On the Practice of Bundling a Free Gift with a Threshold Purchase

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Abstract

This article explores the practice of bundling a free unit of good y (drink) with a threshold purchase of good x (food). Allowing for heterogeneity in consumer drink preferences we characterize the conditions under which the practice is strictly more profitable than linear pricing. An increase in food sales is the source of higher profits. Selection issues reduce profitable opportunities in a heterogeneous population of consumers relative to a homogeneous population. In some cases, an increase in profit from using the practice can be higher when consumers would not otherwise buy the drink. The practice can be Pareto-improving if the seller has limited control over prices.

Keywords: mixed bundling, multi-product nonlinear pricing, quantity discounts, threshold.

JEL: D4, L1

1. Introduction

We study profitability of a selling practice that involves giving a free gift to a consumer who purchases an amount of another good that meets or exceeds a pre-specified threshold. The practice is fairly widespread. One can get a free drink by purchasing a certain minimum amount of food (or by spending a minimum amount of money) at a local fast food operation. Some stores offer gift cards to consumers who buy a minimum amount of a particular good. A buyer of a large bottle of perfume is often rewarded with a duffel bag or a similar free item. At the time of writing this article a chain of supermarkets in the San Francisco Bay Area was offering a deal whereby the consumers who spend a qualifying amount of money within an approximately 5-month period can get a free piece of cookware. Intuitively, a seller offering a free gift anticipates an increase in sales. However, even if the premise is correct it is not immediately obvious whether the benefit from an increase in sales outweighs the cost from giving out another good for free, especially once adverse selection is taken into account. This work is a theoretical investigation of whether the approach of bundling a free gift does indeed lead to an increase in sales and under what conditions the increase in sales is actually profitable.

In our theoretical model a seller offers two goods for sale. Consumers demand continuously varying quantities of one good and have unit demands for the second good. To facilitate exposition we shall often refer to the good demanded in continuous quantities as "food" and to the unit-demand good as "a drink". A motivating example for this notation is a salad bar where food is purchased by weight and drinks can be purchased in fixed quantities. The seller decides whether to sell the goods separately or offer a deal whereby consumers who purchase a minimum threshold amount of food get the drink for free. We refer to the practice of bundling a free gift with a minimum purchase of another good as threshold-based mixed bundling (TBMB).

TBMB combines elements of two widely studied phenomena: non-linear pricing (Buchanan, 1952; Maskin and Riley, 1984; Oi, 1971; Stole, 2007; Varian, 1989) and bundling (McAfee et al., 1989; Schmalensee, 1984; Nalebuff, 2003). A seller offering a free drink with a purchase of a threshold amount of food is essentially giving a discount to high-volume buyers, a common type of non-linear pricing. The discount is in the form of another good provided free of charge. The latter aspect is related to commodity bundling and, in particular, mixed bundling -- a type of bundling where both the bundle and its components are available for purchase. We allow mixed bundling since it is often the case that the free gift can be purchased separately if the consumer fails to meet the minimum threshold.

The bulk of the literature on non-linear pricing and bundling is devoted to studying decisions of sellers with substantial market power: a monopolist or an oligopolist.[1] In our analysis we examine the setting where the seller does not control prices and enjoys a positive markup in the market for food. We do so to get a better insight into the determination of the optimal parameters of TBMB (the optimal threshold) and to better highlight the interaction of this decision with the consumer heterogeneity in drink preferences. This setting is also of interest in its own right. Price taking and positive markups can coexist in various settings. Collusive agreements, resale price maintenance and similar arrangements may prevent firms from exercising control over prices in imperfectly competitive markets. In such cases, the firms can be reluctant to explicitly change prices and prefer to employ TBMB to affect the prices implicitly. Finally, the lack of control over prices poses a bigger challenge to the profitability of the practice and can be viewed as the worst-case scenario.

We formulate a model that allows consumers to differ in their preferences for drinks (e.g., thirst level). We find that TBMB accomplishes a type of price discrimination in the sense that some consumers pay lower unit prices for food than others. Those consumers purchase at least the threshold amount of food and get a discount in the form of a free drink, the response we refer to as "taking the deal". In a typical setup price discrimination helps the seller to extract a larger part of the consumer surplus making some consumers worse off. Because the seller is a price taker, consumers always have a choice of not taking the deal and purchasing the same amount and at the same prices as under linear pricing. Thus consumers are no worse off. The implication is that such practices as TBMB, if profitable, can be welfare-enhancing when sellers have limited control over prices.

The main goal of this study is to explore the conditions under which TBMB is more profitable than the natural benchmark of uniform pricing with unbundled sales (which we refer to as linear pricing). We find that TBMB can indeed be more profitable than linear pricing due to the fact that it induces some consumers to purchase more food than they otherwise would. Thus, the intuition that a free gift makes consumers increase their purchases is borne out in our model. The key to the profitability of TBMB is that by giving out for free a good with a relatively low opportunity cost (drink), the seller can boost sales of a relatively profitable good (food). The rest of the paper is structured as follows. In the next section we describe our assumptions about the consumers' preferences. In Section 3 we derive the seller's profit for the benchmark case of linear pricing. In Section 4 we characterize analytically the conditions under which TBMB is strictly more profitable than a linear pricing schedule. In Section 5 the key results of the paper are summarized and discussed.

2. Consumer preferences

The seller faces a continuum of consumers with mass normalized to 1. Suppose consumers derive utility from three goods: x (food), y (drink), and m (a composite good). Goods x and m can be purchased in any non-negative quantity. Good y can only be purchased in quantities 1 (a drink) or 0 (no drink).

[1]For surveys of the literature see e.g. Varian (1989) and Stole (2007).

We assume that a consumer's utility function is additively separable in all three goods and can vary by thirst level t: $u(x, y, m \mid t) = v(x) + d(y \mid t) + m$. Function $v(x)$ is twice continuously differentiable with $v'(x) > 0$ and $v''(x) < 0$, i.e. the marginal utility of food is positive and diminishing. The marginal utility from the drink is defined as $d_t \equiv d(1 \mid t) - d(0 \mid t) > 0$. It is increasing in the thirst level t: $d_{t'} > d_{t''}$ for $t' > t''$. For simplicity we assume that there are only two thirst levels in the population of consumers: B and NB such that $B > NB$ and, consequently, $d_B > d_{NB}$.[2] The fraction of consumers with the thirst level NB is μ and the fraction of consumers with the thirst level B is $1 - \mu$.

Let the price of y be p_y, the price of x be p_x, the price of m be 1, and the consumer's income be I. Thus, the consumer's budget constraint is $p_x x + p_y y + m \le I$, which holds with equality since the consumer's utility is increasing in all three goods. The consumer's problem is to maximize utility by choosing what amount of food x to purchase and whether to buy the drink or not:

$$\max_{x,y} u(x, y \mid t) = \max_{x,y} \left[v(x) + d(y \mid t) + I - p_x x - p_y y \right]. \tag{1}$$

3. *Linear pricing*

First, suppose that the seller uses a linear pricing schedule to sell the two goods, i.e. the per-unit prices that consumers pay are simply p_x and p_y. Due to additive separability of the utility function, the consumer's food demand is independent from the drink purchasing decision if the income is sufficiently high. The demand $x^* = x^*(p_x)$ is obtained from the first-order condition of (1) with respect to x: $v'(x^*) = p_x$. Given our assumption that $v(x)$ is strictly concave in x, the first-order condition is both necessary and sufficient. Implicitly differentiating the condition we conclude that the demand for food is downward-sloping: $x^{*\prime}(p_x) = -\dfrac{-1}{v''(x^*)} < 0$.

Turning to the consumer's drink purchasing decision, note that the consumer will buy the drink if doing so provides him with a utility level at least as high as that from not buying the drink: $u(x,1 \mid t) \ge u(x,0 \mid t) \Leftrightarrow d_t \ge p_y$. Therefore, the consumer's demand for the drink is given by an indicator function: $y^* = y^*(p_y \mid t) = \mathbf{1}_{(d_t \ge p_y)}$.

Next, we derive the seller's profit. Let c_x and c_y be the constant marginal costs of x and y respectively. We assume that the drink price is such that $d_{NB} < p_y < d_B$, i.e. only the consumers with the high thirst level purchase the drink. Consequently, we refer to the consumers with $t = B$ as *drink buyers* and to the consumers with $t = NB$ as *drink non-buyers*. Thus, the seller's expected profits under linear pricing are given by:

$$\pi_{NB} = [p_x - c_x] x^*(p_x) \tag{2}$$

$$\pi_B = [p_x - c_x] x^*(p_x) + [p_y - c_y] \tag{3}$$

$$\pi_M = [p_x - c_x] x^*(p_x) + [1 - \mu][p_y - c_y],$$

where π_{NB}, π_B, and π_M are respectively profits from serving only drink non-buyers, only drink buyers, and a mixed population. These profits serve as a benchmark in the ensuing analysis of the threshold-based mixed bundling scheme.

4. *Threshold-based Mixed Bundling*

Suppose that the seller offers an optional free-drink deal whereby a consumer gets the drink for free if he purchases an amount of food x equal to or higher than some threshold quantity a.

[2] The abbreviations stand for "buyers" and "non-buyers" as we explain below.

If the amount of food a consumer purchases falls short of the threshold level, $x < a$, he can still buy the drink if he wishes. The seller effectively offers a bundle consisting of the drink and an amount of food that meets or exceeds the threshold. The bundle is discounted compared to buying the two goods separately. The discount is indirect and is implemented as a free gift to the consumers who meet the threshold.

4.1 Determination of the optimal threshold

We start by analyzing the seller's problem of determining the optimal threshold a. Under linear pricing the consumer demand is $x^* = x^*(p_x)$. Intuitively, the seller prefers to set the threshold above x^*. With $a \le x^*$ the seller's profit can only decrease as consumers continue purchasing x^* while getting the drinks for free. The seller can only increase profits by inducing the consumers to purchase more food in exchange for getting free drinks, the response we refer to as "taking the deal". A consumer will take the deal if the utility from doing so is at least as high as from not taking the deal: $u(a,1|t) \ge u(x^*, y^*|t)$, where x^* and y^* are quantities demanded under linear pricing. The consumer's utility from taking the deal is $u(a,1|t) = v(a) + d(1|t) + I - p_x a$. Without taking the deal it is $u(x^*, y^*|t) = v(x^*) + d(y^*|t) + I - p_x x^* - p_y y^*$. Using these definitions, $u(a,1|t) \ge u(x^*, y^*|t)$ can be written as:

$$[v(a) - v(x^*)] - p_x[a - x^*] \ge -B_t \equiv -\min\{d_t, p_y\}, \tag{4}$$

where B_t is the benefit to the consumer of getting the drink for free. Depending on the thirst level the benefit is an increase in the consumer's utility from either not having to pay for the drink he would otherwise buy ($p_y < d_B$), or from getting the free drink he would otherwise not buy ($d_{NB} < p_y$).

Suppose the seller faces a population with the same thirst level t. In this case, she prefers to set a threshold $a_t^* > x^*$ such that (4) holds with equality. To see this, note that the seller's profit increases in a as long as the incentive condition in (4) is satisfied. The condition is slack for $a = x^*$. For $a > x^*$ the left-hand side of (4) decreases with a at the rate of $v'(a) - p_x < 0$. The rate is negative since $a > x^*$, $v'(x^*) = p_x$, and $v(x)$ is strictly concave in x. For some high enough a inequality (4) binds, at which point the consumers are indifferent between taking the deal or not. Thus, by setting $a = a_t^*$ the seller ensures that she gets the highest possible increase in food purchases without losing the customers.

The optimal threshold a_t^* is a function of the price of food p_x and the benefit of the free drink B_t: $a_t^* = a^*(p_x, B_t)$. The effect of those parameters on a_t^* are summarized in the following lemma.

Lemma 4.1 *When consumers' drink preferences are homogeneous, the optimal threshold* $a_t^* = a^*(p_x, B_t)$ *decreases in the price of food* p_x *and increases in the benefit of getting the drink for free,* B_t *(as long as* $d_{NB} < p_y$ *continues to hold).*

Proof. The result follows from implicitly differentiating (4) evaluated at $a_t^* = a^*(p_x, B_t)$ and noting that $a_t^* > x^*$ and $p_x = v'(x^*) > v'(a_t^*)$ by strict concavity of $v(x)$. ∎

Thus, the optimal threshold a_t^* varies depending on whether the consumers are drink buyers or drink non-buyers. A closed form expression for the optimal threshold is not available for commonly used strictly concave utility functions. But since the optimal threshold increases in B_t and $B_B > B_{NB}$ ($p_y > d_{NB}$ by assumption), it follows that $a_B^* > a_{NB}^*$. Therefore, the seller optimally sets a higher threshold when all consumers are drink buyers. When the population of consumers is a mix of drink buyers and drink non-buyers and the seller cannot distinguish between them, then it is easy to see that the optimal threshold is either a_B^* (so that only drink buyers take the deal) or a_{NB}^* (so that all consumers take the deal).

If the seller sets $a > a_B^*$ then no consumer would take the deal as (4) would be violated for both groups. If she sets $a_{NB}^* < a \le a_B^*$ then only drink buyers would take the deal as the threshold would be too high for drink non-buyers. Since lowering a below a_B^* does not attract drink non-buyers but decreases the amount of food that drink buyers purchase, the optimal threshold in that interval is the upper bound a_B^*. If the seller sets $a \le a_{NB}^*$ then both types of consumers will take the deal. Using similar reasoning, the optimal threshold for this interval is a_{NB}^*. Thus, the optimal threshold is either a_B^* or a_{NB}^*.

Which of the two thresholds the seller should choose when the population of consumers is mixed depends on the parameters of the model. If the seller sets the threshold at $a = a_B^*$ her profit is: $\pi_M^d\left(a_B^*\right) = [1-\mu]\{[p_x - c_x]a_B^* - c_y\} + \mu[p_x - c_x]x^*$, as drink buyers take the deal while drink non-buyers purchase their usual amount of food. If she sets the threshold at $a = a_{NB}^*$ her profit is $\pi_M^d\left(a_{NB}^*\right) = [p_x - c_x]a_{NB}^* - c_y$, as everybody takes the deal. Let us denote the difference between the two profits as $\Delta_\pi \equiv \pi_M^d\left(a_B^*\right) - \pi_M^d\left(a_{NB}^*\right)$:

$$\Delta_\pi = [p_x - c_x]\{[1-\mu]a_B^* + \mu x^* - a_{NB}^*\} + \mu c_y. \tag{5}$$

Thus, the seller will set $a = a_B^*$ if $\Delta_\pi > 0$ and $a = a_{NB}^*$ if $\Delta_\pi < 0$ (provided the deal is better than linear pricing). In the next lemma we state how some of the parameters of the model influence the seller's choice between the two alternatives.

Proposition 4.1 *Facing a mixed population of consumers the seller will focus exclusively on drink buyers (i.e. set* $a = a_B^*$*) as the proportion of drink non-buyers approaches zero,* $\mu \to 0$*. She will make the deal attractive to all consumers (i.e. set* $a = a_{NB}^*$*) as* $\mu \to 1$ *(provided the deal is more profitable than linear pricing in a population of only drink non-buyers). Focusing exclusively on drink buyers becomes relatively more attractive to the seller with an increase in* c_x *(unless the seller already focuses on drink buyers),* c_y*, or* p_y*, or a decrease in* d_{NB}*.*

Proof. From (5), $\lim_{\mu\to 0}\Delta_\pi = [p_x - c_x][a_B^* - a_{NB}^*] > 0$, i.e. setting $a = a_B^*$ is more profitable than $a = a_{NB}^*$ when the consumers are mostly drink buyers. On the other hand, $\lim_{\mu\to 1}\Delta_\pi = -[p_x - c_x]\{[a_{NB}^* - x^*]\} + c_y$. As we show later this limit can be written as $-\Delta\pi_{NB}\left(a_{NB}^*\right)$ (see (8)), the negative of the difference between the profits from offering the deal and the profits from linear pricing when all consumers are drink non-buyers. Thus, as long as offering the deal in such a population is better than linear pricing, it follows that $\lim_{\mu\to 1}\Delta_\pi < 0$. Thus, setting $a = a_{NB}^*$ is more profitable than $a = a_B^*$ when the consumers are mostly drink non-buyers.

Since $\dfrac{\partial\Delta_\pi}{\partial c_y} = \mu > 0$, $\dfrac{\partial\Delta_\pi}{\partial p_y} = [p_x - c_x][1-\mu]\dfrac{\partial a_B^*}{\partial p_y} > 0$, and $\dfrac{\partial\Delta_\pi}{\partial d_{NB}} = -[p_x - c_x]\dfrac{\partial a_{NB}^*}{\partial d_{NB}} < 0$, setting $a = a_B^*$ is relatively more attractive than setting $a = a_{NB}^*$ when c_y is high and/or p_y is high and/or d_{NB} is low.

The sign of $\dfrac{\partial\Delta_\pi}{\partial c_x} = -\{[1-\mu]a_B^* + \mu x^* - a_{NB}^*\}$ is ambiguous. Note that $\lim_{\mu\to 1}\dfrac{\partial\Delta_\pi}{\partial c_x} > 0$ and a higher cost of food increases the likelihood that the seller will focus exclusively on drink buyers. However, $\dfrac{\partial\Delta_\pi}{\partial c_x} < 0$ for some μ close to 0.

Suppose $\frac{\partial \Delta_\pi}{\partial c_x} < 0$, but then $\{[1-\mu]a_B^* + \mu x^* - a_{NB}^*\} > 0$ and $\Delta_\pi > 0$, i.e. for these values of μ, the seller already focuses exclusively on drink buyers for any c_x and the sign of the derivative is irrelevant. ■

The proposition states that either of the two thresholds can be optimal for different consumer populations. When there are enough drink buyers, the seller prefers to ignore drink non-buyers. However, if the proportion of drink buyers is small, the seller finds it optimal to target drink non-buyers even though it implies that drink buyers get free drinks as well. Figure (1) illustrates the rest of the proposition for a population with 50 percent drink buyers.[3] The left panel shows the effect of food costs on the optimal threshold. When c_x is low, the seller caters to all consumers by setting $a = a_{NB}^*$. When the costs are high enough, the seller ignores drink non-buyers and sets $a = a_B^*$. The food cost γ' is such that $\Delta_\pi = 0$, i.e. the seller is indifferent between the two thresholds.

The panel also illustrates what happens to the proportion of consumers who take the deal. At $c_x = \gamma'$, the proportion of drink non-buyers taking the deal drops from 1 to 0. The right panel shows the effect of c_y. At $c_y = \gamma''$ (defined as the drink cost at which $\Delta_\pi = 0$) the seller abandons drink non-buyers. The cost of drink has no effect on the fraction of drink buyers taking the deal (as long as it remains profitable for the firm to stay in business).

We have discussed what threshold the seller would set to maximize her profit when offering the drink deal. Next, we characterize the conditions under which the optimally parameterized deal is more profitable than linear pricing. Note that as a direct implication of the price-taking assumption a seller offering the deal can always ensure the same profit as under linear pricing. She always has an option of setting $a > a_B^*$ so that no consumer takes the deal and the outcome is equivalent to linear pricing. For example, in Figure (1) for food costs $c_x \geq \gamma_M$ the seller prefers to set a threshold $a > a_B^*$. Thus, when consumers are homogeneous in their food preferences, the seller's profits with TBMB are at least as high as with linear pricing.

4.2 Conditions for deal profitability

In this section we explore the conditions under which the optimally parameterized deal can be *strictly* more profitable than linear pricing. We define the advantage of the free-drink deal over linear pricing as the difference between the corresponding profits. We consider four possible cases: 1) all consumers are drink buyers; 2) all consumers are drink non-buyers; 3) the seller faces a mix of consumers and sets $a = a_B^*$; and 4) the seller faces a mix of consumers and sets $a = a_{NB}^*$.

When the seller offers the deal to a homogeneous population of consumers of type $t \in \{B, NB\}$ with the corresponding optimal threshold a_t^* her profit is $\pi_t^d(a_t^*) = [p_x - c_x]a_t^* - c_y$, as all the consumers take the deal and get the drink for free.

Thus, when all consumers are drink buyers the deal advantage is given by:

$$\begin{aligned}\Delta\pi_B(a_B^*) &= \pi_B^d(a_B^*) - \pi_B \\ &= [p_x - c_x][a_B^* - x^*] - p_y\end{aligned} \tag{6}$$

[3]Even in the special case with a single hunger level the optimal thresholds cannot be solved for analytically for common utility functions. Thus, we use numerical calculations to generate this and other figures. The default assumptions about functional forms and parameter values are as follows: $v(x) = h\log(x)$, $h = 3$, $p_x = 1$, $p_y = 1$, $c_x = 0.5$, $c_y = 0.5$, $d_{NB} = 0.5$, $d_B = 1.5$, and $I = 15$.

$$= v(a_B^*) - v(x^*) - c_x[a_B^* - x^*], \quad (7)$$

where π_B is the profit from linear pricing given in (3), and the last line is obtained by substituting the constraint in (4) evaluated at a_B^*, and by noting that $B_t = p_y$ for drink buyers. The expression $\Delta\pi_t(a)$ denotes a change in profits when the seller faces consumers with thirst level t and sets the threshold at a.
Similarly, when all consumers are drink non-buyers the deal advantage is:

$$\Delta\pi_{NB}(a_{NB}^*) = [p_x - c_x][a_{NB}^* - x^*] - c_y \quad (8)$$
$$= v(a_{NB}^*) - v(x^*) - c_x[a_{NB}^* - x^*] + [d_{NB} - c_y]. \quad (9)$$

When the seller faces a mix of drink buyers and non-buyers and sets $a = a_B^*$ then only drink buyers take the deal. Hence, the portion of the profits obtained from drink non-buyers is not affected and the deal advantage is due exclusively to drink buyers purchasing more food and getting the drink for free ($[1-\mu]$ fraction of the population):

$$\Delta\pi_M(a_B^*) = \Delta\pi_B(a_B^*)[1-\mu] \quad (10)$$
$$= \{[p_x - c_x][a_B^* - x^*] - p_y\}[1-\mu] \quad (11)$$
$$= \{v(a_B^*) - v(x^*) - c_x[a_B^* - x^*]\}[1-\mu], \quad (12)$$

where subscript M indicates that the population of consumers is mixed.

If instead the seller sets $a = a_{NB}^*$ then all consumers take the deal. The deal advantage in this case is a weighted average of (7) and (9) evaluated at a_{NB}^*:

$$\Delta\pi_M(a_{NB}^*) = \Delta\pi_B(a_{NB}^*)[1-\mu] + \Delta\pi_{NB}(a_{NB}^*)\mu \quad (13)$$
$$= [p_x - c_x][a_{NB}^* - x^*] - \{p_y[1-\mu] + c_y\mu\} \quad (14)$$
$$= v(a_{NB}^*) - v(x^*) - c_x\{a_{NB}^* - x^*\} + [d_{NB} - \{p_y[1-\mu] + c_y\mu\}]. \quad (15)$$

Below we state how the deal advantage over linear pricing is affected by various model parameters.

Lemma 4.2 *The advantage of the deal over linear pricing (if any) is*

- decreasing in c_x regardless of the consumer population composition;
- decreasing in c_y but only when the deal is attractive to drink non-buyers (i.e. either all consumers are drink non-buyers or the seller targets all consumers by setting $a = a_{NB}^*$);
- decreasing in p_y when the deal is only attractive to drink buyers and increasing in d_{NB} when the consumer population is mixed and the deal is attractive to all consumers (provided p_y remains above d_{NB}).

Proof. From the expressions in (7), (9), (12) and (15), the sign of the derivative of the deal advantage with respect to c_x is negative in all cases. From the expressions in (7) and (12), the derivative of the deal advantage with respect to c_y is zero when the deal is only attractive to drink buyers.

When the deal is attractive to drink non-buyers, the derivative is negative, as is clear from (9) and (15). Finally, from the expression in (15), the derivative of the deal advantage with respect to p_y is negative. From (14), the derivative of the deal advantage with respect to d_{NB} is positive. ■

As an example, Figure (2) depicts the effect of c_x on the seller's profits and the deal advantage when all consumers are drink buyers. Naturally, the profits decrease as c_x goes up. In addition, as stated in Lemma (4.2), the deal advantage is also reduced at higher c_x. Figure (2) also illustrates the fact that the deal advantage is non-negative. At high c_x, the seller resorts to setting a threshold $a > a_B^*$ to discourage any consumers from taking the deal.

Lemma (4.2) gives us some idea about the influence of various parameters on the deal advantage. Specifically, lower costs of food and drink as well as a lower price of drink and a higher marginal utility of drink for drink non-buyers have a positive effect in some settings. The following two propositions extend these results and provide sufficient conditions on the parameters that ensure that the deal is strictly more profitable.

First, consider the case where all consumers are drink buyers or the seller targets only drink buyers in a mixed population. Then, from either (6) or (10) the deal advantage $\Delta\pi_B(a_B^*)$ is positive whenever:

$$[p_x - c_x] > \frac{p_y}{a_B^* - x^*}. \tag{16}$$

In other words, the food profit margin must be high enough so that it exceeds the opportunity cost of giving out free drinks, which in this case is related to p_y. It is not immediately obvious from this expression that such a high margin is possible since both sides of the inequality increase in p_x.

Proposition 4.2 *When all consumers are drink buyers or the population of consumers is mixed, the deal is guaranteed to be more profitable than linear pricing if the cost of food* c_x *is low enough (the food profit margin is high enough):*

$$0 < c_x < \gamma_B \equiv \frac{v(a_B^*) - v(x^*)}{a_B^* - x^*}. \tag{17}$$

Proof. If all consumers are drink buyers then the relevant condition is $\Delta\pi_B(a_B^*) > 0$. If the population is a mix then the seller can always ignore drink non-buyers by setting $a = a_B^*$. In this case the condition for deal profitability is $\Delta\pi_M(a_B^*) > 0$. Using (7) and (12) both conditions can be written as (17). Thus, if the cost of food is below some $\gamma_B > 0$, then the deal is more profitable compared to linear pricing in both settings. ■

Since $v(x)$ is strictly concave in x and $v'(x^*) = p_x$ we can further conclude that $\gamma_B < p_x$, i.e. the restriction on food costs is non-trivial and a positive food profit margin is required for the deal advantage to be positive. Proposition 4.2 is illustrated in Figure (2). For low food costs ($c_x < \gamma_B$) the deal is strictly better than linear pricing. For high food costs ($c_x > \gamma_B$) the deal would be less profitable than linear pricing if the seller were to set $a = a_B^*$. However, by setting $a > a_B^*$ the seller ensures the same profit as under linear pricing.

When the population of consumers is a mix of drink buyers and non-buyers and the seller finds it optimal to target both types of consumers, the condition for deal profitability is:

$$[p_x - c_x] > \frac{\{p_y[1-\mu] + c_y\mu\}}{a_{NB}^* - x^*}. \tag{18}$$

The condition is similar to (16) requiring a high enough food profit margin. On the one hand, it seems to be easier to satisfy than (16) since the opportunity cost of giving out free drinks is now a weighted average of the price of drink and the cost of drink, $c_y < p_y$.

However, $a_{NB}^* < a_B^*$ making the ranking ambiguous. The next proposition states that with appropriate restrictions on the cost of food and other parameters, a high enough food profit margin exists.

Proposition 4.3 *When at least some consumers are drink non-buyers and the seller finds it optimal to attract all consumers, the deal is more profitable than linear pricing if* c_x *is low enough provided a parameter restriction on* c_y, p_y, d_{NB}, *and* μ *holds. The restriction requires that* c_y *and/or* p_y *are low enough, and/or that* d_{NB} *and/or* μ *are high enough:*

$$0 < c_x < \gamma_M \equiv \frac{v(a_{NB}^*) - v(x^*)}{a_{NB}^* - x^*} + \frac{d_{NB}}{a_{NB}^* - x^*} - \frac{\mu c_y + [1-\mu]p_y}{a_{NB}^* - x^*} \quad (19)$$

$$c_y \leq \frac{d_{NB} - [1-\mu]p_y}{\mu}. \quad (20)$$

Proof. The relevant condition is $\Delta\pi_M(a_{NB}^*) > 0$. Using (15) the condition can be written as (19).
It is possible that the sum of the last two terms in (19) is negative. Thus, without additional parameter restrictions the set of food costs for which the deal is more profitable can be empty ($\gamma_M < 0$). However, if (20) holds then the sum of these last two terms is positive and the deal is guaranteed to be more profitable than linear pricing for some positive food costs below $\gamma_M > 0$. Furthermore, the left-hand side of (20) is increasing in c_y while the right-hand side is increasing in d_{NB} and μ, and decreasing in p_y. ■

A special case of Proposition 4.3 is a population of only drink non-buyers ($\mu = 1$). Other things equal, this case corresponds to the highest γ_M as defined in (19). We denote this value of γ_M as γ_{NB}. Thus, for all mixed populations ($\mu < 1$) in settings where the seller targets all consumers ($a = a_{NB}^*$) the range of c_x that make the deal more profitable is smaller compared to a homogeneous population of drink non-buyers (i.e. $\gamma_M < \gamma_{NB}$).
The next proposition suggests that in some settings TBMB is more likely to be useful in a population containing consumers who would not buy the drink otherwise.

Proposition 4.4 *When at least some consumers are drink non-buyers, (20) holds, and the proportion of drink non-buyers is close to* 1 *(*$\mu \to 1$*) then the deal is more profitable than linear pricing for a wider range of* c_x *compared to when all consumers are drink buyers:* $\gamma_M > \gamma_B$.

Proof. If conditions (19) and (20) hold then the deal is more profitable than linear pricing in a population of all drink non-buyers ($\mu = 1$). Thus, as $\mu \to 1$ the seller finds it optimal to target all consumers (Proposition 4.1). Therefore, as $\mu \to 1$ the condition for deal profitability is given in (19). Since $a_B^* > a_{NB}^*$ and by strict concavity of $v(x)$, it follows that $\frac{v(a_{NB}^*) - v(x^*)}{a_{NB}^* - x^*} > \frac{v(a_B^*) - v(x^*)}{a_B^* - x^*}$. Thus, condition (20) ensures that $\gamma_M > \gamma_B$. ■

All three propositions are illustrated in Figure (3). Parameter values used to generate the figure are such that in the homogeneous populations as well as in mixed populations with 10 and 50 percent of drink buyers the deal is more profitable than linear pricing for a range of food costs (Propositions 4.2 and 4.3). Furthermore, $\gamma_{NB} > \gamma_M > \gamma_B > 0$. The inequality $\gamma_B > 0$ holds in general as stated in Proposition 4.2. Additionally, $\gamma_{NB} > \gamma_M$ from their definitions implying that the range of food costs where the deal is more profitable is smaller when the population of consumers is a mix rather than all drink non-buyers. The rest of the relationship does not necessarily hold for all parameter values. For the values chosen, however, $\gamma_{NB} > \gamma_B$, i.e. the deal remains more profitable than linear pricing at high c_x in a population of drink non-buyers even though in a population of drink buyers such high food costs would eliminate the deal advantage (Proposition 4.4). This result presents an interesting hypothesis for empirical testing. It suggests that TBMB is more likely to increase profits relative to linear pricing in settings where the consumers do not normally buy the drink. To put it differently, in a population of drink non-buyers, TBMB can increase profits even when the food profit margin is relatively low.

Additionally, $\gamma_M > \gamma_B$ for a similar result in a mixed population with 10 percent of drink buyers. In the mixed population with 50 percent of drink buyers γ_M would be below γ_B if the seller continued to target both types of consumers. However, she decides to ignore drink non-buyers at higher c_x (see Figure (1)) and γ_B becomes the upper bound on food costs at which the deal advantage is positive (Proposition 4.2).
The last proposition tells us that depending on parameter values one or both homogeneous populations (all drink buyers and all drink non-buyers) is better than a given mixed population.

Proposition 4.5 *An increase in profits from TBMB in a population containing both drink buyers and drink non-buyers is lower compared to an increase obtainable in one or both homogeneous populations.*

Proof. For a mixed population of consumers the seller may set $a = a_B^*$ or $a = a_{NB}^*$ depending on which is more profitable. Suppose setting $a = a_B^*$ is more profitable, i.e. the deal is made attractive to drink buyers only. Recall from (10) that $\Delta\pi_M\left(a_B^*\right) = \Delta\pi_B\left(a_B^*\right)[1-\mu]$, i.e. the highest increase in profits from the deal in such a population is a fraction of the increase in profits obtainable if this population were all drink buyers. Therefore, a homogeneous population of drink buyers is preferred to such a mixed population.

Now, suppose setting $a = a_{NB}^*$ is more profitable, i.e. the deal is made attractive to all consumers. Recall from (13) that $\Delta\pi_M\left(a_{NB}^*\right) = \Delta\pi_B\left(a_{NB}^*\right)[1-\mu] + \Delta\pi_{NB}\left(a_{NB}^*\right)\mu$, i.e. the highest increase in profits from the deal in such a population is a weighted average of profit increases in homogeneous populations with the threshold set at a_{NB}^*. If the two terms in the expression are different, then the average is smaller than the higher of the two terms. That is, if $\Delta\pi_B\left(a_{NB}^*\right) < \Delta\pi_{NB}\left(a_{NB}^*\right)$ then $\Delta\pi_M\left(a_{NB}^*\right) < \Delta\pi_{NB}\left(a_{NB}^*\right)$, i.e. the highest increase in profits in the mixed population is lower than the increase in profits when all consumers are drink non-buyers, the result we need to demonstrate. From the corresponding definitions this is indeed true: $\Delta\pi_B\left(a_{NB}^*\right) < \Delta\pi_{NB}\left(a_{NB}^*\right) \Leftrightarrow p_y > c_y$. Therefore, a homogeneous population of drink non-buyers is preferred to such a mixed population. ■

In terms of Figure (3) this result means that for any mixed population of consumers ($0 < \mu < 1$) the deal advantage line lies strictly below the maximum of the two lines corresponding to 0 and 100 percent of drink buyers. The result is not surprising and is due to adverse selection. With multiple types, the deal attracts the type of consumers who will benefit from it the most (drink buyers). Whether the seller decides to target only drink buyers at the expense of drink non-buyers ($a = a_B^*$) or settles for a pooling equilibrium ($a = a_{NB}^*$), the resulting profits are lower compared to one of the two homogeneous populations where adverse selection is not an issue. Note that this proposition does not imply that a particular homogeneous population is always better for the seller than any mixed population.

Finally, note that when the seller is a price taker, if TBMB is more profitable than linear pricing then it is Pareto-improving. Since the seller is a price taker in the markets for the bundle's components and the components are available for sale at those prices even when the seller offers the deal, the consumers are no worse off. Therefore, if the seller is better off, then TBMB is Pareto-improving. A counter-example to this proposition is a situation when the seller has market power. Such a seller is likely to change prices when the deal is offered at the expense of some consumer types. Thus, there is no guarantee that the consumer welfare is not reduced. Similarly, if the seller can prevent consumers from purchasing the components when the bundle is offered, the consumer welfare may decrease.

5. Conclusion and discussion

In this article we look at the practice of bundling a free gift (e.g., a drink) with a minimum threshold purchase of another good (food). We refer to the phenomenon as threshold-based mixed bundling (TBMB). Using TBMB a seller induces consumers to increase their purchases of food. We analyze the profitability of the practice assuming price-taking behavior. When the seller has no control over prices, the consumer welfare is weakly higher with TBMB.

Despite this result, under favorable conditions the seller can obtain higher profits employing TBMB compared to simple linear pricing. In particular, if the food is sold at a sufficiently high markup then the increase in profits from selling more food can outweigh the decrease in profits from giving out the drink for free. Another implication of the model is that when a large fraction of consumers would not normally buy the drink offering the deal can increase profits even when the food profit margin is relatively small.

The analysis of the paper provides some insights into the specifics of the deal offered by a chain of supermarkets in the San Francisco Bay Area. As part of the deal, consumers get a stamp for each 10 dollars spent in the supermarket. A threshold number of stamps can be redeemed for a frying pan and similar items. In this setting, the cookware is the "drink" and the composite purchases at the supermarket are the "food". It can be argued that due to competition the seller has little control over the price of "food" so that we can employ insights from our analysis of a price-taking seller. When deciding on the parameters of the deal the seller can select which good to use as the free gift. Picking one of the high-demand items available at the supermarket would imply that a non-trivial share of customers would be buyers of the good. To benefit from the deal in this case the margin of the composite "food" product has to be relatively high, a requirement that might be difficult to satisfy in the supermarket store business. In contrast, offering premium cookware of a lesser-known brand is more likely to make the deal profitable since the opportunity cost of giving it out as a free gift can be low. On the one hand, the premium status of the good, the correspondingly high retail price at which the cookware is sold during the promotion period, and the fact that a grocery store is typically not the place to purchase premium cookware means that only a small fraction of store customers would be buyers of the good. As a result, the cost of the good rather than its price is the applicable opportunity cost of offering the deal. Furthermore, it is likely that the cost of the cookware to the store is relatively low given the amount of promotion the cookware brand gets as a result of the deal. In light of these observations and the analysis of the paper the choices made by the seller are not surprising.

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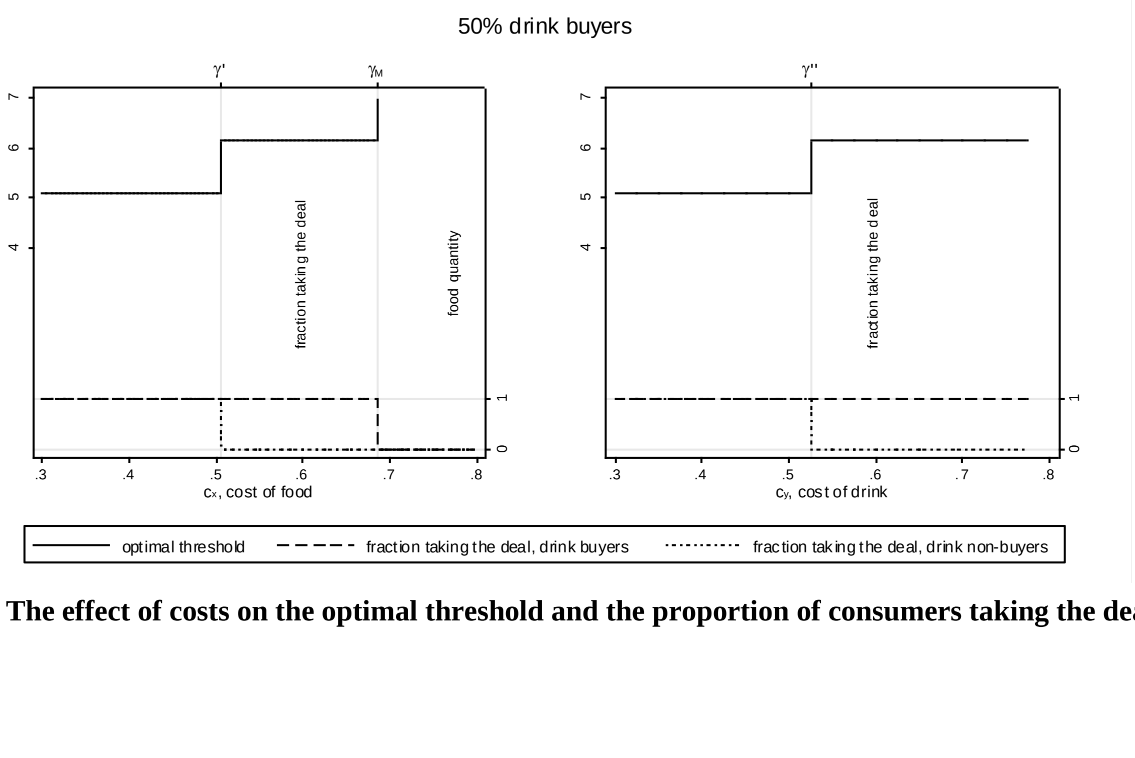

Figure 1: The effect of costs on the optimal threshold and the proportion of consumers taking the deal.

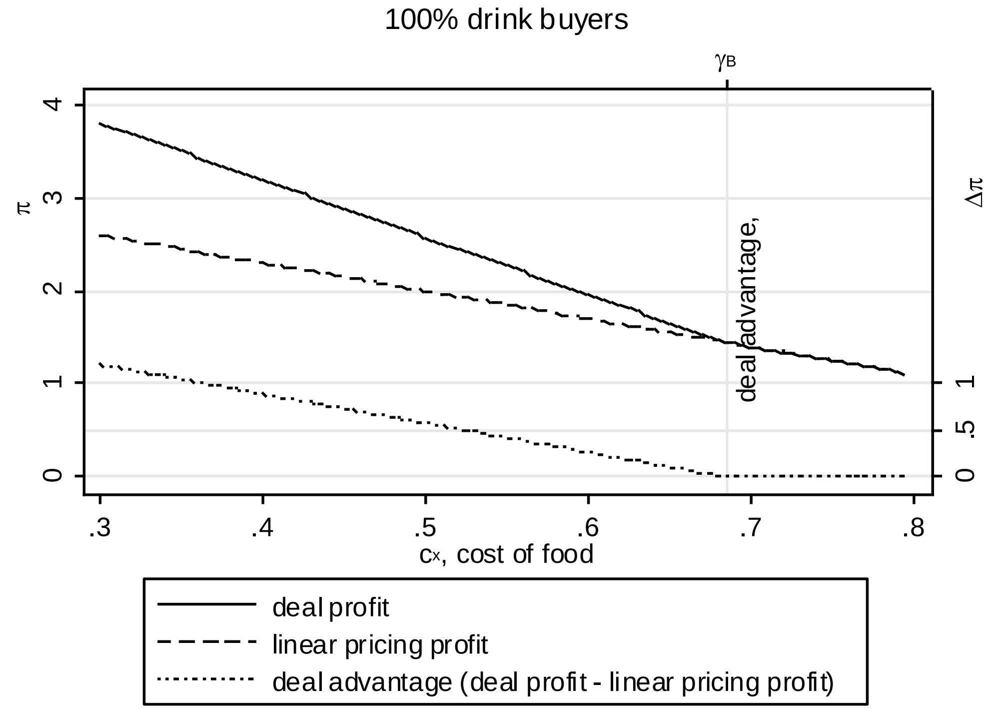

Figure 2: The effect of food costs on the seller's profits and the deal advantage.

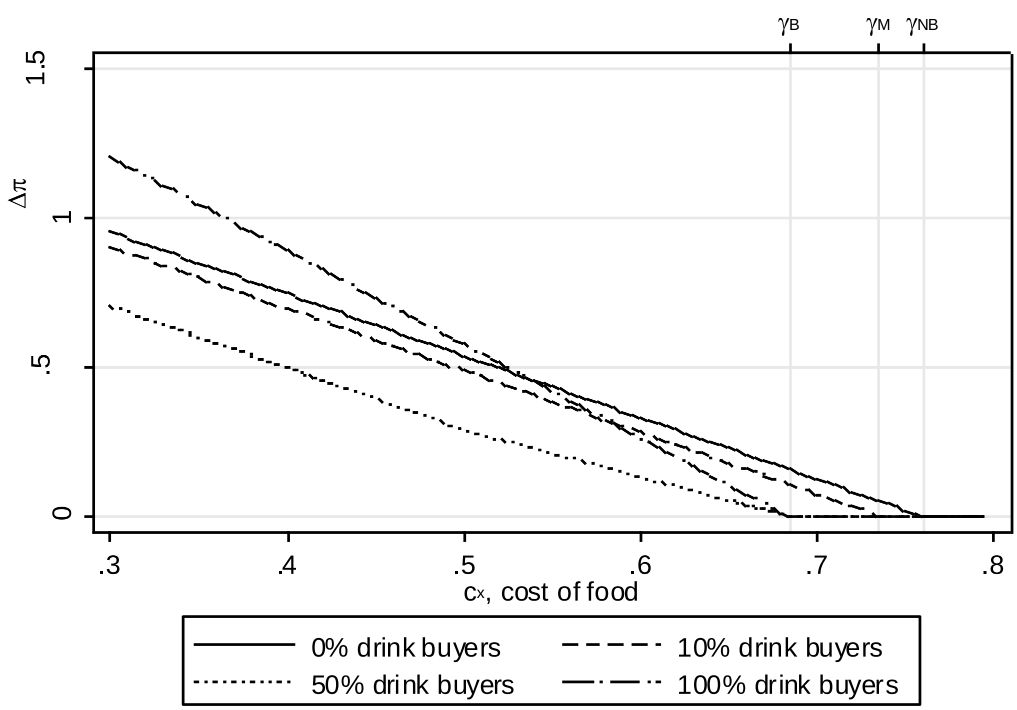

Figure 3: The effect of food costs on the deal advantage, different consumer populations.