Consumer Conformity and Vanity in Vertically Differentiated Markets

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Abstract

Consumers' choice of a good may be motivated by the wish to belong to some group or the search for exclusiveness. Such behavior gives rise to either positive or negative consumption externalities. Although, these externalities are very important in several markets, their impact on firms' quality choice in vertically differentiated markets has not been yet explored. We study firms' decisions in terms of prices and qualities when consumers' utility is affected by the number of consumers who buy the same quality and also partially affected by the number of consumers who buy close qualities. The market outcomes are significantly affected as the low quality firm may prefer to stay out of the market when the consumption externality is strong. Moreover, under some conditions on the strength of the consumption externality and on consumers' sensitivity to firms' quality difference, we prove that product differentiation may not be maximal as in the standard model.

Keywords: *Vertical differentiation, Consumption externalities, Price competition, Quality choice*

1. Introduction

The decision to buy a good depends not only upon the intrinsic quality of this good but may also be positively or negatively affected by the consumption choice of other consumers. The observation of some markets like the markets for beverages, electronic devices, sports, entertainments... suggests that they are characterized by conformity in consumers' behavior. Conformity is generally defined as the tendency that people have to comply with the group norm (Lascu and Zinkhan, 1999). People usually conform because they want to be liked and accepted by a certain group. Some other markets like the markets for luxury goods: perfumes, sport cars... are characterized by vanity in consumers' behavior. In this case, consumers have strong individualistic values and need exclusiveness and prestige. While conformity gives rise to positive consumption externalities, vanity implies negative consumption externalities[1].

The objective of this paper is to investigate firms' strategic decisions in the framework of a vertically differentiated market when consumers' behavior is characterized either by conformity or by vanity. We mainly focus on how the existence of such consumption externalities affects firms' decisions. How are firms' prices and market shares compared to the standard vertical differentiation model without consumptions externalities? Does maximal differentiation still occur?

The originality of the paper is twofold. First, our model allows to understand one important firms' decision which is the choice of the quality to produce. To our knowledge, when consumption externalities are introduced into a product differentiation model, only the price formation has been studied while the choice of qualities or locations has been neglected by assuming exogenous locations or qualities. Second, we suppose that a consumer's utility is affected by the number of consumers who buy the same quality and is also "partially" affected by the number of consumers who buy close qualities. More precisely, we assume that the similarity degree between consumers depends on how close the qualities they choose are. The closer the qualities, the higher the similarity degree and therefore the more important the consumption externality. Hence, we clearly combine the vertical differentiation model and the consumption externality model. This model leads as will be shown later to new results about firms' decisions.

[1] See Veblen (1899), Duesenberry (1949) and Leibenstein (1950) for a more detailed explanation of such behaviors.

Only some recent papers combine product differentiation and consumption externalities. Grilo et al (2001) analyze firms' pricing strategy when a consumer behavior is characterized either by conformity or by vanity. They propose a spatial model of product differentiation that may deal with both horizontal and vertical differentiation depending on exogenously fixed firms' locations. Consumers belong to the same group if they buy the product of the same firm. Therefore, even if firms' products are very similar (firms are very close in the product space), their respective customers do not affect the utility of each other. Contrary to Grilo et al (2001), we do not compel consumers to buy exactly the same quality to be perceived as similar. It seems more reasonable to assume that consumers buying different but close qualities also have some impact on each other.

Ghazzai and Lahmandi-Ayed (2009) study a similar model where consumers' behavior is characterized by conformity and where consumers belong to the same social network if they buy products exhibiting close characteristics. A compatibility interval is exogenously fixed and defines how close products' characteristics should be to be perceived by consumers as compatible i.e. consumers buying them belong to the same group. They study a game where an incumbent produces the highest possible quality and where a potential entrant must decide to be either compatible or incompatible with the incumbent depending on whether it chooses a quality in the compatibility interval or not. Conformity in their model is a 0 or 1 variable as the products are either fully compatible or fully incompatible. We consider a more general model where similarity depends more smoothly on the distance between firms' qualities.

Jonard and Shenk (2004) study a circular model of horizontal differentiation. They suppose that firms are closer in the product space if they are compatible. They study the strategy of two incumbent firms facing the threat of a potential entrant. They find that strong externalities can favor entry, as merging networks and accommodating entry can be preferred by the incumbents. Compatibility and incompatibility in their model imply exogenously given locations for firms and as in Grilo et al (2001) and Ghazzai and Lahmandi-Ayed (2009), compatibility is a 0 or 1 variable.[2]

The paper shows that if vanity characterizes consumers' behavior then maximal differentiation always occurs at equilibrium like in the standard model. However, prices are higher and vanity decreases the advantage of the high quality firm in terms of market shares.

Under conformity, firms' decisions are more distinct from the standard model. If consumers are very conformist then only the high quality firm is active at price equilibrium and only one social group exists as consumers buy the same quality. This allows to confirm casual observation that suggests that when conformity characterizes some markets like garments, entertainments, sports...only a small number of producers share the market and there is an emergence of a common standard. If consumers are not very conformist, both firms are active at price equilibrium but price competition is fiercer and results in lower equilibrium prices than in the standard vertical model.

When choosing qualities, one firm always chooses the highest possible quality. The other firm either chooses the lowest possible quality and maximal differentiation results at equilibrium or it chooses a quality inside the interval of qualities and differentiation between firms' products is not maximal. The low quality firm's choice depends on the strength of the consumption externality and on the magnitude by which a change in the difference between firms' qualities reverberates on the perception of similarity between consumers. In fact, if an increase in the difference between firms' qualities highly decreases consumers' similarity degree (linear similarity function), maximal differentiation always results at equilibrium. However, if an increase in the difference between firms' qualities only slightly reverberates on the similarity degree between consumers (quadratic similarity function) then the low quality firm may chooses an interior solution.

In the rest of the paper, section 2 describes the model. In section 3, we characterize the demand of each firm.

[2] There is a wide body of literature addressing network goods for which conumers' preferences depend on the clientele size. As in the case of conformity, the willingness to pay for such a good increases with the number of customers who buy the same good. More precisely, the utility of a consumer increases with the number of customers who are connected to the same network. Note however that the reasons for this externality is technological rather that social. See Economides and Flyer (1998), Farrell and Saloner (1985) and (1992), Katz and Shapiro (1985)...

We determine the price equilibrium in section 4 and firms' quality choice for two particular similarity functions in section 5. We conclude in section 6. Appendix 1 presents some results with a general similarity function. All proofs are given in Appendix 2.

2. The Model

We consider a linear model of vertical differentiation with consumption externalities. There are two firms, each producing one quality of a particular product. Each firm i chooses its quality in the segment $[\underline{q}, \overline{q}]$. Let p_i, $i=1$ or 2 be the price set by firm i, q_i its quality, N_i its demand and y_i the externality affecting firm i's consumers or firm i's network size. Without loss of generality, we assume that firm 1 produces a higher quality than firm 2 i.e. $q_1>q_2$.[3] Let $\Delta q = q_1-q_2$ denotes the difference between firms' qualities.

Consumers are characterized by their intensity of preference for quality θ. They are uniformly distributed on $[\underline{\theta}, \bar{\theta}]$ with the density function $g(\theta) = \frac{1}{\bar{\theta}-\underline{\theta}}$. Thus, the number of consumers is normalized to 1. The utility of a consumer θ who buys a unit of product from firm i is given by:

$$u_i(\theta)=K- p_i+ \theta\, q_i+ \omega\, y_i \qquad (1)$$

In the utility function (1), K stands for the gross intrinsic utility a consumer derives when consuming one unit of the product. We assume that K is sufficiently large to ensure that all consumers prefer buying rather than not buying [4]. The next two terms represent the standard utility function in a vertically differentiated market introduced by Mussa and Rosen (1978). The last term represents the consumption externality. The intensity of the consumption externality is given by ω. When $\omega<0$, consumers' behavior is characterized by vanity as a consumer's utility decreases with respect to the network size. The larger $|\omega|$, the more individualistic are consumers. When $\omega>0$, consumers' behavior is characterized by conformity. The larger ω, the more conformist are consumers. The network size y_i depends on how close the firms' qualities are. More precisely, the network size of firm i is given by:

$$y_i=N_i+ f(\Delta q)\, N_j \qquad (2)$$

Equation (2) links product differentiation and consumption externalities. In fact, the function $f(\Delta q)$ may be considered as a similarity function. It satisfies the following conditions:

$$f(\overline{q}-\underline{q}) = 0, f(0) = 1 \; and \; f^{'}(\Delta q) < 0 \; for \; all \; \Delta q \in [0, \overline{q} - \underline{q}] \qquad (3)$$

The network size of firm i consists of its own sales N_i to which we add a fraction of its competitor's sales N_j i.e. consumers are not only affected by the number of consumers who buy the same quality but also "partially" affected by the number of consumers who buy close qualities. Condition (3) implies that firms are fully incompatible i.e. customers buying their products do not belong to the same social group if the difference between their qualities is maximal ($\overline{q} - \underline{q}$). In this case, the network size of each firm is equal to the firm's sales N_i . Firms are fully compatible i.e. customers buying their products belong to the same social group if they produce the same quality. In this case both firms have the same network size $y_1=y_2=1$. If $0< \Delta q <\overline{q} - \underline{q}$, firms are partially compatible. The network size of each firm depends on the similarity function $f(\Delta q)$. The closer the firms' qualities, the larger the similarity degree $f(\Delta q)$ and the more important the consumption externality. Hence, we do not compel consumers to buy exactly the same quality to be perceived as similar. Their similarity degree depends on how close the qualities they choose are.

The objective of the paper is to characterize the subgame perfect Nash equilibrium of the game described by the following steps:

[3] The analysis of firms' quality choice shows that firms will not choose identical qualities.

[4] This assumption is discussed in the last paragraph of section 4.

1. Firms choose their qualities choosing at the same time the similarity degree between their customers.[5]
2. Firms set prices simultaneously.
3. Each consumer decides which quality to buy.

The game is solved by backward induction. We first determine the demand of each firm as a function of p_1, p_2 and Δq. Then, we find the price equilibrium and finally the quality choice.
We assume that $\bar{\theta} > 2\underline{\theta}$. This is the condition to have two active firms at equilibrium in a standard vertical differentiation model. As we will prove later, activity of both firms in our model requires an additional condition on qualities under conformity.

To focus on the objective of this paper, marginal production costs are set equal to zero for both firms.[6]

3. Demand Characterization

We determine consumers' choice depending on firms' prices and qualities. We may assume that consumers play a Nash equilibrium at the third stage of the game. In fact, each consumer has to choose the quality that maximizes his/her utility taking as given the decisions of all other consumers. We prove that in any Nash equilibrium consumers group according to their type θ. In fact, each firm's demand is necessarily an interval and intervals are ordered as in the standard case: The lowest θ buy the lowest quality and the highest θ buy the highest one[7]. Thus, there are three possible types of Nash equilibria:

- Only firm 1 is active: N_1=1 and N_2=0. This is called a type 1 Nash equilibrium.
- Only firm 2 is active: N_1=0 and N_2=1. This is called a type 2 Nash equilibrium.
- Both firms have positive sales. This is called a type 3 Nash equilibrium. As consumers group according to their type $\theta, N_2 = \frac{\hat{\theta}-\theta}{\bar{\theta}-\underline{\theta}}$ and $N_1 = \frac{\bar{\theta}-\hat{\theta}}{\bar{\theta}-\underline{\theta}}$. The marginal consumer $\hat{\theta}$ indifferent between q_1 and q_2 is necessarily given by $u_1(\hat{\theta}) = u_2(\hat{\theta})$, which is equivalent to

$$\hat{\theta} = \frac{(p_1 - p_2)(\bar{\theta} - \underline{\theta}) - \omega(\bar{\theta} + \underline{\theta})(1 - f(\Delta q))}{\Delta q(\bar{\theta} - \underline{\theta}) - 2\omega(1 - f(\Delta q))} \quad (4)$$

  if $\Delta q(\bar{\theta} - \underline{\theta}) - 2\omega(1 - f(\Delta q)) \neq 0$.[8]

The price pairs for which $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$depend on the sign of the denominator of (4). If $\Delta q(\bar{\theta} - \underline{\theta}) - 2\omega(1 - f\Delta q<0$ which may only happen under conformity ($\omega>0$), we can easily prove that we have multiple equilibria at the last stage of the game. To rule out the possibility of multiple equilibria under conformity, we assume the following:

$$\text{Under conformity i.e. when } \omega > 0, f'(\Delta q) > -\frac{(\bar{\theta}-\underline{\theta})}{2\omega} \text{ for all } \Delta q \in (0, \bar{q} - \underline{q}] \quad (5)$$

By condition (5), we impose a lower bound on the curvature of the similarity function $f(\Delta q)$ when consumers' behavior is characterized by conformity. We ensure that the similarity degree $f(\Delta q)$ does not drop too fast with respect to the firms' quality difference Δq. We also link by condition (5) consumers' sensitivity to firms' quality difference represented by $f'(\Delta q)$ and the intensity of the consumption externality ω. The higher ω, the higher the lower bound imposed on $f'(\Delta q)$ and the flatter the curve of $f(\Delta q)$. Hence, if consumers care too much about the size of their network than they are less sensitive to quality difference.

For fixed prices and qualities, we determine in lemma (1) under which conditions the different Nash equilibria prevail.

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[5] Recall that the similarity degree depends on the difference between firms' qualities.

[6] The analysis holds when marginal costs are constant w.r.t. quantity and quality.

[7] The proof is given in Ghazzai and Lahmandi-Ayed (2009). Baake and Boom (2001) also obtain the same result. See also Grilo et al (2001) for similar results with a different consumption externality and see Gabszwicz and Thisse (1979) and (1980) and Shaked and Sutton (1982) and (1983) for similar results without network effects.

[8] If $\Delta q(\bar{\theta} - \underline{\theta}) - 2\omega(1 - f(\Delta q)) = 0$, a type 3 Nash Equilibrium never exists. Only type 1 or type 2 Nash equilibria may prevail.

Lemma 1 *Three types of Nash equilibria may exist:*

- *A type 1 Nash equilibrium, where only firm 1 is active, exists if and only if* $p_1\text{-}p_2 \leq \underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big)$.
- *A type 2 Nash equilibrium, where only firm 2 is active, exists if and only if* $p_1\text{-}p_2 \geq \bar{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big)$.
- *A type 3 Nash equilibrium, where both firms are active, exists if and only if* $\underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) < p_1\text{-}p_2 < \bar{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big)$.

Note that in the last point of lemma (1), when consumers' behavior is characterized by conformity, condition (5) ensures that $\bar{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) > \underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big)$ under conformity[9]. Under vanity, the previous inequality is always true as ω is negative.

Using lemma (1), firms' demands are given by equations (6) and (7).

$$N_1 = \begin{cases} 1 & if \quad p_1 \leq p_2 + \underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) \\ \dfrac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}} & if \quad p_2 + \underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) < p_1 < p_2 + \bar{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) \\ 0 & if \quad p_1 \geq p_2 + \bar{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) \end{cases} \tag{6}$$

$$N_2 = \begin{cases} 1 & if \quad p_2 \leq p_1 - \bar{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) \\ \dfrac{\hat{\theta} - \underline{\theta}}{\bar{\theta} - \underline{\theta}} & if \quad p_1 - \bar{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) < p_2 < p_1 - \underline{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) \\ 0 & if \quad p_2 \geq p_1 - \underline{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) \end{cases} \tag{7}$$

where $\hat{\theta}$ characterizes the consumer indifferent between q_1 and q_2. The expression of $\hat{\theta}$ is given by (4). [10]

4. Price Equilibrium

We now solve the second step of the game to find firms' pricing decisions for a given quality difference Δq. Under conformity, we prove that the necessary condition $\bar{\theta} > 2\underline{\theta}$ that ensures the activity of two firms in the standard vertical differentiation model without consumption externalities is no more sufficient. A new condition on firms' qualities is needed. Two cases may emerge under conformity at price equilibrium: either two firms are active or only firm 1 is active. In the latter case, the network becomes relatively more important in consumers' decision, price competition is tough because firms have close qualities and compels firm 2 to stay out of the market. Under vanity, however, there is always room for two firms in the market and price competition is relaxed. Lemma (2) gives the price equilibrium and proposition (1) summarizes our findings about the price stage of the game.

In this section, we will say that consumers' behavior exhibits:

- Vanity if $\omega < 0$.
- Weak conformity if $\omega > 0$ and $\Delta q > \frac{3\omega(1-f(\Delta q))}{\bar{\theta}-2\underline{\theta}}$.
- Strong conformity if $\omega > 0$ and $\Delta q \leq \frac{3\omega(1-f(\Delta q))}{\bar{\theta}-2\underline{\theta}}$

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[9] If condition (5) is not satisfied i.e when ω is large, then for some Δq, we may have $\Delta q(\bar{\theta} - \underline{\theta}) - 2\omega\big(1 - f(\Delta q)\big) < 0$ and consequently the last condition of lemma (1) becomes $\bar{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) < p_1\text{-}p_2 < \underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big)$. We easily prove then that there is a multiplicity of equilibria at the last stage of the game. In fact, the conditions for the existence of the three types of equilibria can be simultaneously satisfied. We also prove that there is no room for two active firms in the market at the price stage of the game. The full proof can be provided upon request.

[10] The proof of equations (6) and (7) is immediate from the conditions of lemma (1). A very similar proof is provided in Ghazzai and Lahmandi-Ayed (2009).

Lemma 2 *Two firms are active either under vanity or under weak conformity. The equilibrium prices and profits are given by:*

$$\begin{cases} p_1 = \frac{2\bar{\theta} - \underline{\theta}}{3}\Delta q - \omega(1 - f(\Delta q)) \\ p_2 = \frac{\bar{\theta} - 2\underline{\theta}}{3}\Delta q - \omega(1 - f(\Delta q)) \end{cases}$$

$$\begin{cases} \pi_1 = (\frac{2\bar{\theta} - \underline{\theta}}{3}\Delta q - \omega\big(1 - f(\Delta q)\big))^2 \frac{1}{\Delta q\big(\bar{\theta} - \underline{\theta}\big) - 2\omega(1 - f(\Delta q))} \\ \pi_2 = (\frac{\bar{\theta} - 2\underline{\theta}}{3}\Delta q - \omega\big(1 - f(\Delta q)\big))^2 \frac{1}{\Delta q\big(\bar{\theta} - \underline{\theta}\big) - 2\omega(1 - f(\Delta q))} \end{cases}$$

Only firm 1 is active under strong conformity. The equilibrium prices and profits are given by:

$$\begin{cases} p_1 = \underline{\theta}\Delta q - \omega(1 - f(\Delta q)) \\ p_2 = 0 \end{cases}$$

$$\begin{cases} \pi_1 = \underline{\theta}\Delta q - \omega(1 - f(\Delta q)) \\ \pi_2 = 0 \end{cases}$$

Under vanity, the equilibrium prices increase with respect to the intensity of the consumption externality $|\omega|$ and are higher than the prices in a standard vertical differentiation model. The more individualistic are consumers, the more relaxed is the competition between firms on the market shares as consumers prefer firms with a small number of customers. Firms can charge a higher price as a counterpart of a small clientele. From straightforward calculations, we check that the market share of the high quality firm decreases with respect to $|\omega|$ and the market share of the low quality firm increases with respect to $|\omega|$. As in the standard case, the high quality firm has always a bigger market share than the low quality firm. However, this advantage in terms of market share is less important than in the standard case. This means that when consumers want some exclusivity they are willing to buy a lower quality as it allows to decrease the negative effect of the consumption externality.

Under weak conformity, the equilibrium prices decrease with respect to the intensity of the consumption externality ω and are smaller than the prices in a standard vertical differentiation model without consumption externalities. The more important the consumption externality intensity, the fiercer the competition between firms on the market shares. Prices decrease as the price competition is tougher[11]. We also have from straightforward calculations that the market share of the high quality firm increases with respect to the intensity of the consumption externality ω. Compared to the standard model, conformity increases the advantage of the high quality firm in terms of market shares.

As the desire for conformity keeps rising, the high quality firm can capture all the demand (strong conformity). In this case, the quality difference is small relative to the consumption externality intensity. In fact, the high quality firm with an appropriate choice of its price (a limit price) can compel the low quality firm to stay out of the market. As consumers evaluate the products mainly in terms of their network size, monopolization is the optimal pricing strategy and the best choice for consumers is to buy the same product to make sure they belong to the same group. Note that in this case firm 1's price depends on firms' quality difference Δq and consequently on firm 2's quality choice. When for any quality choice firm 2 is better off when it stays out of the market and as firm 1's profit increases with respect to q_1, any pair of qualities entailing firm 1 choosing $\bar{q}$ is an equilibrium pair.

Proposition (1) compares the outcomes of the second stage of the game in terms of prices and market shares to the standard model without consumption externalities.

Proposition 1 *Compared to the standard model, vanity yields to higher prices and decreases the market share of the high quality firm. However, as in the standard case, the high quality firm has a bigger market share than the low quality firm. On the contrary, weak conformity leads to lower prices and strengthens the advantage of the high quality firm in terms of market shares. This firm even captures the demand of all consumers under strong conformity.*

[11] This effect was also described in Baake and Boom (2001) in the framework of a vertically differentiated market and Navon, Shy and Thisse (1995) in the framework of a horizontally differentiated market.

Note, however, that the results in lemma (2) and proposition (1) strongly depend on the assumption that the market is covered i.e. all consumers buy one unit of the product. Under conformity, the assumption of a covered market is not crucial since conformity leads to lower prices than in the standard model and consequently increases the utility of purchasing. The parameters of the model may therefore be easily adjusted so that the market is endogenously covered at equilibrium. The conditions that ensure that the market is covered in the standard case also ensure that the market is covered under conformity. In Wauthy (1996), it has been proved that if $\frac{\overline{\theta}}{\underline{\theta}} \epsilon (2{,}5)$ and $\underline{q} > \overline{q}\,\frac{\overline{\theta}-2\underline{\theta}}{\overline{\theta}+\underline{\theta}}$ then the market is endogenously covered at equilibrium without a corner solution. These two conditions are sufficient to ensure that the market is covered without a corner solution under conformity as the utility of purchasing is higher under conformity. Under vanity, prices are higher than in the standard case. Thus, the utility of purchasing decreases compared to the standard case. Without the assumption of a covered market, some consumers may decide not to buy. These consumers are necessarily those with low preferences for quality.

5. *Quality Choice*

We now study the quality choice of firms which corresponds to the solution of the first step of the game. We consider two particular similarity functions: a linear similarity function and a quadratic similarity function. When the similarity function is linear, the perceived similarity degree between firms' customers proportionally decreases with respect to firms' quality difference. With a quadratic similarity function, consumers are supposed to be less sensitive to quality difference than in the linear case because when the quality difference increases, the decrease in the perceived similarity degree is less important than with a linear similarity function. We prove that under vanity maximal differentiation occurs in both cases. Under conformity, when two firms are active and when the similarity function is quadratic, an interior solution may exist i.e. differentiation between firms' products is not maximal at equilibrium. The quality choice with a general similarity function could not be completely characterized. Appendix 1 provides some partial results about quality choice with a general similarity function.

5.1. Quality Choice with a Linear Similarity Function

Taking into account condition (3), the similarity function is given by:

$$f(\Delta q) = 1 - \frac{\Delta q}{\overline{q} - \underline{q}} \qquad (8)$$

As mentioned previously, the similarity degree between consumers proportionally decreases with respect to firms' quality difference. Under conformity, condition (5) becomes $\overline{q} - \underline{q} > \frac{2\omega}{\overline{\theta}-\underline{\theta}}$.[12] The firms' quality decisions are given by proposition (2).

Proposition 2 *When the similarity function is linear, we have the following results:*

- *Under vanity, firm 1 chooses $\overline{q}$, firm 2 chooses $\underline{q}$ and maximal differentiation results at equilibrium.*
- *Under conformity, if $\omega < \frac{\overline{\theta}-2\underline{\theta}}{3}(\overline{q} - \underline{q})$, firm 1 chooses $\overline{q}$, and firm 2 chooses $\underline{q}$. Maximal differentiation results at equilibrium. Otherwise, only firm 1 is active and produces $\overline{q}$.*

Two effects influence the quality choice of firms: the price effect and the network effect. The higher the difference between firms' qualities, the more relaxed the price competition and the weaker the network externality. Under vanity, as consumers prefer small networks, both effects favor maximal differentiation. Under conformity, these two effects are opposite as a weak network externality decreases consumers' valuation of a product. Firm 2 is active only if the intensity of the consumption externality ω is not very large (weak conformity). When active, Firm 2 always chooses the lowest possible quality thus maximizing the differentiation between firms' products as in the standard vertical model. Here, consumers are very sensitive to quality difference. The change in the perception of the similarity degree between consumers is as fast as the change in product differentiation.

[12] Recall that if condition (5) is not satisfied, we have two possible equilibria under conformity at the last stage of the game where only one firm is active.

When the similarity function is linear, the price effect always outweighs the network effect. Maximal differentiation occurs and two social groups exist at equilibrium.

5.2. Quality Choice with a Quadratic Similarity Function

A similarity function that satisfies condition (3) is given by:

$$f(\Delta q) = 1 - \frac{\Delta q^2}{(\overline{q} - \underline{q})^2} \tag{9}$$

Under conformity, condition (5) is equivalent to $\overline{q} - \underline{q} > \frac{4\omega}{\overline{\theta} - \underline{\theta}}$. In their perception of similarity, consumers are less sensitive to quality difference than with a linear similarity function.

Let us denote by $\Lambda = \sqrt{\frac{\frac{11}{3}\overline{\theta} + \frac{5}{3}\underline{\theta}}{\overline{\theta} - \underline{\theta}}}$. [13] Firms' quality choices are given by proposition (3).

Proposition 3 *When the similarity function is quadratic, we have the following results:*

- *Under vanity, firm 1 chooses $\overline{q}$, firm 2 chooses $\underline{q}$ and maximal differentiation results at equilibrium.*
- *Under conformity, If $\omega > \frac{3-\Lambda}{8}\left(\overline{q} - \underline{q}\right)\left(\overline{\theta} - \underline{\theta}\right)$ then firm 1 chooses $\overline{q}$ and firm 2 chooses $\overline{q} - (3 - \Lambda)\frac{\left(\overline{q}-\underline{q}\right)^2\left(\overline{\theta}-\underline{\theta}\right)}{8\omega} > \underline{q}$. Thus, quality differentiation is not maximal. If $\omega \leq \frac{3-\Lambda}{8}\left(\overline{q} - \underline{q}\right)\left(\overline{\theta} - \underline{\theta}\right)$, firm 1 chooses $\overline{q}$, firm 2 chooses $\underline{q}$ and quality differentiation is maximal.*

Under vanity, maximal differentiation always occurs as maximal differentiation relaxes price competition and also decreases the consumption externality. Under conformity, two equilibria may emerge:

- When the conformity desire is strong enough (large enough), both firms are active and product differentiation is not maximal.
- When the conformity desire is weak (small ω), both firms are active and product differentiation is maximal.

When the similarity function is concave as given by function (9), firm 2 can slightly decrease its quality and this decrease will not completely reverberate on the perceived similarity degree and thus on the consumption externality. In fact, firm 2 can relax price competition by decreasing its quality and keep at the same time the consumption externality sufficiently high. The choice of an interior solution by firm 2 requires a strong enough consumption externality intensity ω. This is equivalent to say that the quality segment must not be very large compared to the intensity of the consumption externality and therefore firm 2 can not relax price competition substantially by choosing maximal differentiation. The concavity of the similarity function and a sufficiently strong ω (but not too strong)[14] lead jointly to the choice of an interior solution where firm 2 does not choose maximal differentiation and therefore does not relax completely price competition but compensates the price effect by the gain in the network size. If ω is small, maximal differentiation is the optimal solution of the game as the price effect outweighs the network effect.

Since the results are derived under specific similarity functions, one may wonder about their generality. Our intuition is that under vanity and with the assumption of a covered market, maximal differentiation will result because of a relaxed price competition and a lower consumption externality. Under conformity two factors may favor the choice by the low quality firm of a quality inside the quality segment: The intensity of the consumption externality ω and the concavity of the similarity function. In fact, ω must be neither very weak nor very strong. If ω is small, it is better to choose maximal differentiation as the consumption externality is weak.

[13] Using the fact that $\overline{\theta} > 2\underline{\theta}$, we easily prove that Λ is less than 3.

[14] From condition (5), we have that $\omega < \frac{\left(\overline{q}-\underline{q}\right)\left(\overline{\theta}-\underline{\theta}\right)}{4}$.

If ω is very large than the low quality firm will be excluded from the market. Moreover, as we have seen from the two particular cases studied above, the concavity of the similarity function seems to play an important role in the choice of the low quality firm. The more concave the similarity function, the less sensitive consumers are to a change in firms' quality difference Δq and the lower the decrease in the similarity degree and in the consumption externality will be. Thus, the low quality firm can slightly decrease its quality and beneficiate at the same time from sufficiently large network. In fact, the similarity function has to be "sufficiently" concave in order to favor the choice of an interior quality by the low quality firm.

6. Conclusion

We have shown that the consideration of consumption externalities into a vertical differentiation model has a significant impact on the market outcomes. Under conformity, we proved that firms' prices are lower than prices in the standard case and conformity increases the advantage of the high quality firm in terms of market shares. If the consumption externality intensity is very high, only one firm is active at price equilibrium. Otherwise, two firms are active. Firms' quality choice depends on the similarity function and on the intensity of the consumption externality. Unlike the standard case, maximal differentiation does not always occur. When the consumption externality is strong enough (but not very strong) and when consumers are not very sensitive to a change in firms' quality difference, differentiation between firms' product is not maximal. Under vanity, we have proved that as in the standard case firms choose maximal differentiation. However, prices are higher than in the standard case and the advantage of the high quality firm in terms of market shares is reduced.

There are different ways to improve the obtained results: First, we can consider a game with a general similarity function. The difficulty in finding the equilibrium configurations comes from the non-obvious analysis of the firms' profits. Then, we may consider the case where the number of firms in the market is endogenous. Finally, we may study the case where individuals do not react to all other individuals choices but only react to the consumption of some specific individuals.

Appendix 1

Price Equilibrium and Quality Choice with a General Similarity Function

In this section, we provide some results about the price equilibrium and the quality choice with a general similarity function. The price equilibrium was fully characterized is section 4. Proposition (4) provides sufficient conditions on the curvature of the similarity function $f(\Delta q)$ that ensure the activity of either two firms or only one firm at price equilibrium under conformity. We provide in proposition (5) the necessary and sufficient conditions on the similarity function $f(\Delta q)$ that lead to maximal differentiation.

Proposition 4 *Under conformity, if* $f'(\Delta q) > -\frac{\bar{\theta}-2\underline{\theta}}{3\omega}$ *for all* $\Delta q \in (0.\bar{q}-\underline{q}]$ *then two firms are active at price equilibrium. If* $f'(\Delta q) < -\frac{\bar{\theta}-2\underline{\theta}}{3\omega}$ *for all* $\Delta q \in (0.\bar{q}-\underline{q}]$ *then only one firm is active at price equilibrium. The equilibrium prices and profits are given by lemma (2).*

Proof. Immediate. Denote by $g(\Delta q) = \Delta q - \frac{3\omega}{\bar{\theta}-2\underline{\theta}}(1-f(\Delta q))$. We have that $g'(\Delta q) = 1 + \frac{3\omega}{\bar{\theta}-2\underline{\theta}} f'(\Delta q)$. If $f'(\Delta q) > -\frac{\bar{\theta}-2\underline{\theta}}{3\omega}$ then $g'(\Delta q) > 0$ and $g(\Delta q) > g(0) = 0$ for all $\Delta q \in (0.\bar{q}-\underline{q}]$. ∎

From proposition (4), we conclude that if the similarity degree decreases too quickly with respect to quality difference then only one firm can be active at price equilibrium. In this case, consumers are very sensitive to firms' quality difference and consumers who buy different qualities have only a weak effect on each other. Therefore, consumers buy the same quality and only one firm is active at price equilibrium.

Proposition 5 *When two firms are active, they choose maximal differentiation i.e. firm 1 chooses* $\bar{q}$ *and firm 2 chooses* $\bar{q}$ *under vanity or under conformity if and only if:*

$$2\frac{1+\frac{3\omega f'(\Delta q)}{2\bar{\theta}-\underline{\theta}}}{\Delta q - \frac{3\omega(1-f(\Delta q))}{2\bar{\theta}-\underline{\theta}}} > \frac{1+\frac{2\omega f'(\Delta q)}{\bar{\theta}-\underline{\theta}}}{\Delta q - \frac{2\omega(1-f(\Delta q))}{\bar{\theta}-\underline{\theta}}} \qquad (10)$$

and

$$2\frac{1+\dfrac{3\omega f'(\Delta q)}{\bar{\theta}-2\underline{\theta}}}{\Delta q-\dfrac{3\omega(1-f(\Delta q)}{\bar{\theta}-2\underline{\theta}}} > \frac{1+\dfrac{2\omega f'(\Delta q)}{\bar{\theta}-\underline{\theta}}}{\Delta q-\dfrac{2\omega(1-f(\Delta q)}{\bar{\theta}-\underline{\theta}}} \qquad (11)$$

When equation (10) is met, firm 1's profit is increasing with respect to the quality difference Δq. Therefore, firm 1 will choose to produce $\bar{q}$. When equation (11) is satisfied, firm 2's profit is increasing with respect to Δq and firm 2 will produce $\underline{q}$. The market outcome corresponds to maximal differentiation as in the standard case when both conditions (10) and (11) are met.

Note that when the similarity function is linear these two conditions are always satisfied and firms' profits are increasing with respect to the quality difference. When the similarity function is concave, we prove that under conformity condition (10) is satisfied and firm 1 prefers maximal differentiation but condition (11) is not necessarily satisfied and maximal differentiation may not occur. Under vanity and given a concave similarity function, we prove that condition (11) is satisfied and firm 2 prefers maximal differentiation but firm 1 may not prefer maximal differentiation as condition (10) is not necessarily satisfied.

Appendix 2

Proof of Lemma 1. We study the best strategy of a consumer θ depending on the choices of other consumers. We have to distinguish three cases as we have three possible types of Nash equilibria.

First Case: Type 1 Nash equilibrium

Consider some consumer $\theta\epsilon[\underline{\theta},\overline{\theta}]$.

$u_1(\theta)-u_2(\theta)=-p_1+p_2+\theta\Delta q+\omega\big(1-f(\Delta q)\big)(N_1-N_2)$.

If all the other consumers buy q_1 then N_1=1 and N_2=0.

Consumer θ also prefers q_1 if $-p_1+p_2+\theta\Delta q+\omega\big(1-f(\Delta q)\big)>0$. An equilibrium where only firm 1 is active exists if $-p_1+p_2+\theta\Delta q+\omega\big(1-f(\Delta q)\big)>0$ for every $\theta\epsilon\big(\underline{\theta},\overline{\theta}\big]$. Consumer $\underline{\theta}$ either prefers q_1 or is indifferent between both qualities. We have then

$$p_1-p_2\leq\underline{\theta}\Delta q+\omega\big(1-f(\Delta q)\big).$$

Second Case: Type 2 Nash equilibrium

If consumer θ supposes that only firm 2 is active then N_1=0 and N_2=1.

Consumer θ also prefers q_2 if $u_1(\theta)-u_2(\theta)=-p_1+p_2+\theta\Delta q-\omega\big(1-f(\Delta q)\big)<0$.

An equilibrium where only firm 2 is active exists if $-p_1+p_2+\theta\Delta q-\omega\big(1-f(\Delta q)\big)<0$ for every $\theta\epsilon\big(\underline{\theta},\overline{\theta}\big]$. Consumer $\overline{\theta}$ either prefers q_2 or is indifferent between both qualities. We have then

$$p_1-p_2\geq\overline{\theta}\Delta q-\omega\big(1-f(\Delta q)\big).$$

Third Case: Type 3 Nash equilibrium

If a consumer θ supposes that both firms are active, then he/she knows that there exists a marginal consumer $\hat{\theta}\epsilon(\underline{\theta},\bar{\theta})$ indifferent between q_1 and q_2 (otherwise only one firm is active) and that the market is divided according to the rule:

- Consumers in $[\underline{\theta},\hat{\theta})$ buy q_2.
- Consumers in $(\hat{\theta},\overline{\theta}]$ buy q_1.

We prove that consumer θ has no interest to deviate from the specified rule. The marginal consumer $\hat{\theta}$ is such that $u_1\big(\hat{\theta}\big)=u_2\big(\hat{\theta}\big)$.

$u_1(\theta)-u_2(\theta)=u_1(\theta)-u_2(\theta)-(u_1\big(\hat{\theta}\big)-u_2\big(\hat{\theta}\big))=\Delta q(\theta-\hat{\theta})$.Thus $u_1(\theta)-u_2(\theta)$ has the same sign as $(\theta-\hat{\theta})$ and consumer θ behaves according to the rule.

An equilibrium with two active firms exists if and only if $\underline{\theta}<\hat{\theta}<\overline{\theta}$ which is equivalent to the last condition cited in the lemma.■

Proof of Lemma 2. From equations (6) and (7), we have that firm 1's profit is given by:

$$\pi_1 = \begin{cases} p_1 & if \quad p_1 \le p_2 + \underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) \\ p_1 \dfrac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}} & if \quad p_2 + \underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) < p_1 < p_2 + \bar{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) \\ 0 & if \quad p_1 \ge p_2 + \bar{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) \end{cases}$$

Firm 2's profit is given by:

$$\pi_2 = \begin{cases} p_2 & if \quad p_2 \le p_1 - \bar{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) \\ p_2 \dfrac{\hat{\theta} - \underline{\theta}}{\bar{\theta} - \underline{\theta}} & if \quad p_1 - \bar{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) < p_2 < p_1 - \underline{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) \\ 0 & if \quad p_2 \ge p_1 - \underline{\theta}\Delta q - \omega\big(1 - f(\Delta q)\big) \end{cases}$$

From straightforward calculations, the best reply correspondences are thus given by:

$$R_1(p_2) = \begin{cases} \dfrac{p_2 + \bar{\theta}\Delta q - \omega(1 - f(\Delta q))}{2} & if \quad p_2 \le \big(\bar{\theta} - 2\underline{\theta}\big)\Delta q - 3\omega(1 - f(\Delta q)) \\ p_2 + \underline{\theta}\Delta q + \omega(1 - f(\Delta q)) & if \quad p_2 > \big(\bar{\theta} - 2\underline{\theta}\big)\Delta q - 3\omega(1 - f(\Delta q)) \end{cases}$$

$$R_2(p_1) = \begin{cases} Any\ price & if \quad p_1 \le \underline{\theta}\Delta q + \omega(1 - f(\Delta q)) \\ \dfrac{p_1 - \underline{\theta}\Delta q - \omega(1 - f(\Delta q))}{2} & if \quad \underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) < p_1 \le \big(2\bar{\theta} - \underline{\theta}\big)\Delta q - 3\omega(1 - f(\Delta q)) \\ p_1 - \bar{\theta}\Delta q + \omega(1 - f(\Delta q)) & if \quad p_1 > \big(2\bar{\theta} - \underline{\theta}\big)\Delta q - 3\omega(1 - f(\Delta q)) \end{cases}$$

Under strong conformity, the best reply correspondences intersect at

$$\begin{cases} p_1 = \underline{\theta}\Delta q + \omega\big(1 - f(\Delta q)\big) \\ p_2 = 0 \end{cases}$$

Under vanity or weak conformity, the best reply correspondences intersect at

$$\begin{cases} p_1 = \dfrac{2\bar{\theta} - \underline{\theta}}{3}\Delta q - \omega\big(1 - f(\Delta q)\big) \\ p_2 = \dfrac{\bar{\theta} - 2\underline{\theta}}{3}\Delta q - \omega\big(1 - f(\Delta q)\big) \end{cases}$$

Remark: The proof of lemma (2) could be performed in a more intuitive way. For the equilibrium with two active firms, it is required that both firms set positive prices and that FOCs and SOCs are satisfied. It is then straightforward to check that when ω is positive and large enough, no such equilibrium is possible as firms would have to set negative prices. For a monopoly equilibrium where the active firm is either firm 1 or firm 2, it is required that 1) the price of the non-active firm is zero 2) the monopolist sets the highest possible price which allows to monopolize the market and 3) the monopolist does not have any incentive to increase its price. We easily conclude then that under conformity and given condition (3) firm 2 could never monopolize the market as it has to set a negative price while firm 1 could.■

Proof of Proposition 2. Substituting $f(\Delta q)$ by its expression, when two firms are active, firms' profits are given by:

$\pi_1 = \Delta q(\frac{2\bar{\theta}-\underline{\theta}}{3} - \frac{\omega}{\bar{q}-\underline{q}})^2 \frac{\bar{q}-\underline{q}}{(\bar{q}-\underline{q})(\bar{\theta}-\underline{\theta})-2\omega}$ and $\pi_2 = \Delta q(\frac{\bar{\theta}-2\underline{\theta}}{3} - \frac{\omega}{\bar{q}-\underline{q}})^2 \frac{\bar{q}-\underline{q}}{(\bar{q}-\underline{q})(\bar{\theta}-\underline{\theta})-2\omega}$.

Firms' profits are increasing with respect to Δq. Under vanity, maximal differentiation results at equilibrium.

Under conformity, the condition $\Delta q > \frac{3\omega(1-f(\Delta q))}{\bar{\theta}-2\underline{\theta}}$ ensuring the activity of both firms at price equilibrium is equivalent to $\Delta q > \frac{3\omega\Delta q}{(\bar{\theta}-2\underline{\theta})(\bar{q}-\underline{q})}$ which is equivalent to $\bar{q} - \underline{q} > \frac{3\omega}{(\bar{\theta}-2\underline{\theta})}$ as $\Delta q > 0$. Expressed differently this inequality can also be written as $\omega < \frac{\bar{\theta}-2\underline{\theta}}{3}\big(\bar{q} - \underline{q}\big)$ and maximal differentiation is the optimal strategy.

If $\omega \geq \frac{\bar{\theta}-2\underline{\theta}}{3}\left(\bar{q}-\underline{q}\right)$ then for every $\Delta q \geq 0$, only the high quality firm is active. It will choose the highest possible quality $\bar{q}$ as its profit is increasing with respect to q_1. ■

Proof of Proposition 3. Substituting $f(\Delta q)$ by its expression, when two firms are active, firms' profits are given by:

$\pi_1 = \Delta q(\frac{2\bar{\theta}-\underline{\theta}}{3} - \frac{\omega\Delta q}{\left(\bar{q}-\underline{q}\right)^2})^2 \frac{1}{(\bar{\theta}-\underline{\theta})-2\omega\frac{\Delta q}{\left(\bar{q}-\underline{q}\right)^2}}$ and $\pi_2 = \Delta q(\frac{\bar{\theta}-2\underline{\theta}}{3} - \frac{\omega\Delta q}{\left(\bar{q}-\underline{q}\right)^2})^2 \frac{1}{(\bar{\theta}-\underline{\theta})-2\omega\frac{\Delta q}{\left(\bar{q}-\underline{q}\right)^2}}$.

We check that for every $\Delta q \in \left(0, \bar{q}-\underline{q}\right]$, $\frac{\partial\pi_1}{\partial\Delta q} > 0$ therefore firm 1 will choose the highest quality $\bar{q}$.

From tedious but straightforward calculations, we have that $\frac{\partial\pi_2}{\partial\Delta q}$ has two roots:

$$\Delta q_{a,b} = \frac{\left(\bar{q}-\underline{q}\right)^2(\bar{\theta}-\underline{\theta})}{8\omega}(3 \mp \Lambda).$$

Under vanity, these two roots are negative and for every $\Delta q \in \left(0, \bar{q}-\underline{q}\right]$, $\frac{\partial\pi_2}{\partial\Delta q}$>0. Therefore firm 2 chooses the lowest possible quality $\underline{q}$.

Under conformity, $\Delta q_{a,b}$ are both positive and the variation table of firm 2's profit is given by

Δq	0		Δq_a		Δq_b		$+\infty$
$\frac{\partial\pi_2}{\partial\Delta q}$		+	0	-	0	+	
$\pi_2(\Delta q)$		$\nearrow$		$\searrow$		$\nearrow$	

When the similarity function is quadratic, the condition ensuring the activity of two firms at price equilibrium becomes $\Delta q < \frac{(\bar{\theta}-2\underline{\theta})(\bar{q}-\underline{q})^2}{3\omega}$. As $\Delta q < \bar{q}-\underline{q}$, we now distinguish two cases:

- Case 1: if $\omega < (\bar{q}-\underline{q})\frac{(\bar{\theta}-2\underline{\theta})}{3}$ then $\bar{q}-\underline{q} < \frac{(\bar{\theta}-2\underline{\theta})\left(\bar{q}-\underline{q}\right)^2}{3\omega}$ and for every $\Delta q \in \left(0, \bar{q}-\underline{q}\right]$, two firms can be active. We prove that firm 2 either chooses $\underline{q}$ or an interior solution.
- Case 2: if $\omega \geq (\bar{q}-\underline{q})\frac{(\bar{\theta}-2\underline{\theta})}{3}$ then $\bar{q}-\underline{q} \geq \frac{(\bar{\theta}-2\underline{\theta})\left(\bar{q}-\underline{q}\right)^2}{3\omega}$. As $\Delta q < \frac{(\bar{\theta}-2\underline{\theta})(\bar{q}-\underline{q})^2}{3\omega}$, firm 2 always chooses an interior solution.

Case 1: To find firm 2's optimal quality, we first prove that $\Delta q_b > \bar{q}-\underline{q}$ (step 1), then we examine the relative positions of Δq_a and $\bar{q}-\underline{q}$ (step 2).

Step 1: We have that $\Delta q_b > \frac{3\left(\bar{q}-\underline{q}\right)^2(\bar{\theta}-\underline{\theta})}{8\omega}$ thus $\frac{\Delta q_b}{\bar{q}-\underline{q}} > \frac{3(\bar{q}-\underline{q})(\bar{\theta}-\underline{\theta})}{8\omega} > \frac{9}{8}\frac{\bar{\theta}-\underline{\theta}}{\bar{\theta}-2\underline{\theta}} > \frac{9}{8} > 1$ and therefore $\Delta q_b > \bar{q}-\underline{q}$. As Δq never exceeds $\bar{q}-\underline{q}$, firm 2's optimal quality choice depends on the relative position of $\bar{q}-\underline{q}$ and Δq_a.

Step 2: We have $\frac{\Delta q_a}{\bar{q}-\underline{q}} = \frac{(\bar{q}-\underline{q})(\bar{\theta}-\underline{\theta})}{8\omega}$(3-$\Lambda$). Thus,

- If $\bar{q}-\underline{q} < \frac{8\omega}{(3-\Lambda)(\bar{\theta}-\underline{\theta})}$ or equivalently $\omega > \frac{3-\Lambda}{8}(\bar{q}-\underline{q})(\bar{\theta}-\underline{\theta})$ then $\Delta q_a < \bar{q}-\underline{q}$ and the optimal quality choice is $q_2 = \bar{q} - (3-\Lambda)\frac{\left(\bar{q}-\underline{q}\right)^2(\bar{\theta}-\underline{\theta})}{8\omega}$ (interior solution) which is a global maximum of firm 2's profit on $\left[\underline{q}, \bar{q}\right]$.

- If $\overline{q} - \underline{q} \geq \frac{8\omega}{(3-\wedge)(\bar{\theta}-\underline{\theta})}$ or equivalently if $\omega \leq \frac{3-\wedge}{8}(\overline{q} - \underline{q})(\bar{\theta} - \underline{\theta})$ then $\Delta q_a \geq \overline{q} - \underline{q}$ and firm 2's profit is an increasing function of the difference between the firms' qualities. Thus, the optimal quality is such that differentiation between firms is maximal and firm 2 chooses $\underline{q}$.

Case 2: To have two active firms at price equilibrium, the quality difference Δq should never exceed $\frac{\overline{\theta}-2\underline{\theta}}{3\omega}(\overline{q} - \underline{q})^2$. We prove from straightforward calculations that $\Delta q_b > \frac{\overline{\theta}-2\underline{\theta}}{3\omega}(\overline{q} - \underline{q})^2$ and that $\Delta q_a < \frac{\overline{\theta}-2\underline{\theta}}{3\omega}(\overline{q} - \underline{q})^2$. Therefore, Δq_a maximizes firm 2 profit's for every $\Delta q \in \left(0, \frac{\overline{\theta}-2\underline{\theta}}{3\omega}(\overline{q} - \underline{q})^2\right]$. The best choice from firm 2 is then $q_2 = \overline{q} - (3 - \wedge)\frac{\left(\overline{q}-\underline{q}\right)^2(\overline{\theta}-\underline{\theta})}{8\omega}$ (interior solution). ■

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