solar and wind energy in Ghana, Bulletin of Energy Research Group 1, 25, Ghana

E. Jackson and F.O. Akuffo, (1990) Angstrom - Page type correlation between monthly average daily global irradiation on the horizontal and the monthly average relative duration of sunshine. Kumasi,Energy conservation and management.

F.O. Akuffo and A. Brew-Hammond, (1993) The frequency distribution of daily global irradiation at Kumasi, Solar Energy 50 (Pages 145-154)

E. Yilmaz and *et al* (2006).Statistical analysis of solar radiation data for the city of Valparaiso in the coast region of Chile. Energy Sources,Volume 29. Pages 71 - 83

Oturanc and *et al (*2003) Statistical analysis on daily solar radiation Energy source volume 25. Pages 89 – 97

R. E. Walpole et al., (2007) Probability and statistics for Engineering & Sciences, Pearson Education, Inc., 8th Edition.United state

B. Bowerman et al., (1997) Applied Statistics, Von Hoffmann Press Incorporated. First Edition United states

Ramakant Khazanie (2001); Statistics in a world of Applications, Harper Collins college Publishers, Fourth Edition United states

J. L. Devore; (2007) Probability and statistics for Engineering and the Sciences, Duxbury Press, third Edition. United state.

Kenneth H. Rosen;Discrete Mathematics and its Application, Fourth edition
MacGraw-Hill.United state.

Laurence. D. Hoffman (1995); Finite mathematics with calculus, Prudential Securities, Inc., second Edition. United States

Mark Hankins, (1995) Small Solar electric System for Africa. Second Edition Commonwealth Secretariat(United Kingdom)

Robert Bartoszynski, Magdelena Niewiadomska-Bugaj (1996) Probability and Statistical inference: JohnWily and Sons,Inc.United State

J Crawshaw and J Chambrs (2004); Concise course in advanced level statistics, fourth edition.Stanley Thorns(publishers) Ltd.

Anderson et al (1993); Introduction to statistics, concepts and applications third edition.West publishing company-United state

Levine et al (2000) Business statistics, A first course second edition,Prenttice Hall, Inc-United state

Bernard Rosner (2000), Fundamentals of biostatistics Fifth edition,Duxbury Thomson learning –united state

Apendix

Table 1: Test of Hypothesis between Mean Solar Irradiation of January and February

	Month	sample size (n)	Mean($\bar{x}$)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	FEB	1500	675	50625

RESULTS

Z =	-0.448966081	Hypothesized difference (Do)	0
$\mid Z_{\alpha/2} \mid$=	1.96	Mean difference	-592.2117801
Decision:	Accept Ho (since Z > -1.96)		
Conclusion:	There is no difference between the population means		

Table 3: Test of Hypothesis between Mean Solar Irradiation of January and March

	Month	sample size (n)	Mean($\bar{x}$)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	MARCH	1500	37.14205451	200752.5614

RESULTS

Z =	0.008801078	Hypothesized difference (Do)	0
$\mid Z_{\alpha/2} \mid$=	1.96	Mean difference	45.64616544
Decision:	Accept Ho (since Z < 1.96)		
Conclusion:	There is no difference between the population means		

Table 4: Test of Hypothesis between Mean Solar Irradiation of January and April

	Month	sample size (n)	Mean($\bar{x}$)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	APRIL	1500	36.42145334	146625.8932

RESULTS

Z =	0.012233982	Hypothesized difference (Do)	0
$\mid Z_{\alpha/2} \mid$=	1.96	Mean difference	46.36676661
Decision:	Accept Ho (since Z < 1.96)		
Conclusion:	There is no difference between the population means		

Table 4: Test of Hypothesis between Mean Solar Irradiation of January and May

	Month	sample size (n)	Mean($\bar{x}$)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	MAY	1500	34.30471912	169874.2951

RESULTS

Z =	0.011044822	Hypothesized difference (Do)	0
$\lvert Z_{\alpha/2} \rvert$=	1.96	Mean difference	48.48350083
Decision:	Accept Ho (since Z < 1.96)		
Conclusion:	There is no difference between the population means		

Table 6: Test of Hypothesis between Mean Solar Irradiation of January and June

	Month	sample size (n)	Mean($\bar{x}$)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	JUNE	1500	404	1616

RESULTS

Z =	-1.766656266	Hypothesized difference (Do)	0
$\lvert Z_{\alpha/2} \rvert$=	1.96	Mean difference	-321.2117801
Decision:	Accept Ho (since Z > -1.96)		
Conclusion:	There is no difference between the population means		

Table 7: Test of Hypothesis between Mean Solar Irradiation of January and February

	Month	sample size (n)	Mean($\bar{x}$)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	JULY	1500	308	616

RESULTS

Z =	-1.267513639	Hypothesized difference (Do)	0
$\lvert Z_{\alpha/2} \rvert$=	1.96	Mean difference	-225.2117801
Decision:	Accept Ho (since Z > -1.96)		
Conclusion:	There is no difference between the population means		

Table 8: Test of Hypothesis between Mean Solar Irradiation of January and August

	Month	sample size (n)	Mean($\bar{x}$)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	AUG	1500	312	936

RESULTS

Z =	-1.283314174	Hypothesized difference (Do)	0
$\vert Z_{\alpha/2} \vert$=	1.96	Mean difference	-229.2117801
Decision:	Accept Ho (since Z > -1.96)		
Conclusion:	There is no difference between the population means		

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Table 9: Test of Hypothesis between Mean Solar Irradiation of January and September

	Month	sample size (n)	Mean($\bar{x}$)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	SEPT	1500	306	612

RESULTS

Z =	-1.256322605	Hypothesized difference (Do)	0
$\vert Z_{\alpha/2} \vert$=	1.96	Mean difference	-223.2117801
Decision:	Accept Ho (since Z > -1.96)		
Conclusion:	There is no difference between the population means		

Table 10: Test of Hypothesis between Mean Solar Irradiation of January and October

	Month	sample size (n)	Mean($\bar{x}$)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	OCT	1500	241	868

RESULTS

Z =	-0.886935958	Hypothesized difference (Do)	0
$\vert Z_{\alpha/2} \vert$=	1.96	Mean difference	-158.2117801
Decision:	Accept Ho (since Z > -1.96)		
Conclusion:	There is no difference between the population means		

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Table 11: Test of Hypothesis between Mean Solar Irradiation of January and November

	Month	sample size (n)	Mean(x̄)	Variance(s^2)
Sample 1	JAN	1500	82.78821995	6853.889362
Sample 2	NOV	1500	494	988

RESULTS

Z =	-2.299895326	Hypothesized difference (Do)	0
$\lvert Z_{\alpha/2} \rvert$=	1.96	Mean difference	-411.2117801
Decision:	Reject Ho in favour of Ha (since Z < -1.96)		
Conclusion:	There is a difference between the population means		

Table 12: Test of Hypothesis between Mean Solar Irradiation of November and December

	Month	sample size (n)	Mean(x̄)	Variance(s^2)
Sample 1	NOV	1500	494	988
Sample 2	DEC	1500	512	2048

RESULTS

Z =	-0.30658722	Hypothesized difference (Do)	0
$\lvert Z_{\alpha/2} \rvert$=	1.96	Mean difference	-18
Decision:	Accept Ho (since Z > -1.96)		
Conclusion:	There is no difference between the population means		

Table 13: Test of Hypothesis between Mean Solar Irradiation of November and January

	Month	sample size (n)	Mean(x̄)	Variance(s^2)
Sample 1	NOV	1500	494	988
Sample 2	JAN	1500	82.78821995	6853.889362

RESULTS

Z =	2.299895326	Hypothesized difference (Do)	0
$\lvert Z_{\alpha/2} \rvert$=	1.96	Mean difference	411.2117801
Decision:	Reject Ho in favour of Ha (since Z > 1.96)		
Conclusion:	There is a difference between the population means		

Table 18: Test of Hypothesis between Mean Solar Irradiation of Jan-Oct and Nov-Dec

INPUTS	
Level of Significance	0.05

	Month	***sample size (n)***	***Mean($\bar{x}$)***	***Variance(s^2)***
Sample 1	jan-oct	1500	428	856
Sample 2	nov-dec	1500	326	652

RESULTS			
Z =	3.671313969	***Hypothesized difference (Do)***	0
$\lvert Z_{\alpha/2} \rvert$ =	1.96	***Mean difference***	102
Decision:	Reject Ho in favour of Ha (since Z > 1.96)		
Conclusion:	There is a difference between the population means		

The figure 1 Mean hourly solar irradiation of the Months