Determination of the Optimal Strategy in *First Prize Private Value Auctions:* A Theoretical and Experimental Approach

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Abstract

In this paper firstly we explain auction types and define their general characteristics in the Introduction. Then, we focus on First Prize Sealed Private Value Auction and construct the necessary related mathematical model for this kind of auction according to the number of players in the auction. With these theoretical models we search for the strategy to find the optimal bid amount whose objective is to maximize the payoff from the auction. After completing these theoretical approaches to find an optimal strategy of First Prize Sealed Private Value Auction and forming formulas for it, we begin with the experimental part where we conduct some experiments with real players. In these experiments we find the opportunity to test our theoretical results in a real environment. Lastly, we compare the experiment consequences with the theoretical ones in the Conclusion.

Keywords: First prize private value auctions, optimal strategy in auctions

1. Introduction

The dictionary definition of auction is the public sale where items are sold to the highest bidder. Auctions have been used to buy and sell goods since prehistory and, even today, auctions are used quite frequently. There are several excellent surveys of the auction theory literature available, including Engelbrecht-Wiggans(1980), Maskin and Riley(1985), Milgrom(1985,1987,1989), McAfee and McMillan(1987), Riley(1989), Wilson(1992), Wolfsetter(1996), Kelmperer(1999) and Menezes at all (2005). Local municipalities and oil companies can be shown as significant examples of their usage: local municipalities use auctions to hire contractors for specific projects while major oil companies use them regularly to lease offshore oils (Klemperer, 2004). Furthermore, even governments sometimes benefit from auctions, such as the United States Government which auctioned licenses to use electromagnetic spectrum for personal communications services such as mobile telephones, two-way paging, portable fax machines, and wireless computer networks in July 1994. This auction is accepted as one of the largest auctions worldwide (McAfee and McMillan, 1996).

Nowadays, most people find auctions exiting because of the win/lose nature of the competition. Especially with the huge progress of Internet, online trading sites and Internet newsgroups offer a fascinating glimpse into the various ways that collectibles can be sold at auctions. As examples of auction web sites, according to usage ratings, *ebay.com, auctionfire.com, ubid.com, my-bids.com, auctions.yahoo.com, etc* are the most popular ones worldwide while *gittigidiyor.com, aldimgitti.com, alinsatin.com, alcamsatcam.com, etc.* are very common in Turkey.

If we consider the main types of auctions, we may observe that Private and Common Value Auctions constitute the two main classes of auctions. In *Private Value Auction,* bidders only know their own valuation of the item, but they may have some idea about the valuation of the other bidders, whereas, in *Common Value Auction* the underlying common value of the prize is not known by the bidders (Menezes and Monterio, 2005).

There are many different subcategories of auctions. The bidder can make their bids simultaneously and put them in sealed envelopes, that is called as *Sealed-Bid Auction* or they may bid sequentially, with the auctioneer calling out the successive bids that is called as *Sequent-Bid Auction*. Additionally, auctions can also be classified regarding the pay amount of the winner as First or Second Prize Auction.

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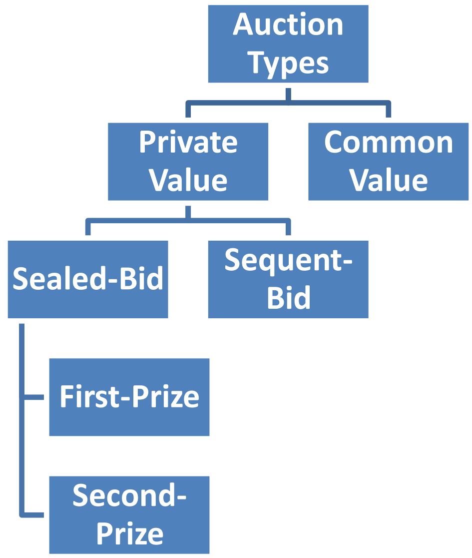
The auction where the winner pays the winning bid is called as *First Prize Auction,* while the one where the winner pays the second highest bid is called as *Second Prize Auction* (Milgrom, 2004).

Figure 1. Subcategories of auctions



Source: Milgrom (2004)

Tan (1992) compares the equilibria of the first- and second-price auctions, assuming that firms always bid own valuations in the second-price auction. He considers a procurement model where firms invest in R&D expenditure prior to the bidding stage. He shows that revenue equivalence holds between first and second price auction in the symmetric equilibrium with decreasing returns to scale.

Stegeman (1996) has shown that the second-price auction, with no entry fee and a reserve price equal to the seller's valuation, has strong efficiency properties in the presence of participation costs. He also explains that if the efficient equilibria of the second-price auction are asymmetric, then the first-price auction has no efficient equilibrium.

Lunander (1999) examines both first and second prize common value sealed bid auctions of government procurements which are organized according to European Community directives. He especially focuses on the outcome difference between first and second prize models and to analyze the difference. He points out the superiority of the second prize auctions, especially, if the number of bidders is a few and there is a high dispersion among bidders' signals, second prize models are more beneficial than the first prize ones.

Holt (1980) presents a game-theoretic analysis of the competition for procurement contracts under different auction procedures. He shows that when bidders are risk averse the first-price auction generates higher revenues since bidders then shade their bids less.

2. Theoretical Background

As we discussed above, in Sealed-Bid Private Auction the bidders only know their own valuation of the item and they have a vague idea about the valuation of the other bidders. So this paper pertains to the case where individual valuations of players differ randomly where each one knows only his own private value and the value range of the other ones. The simplest model of a private value auction is one where bidders' values are independent draws from a distribution that is uniform in the sense that each money amount in a specified interval is equally likely. (Klemperer, 2004)

In this paper we study the First Prize Auction model, so the winner pays his bid which is the highest. In the first step, we form a model of two players, and then in the second step we extend this to a three player model. Finally, we create a general model of N players where we follow the system which is used in two and three player models. At the end of each model, we conclude a function which determines the optimal bid amount maximizing payoff.

2.1. First Prize Private Value Auction with two players

In this auction model, the players' vague ideas are shown by the value interval of the item and that means the value of the rival player can lie at any point on this interval. Now suppose that the auctioned item has the value range between m and M where m is the minimum value, while M the maximum. Let v be the value and b the bid of a particular player. Since the personal value of the rival belongs to this interval $[m, M]$ and there is just one rival, the bid of this competitor can be anywhere on this interval. So, b the bid of the player can be at any point on $[m, M]$. Under these circumstances, with uniform distribution assumption, the function which shows the winning probability would be

$$p(b)= (b-m)/(M-m)\ ,\ m \leq b \leq M \qquad \text{(Equation 1)}$$

Regarding this function, it is obvious that a bid of m which is the lowest one has no chance to win since $p(m)=0$ and a bid of M always win the auction, that is why $p(M)=1$. Here, it can be observed that $p(b)$ values differ in $[0,1]$ dependent on b, that means in mathematical terms, the range of this function is $[0,1]$ where 0 is zero probability, while 1 is hundred percent. And it is clear that the winning probability p increases as the bid amount b increases.

Now, we consider earning of this player; since he wants to earn a positive amount, the bid of the player must always be less than his value, that means $b<v$. And earning of the player can be defined as $v-b$, which is always positive.

Finally, we can formulate the players' expected payoff function $E(b)$ which dependents on b as follows,

$$E(b)=(v-b)\ [(b-m)/(M-m)]=[-b^2 + b(m+v) - vm]/(M-m)\ ,\ m \leq b \leq M \qquad \text{(Equation 2)}$$

In Private Value Auction, we cannot change our valuation but we may set our bid freely, because of this, in our model the bid amount must be the decision variable which depends on other variables. Therefore, we must find an optimal bid amount, say b^*, which yields maximum payoff, in math terms, which maximizes the Expected Function $E(b)$. To fulfill this objective, we should take the first derivative of $E(b)$ and then equate to zero:

$$E'(b) = (-2b+m+v)/(M-m)=0 \qquad \text{(Equation 3)}$$
$$b^* = m/2 + v/2 \qquad \text{(Equation 4)}$$

Now, we have to show that this optimal bid amount b^* really maximizes the $E(b)$ instead of minimizing. This can be clarified by the sign of the $E''(b^*)$ where we put b^* into the second derivative of $E(b)$. According to elementary Calculus rules, b^* is the maximizing one if $E''(b^*)$ is always negative. So,

$$E''(b)= -2/(M-m) \qquad \text{(Equation 5)}$$
$$E''(b^*)= -2/(M-m) \qquad \text{(Equation 6)}$$

Since $M>m$, $M-m$ is always positive, the term $-2/(M-m)$ is always negative, that means b^* maximizes the $E(b)$.
In summary, in a First Prize Private Value Auction with two players with a value range $[m, M]$ where m is the minimum and M is the maximum personal value of the players, the optimal bid b^* of each player is half of the sum of his personal value v and the minimum value m and it is independent of M.

2.1. First Prize Private Value Auction with three players

To analyze this situation we must define some new variables v_1 , v_2 , v_3 as personal values and b_1 , b_2 , b_3 as the bids of three players. Here v_1 , v_2 and v_3 belong to the interval $I=[m,M]$ and b_1 , b_2 and b_3 have a uniform distribution on this interval. That means, in such an auction with uniform distribution of personal values to three players each one will suppose that each b_i $(i=1,2,3)$ one by one lies on a special part of this interval which is constructed with the separation of it into three same length intervals I_1 , I_2 and I_3.
That is to say: $I= I_1 \cup I_2 \cup I_3$ and $|I_1|=|I_2|=|I_3|$, so these intervals are:

$$I_1=[m, m+(M-m)/3]= [m\ , (2m+M)/3] \qquad \text{(Equation 7)}$$
$$I_2=[\ m+(M-m)/3, m +2(M-m)/3] = [\ (2m+M)/3\ , (m+2M)/3\ ] \qquad \text{(Equation 8)}$$
$$I_3=[\ m+2(M-m)/3, m+3(M-m)/3] = [(m+2M)/3\ , M\ ] \qquad \text{(Equation 9)}$$

Because of the result of the uniformity, each player supposes, whether it is really true or not, all bids b_1 b_2 and b_3 lie relatively in these three intervals I_1 , I_2 and I_3 . According to this assumption, the bid of the interval I_3 wins the auction. That means, any bid on I_3 wins the auction.

Because of this, it is illogical to bid more than *(m+2M)/3* which is the starting point of I_3. Due to this supposition, it is clear that after this point, higher bids create less earning since personal value does not change but paying bid amount increases. Therefore, by using the above assumption, we come to these significant consequences to build the probability function, that a bid of *(m+2M)/3* wins the auction hundred percent while a bid of m never wins and *b* can be uniformly at any point on this interval. So, the probability function is:

$$p(b)=(b-m)/\,[(m+2M)/3-m] = [3(b-m)]/[2(M-m)]\ ;\ m\leq b\leq (m+2M)/3 \quad \text{(Equation 10)}$$

Here we must take care of three players, because this probability function *p(b)* only shows the probability of one player's, for simplicity say the first player's, winning chance against the second one. But, even if the bid of the first one is higher than the second one, it is not enough for the first player to win the auction, because there is a third player and his bid must exceed the third's one, too. For this reason if we define these two conditions as two independent events *A* and *B*, where *A* is the event that the bid of first player is higher than the second one, while *B* is the event that the bid of first player is higher than the third one.

It is obvious that, both *A* and *B* must happen together for the first player to win the auction. Since *A* and *B* are independent events, according to elementary probability rules, the probability of the situation where both *A* and *B* happen is the product of the probabilities of *A* and *B*. In math terms, that is to say:

$$p(A \cap B) = p(A)\, p(B) \quad \text{(Equation 11)}$$

And because all the personal values are distributed uniformly at first, this problem has a symmetric shape, that means *p(A)* and *p(B)* are equal. As we recognize *b* as the bid of the first player for simplicity, then it is obvious that

$$p(b)=p(A)=p(B) \quad \text{(Equation 12)}$$

And from there if we put *(12)* into the *(11)* we find, the win probability of the first player, as

$$p(A \cap B) = [p(b)]^2= p^2(b)= [9(b-m)^2]/[4(M-m)^2] \quad \text{(Equation 13)}$$

Earning of the player is again *v-b* and this is positive any time where *v* is the personal value of the first player instead of v_1 just for simplicity. So the Expected Payoff Function is

$$E(b)= (v-b)\, [9(b-m)^2]/[4(M-m)^2]\ ;m\leq b\leq (m+2M)/3 \quad \text{(Equation 14)}$$

To find the optimal bid b^* which maximizes this Expected Function, we take the first derivative of this function by using the product derivative rule and then equal it to zero,

$$E'(b)=0 \rightarrow E'(b) = \{9/[4(M-n)^2]\}\, (b-m)[2(v-b)-(b-m)] =0 \quad \text{(Equation 15)}$$

Since $M\neq n \rightarrow M-n\neq 0$, the term $9/[4(M-n)^2\,]$ is never zero, so

$$(b-m)[2(v-b)-(b-m)] =0 \quad \text{(Equation 16)}$$
$$\rightarrow b^*_1=m \quad \text{(Equation 17)}$$
$$\rightarrow b^*_2=(m+2v)/3 \quad \text{(Equation 18)}$$

Here b^*_1 and b^*_2 are both extremes of *E(b)* and they both belong to the related interval *[m,(m+2M)/3]***.** It is obvious that b^*_1 is on this interval and for b^*_2 , if we remember that *M* is the maximum value of the item, and for this reason, no personal value can exceed it, that is to say $v\leq M$ **,** we can see the inequality $b^*_2 =(m+2v)/3\leq(m+2M)/3$ which shows that b^*_2 lies on it, too. Now we should take the second derivative and put both b^*_1 and b^*_2 into it, and finally, look at their sign to determine whether b^*_1 or b^*_2 is the maximizing one,

$$E''(b)= \{9/[4(M-n)^2]\}\, (-6b+2v+4m) \quad \text{(Equation 19)}$$
$$E''(b^*_1)=E(m)= \{9/[2(M-n)^2]\}(v-m) \quad \text{(Equation 20)}$$
$$E''(b^*_2\,)=E((m+2v)/3)= -\{9/[2(M-n)^2]\}(v-m) \quad \text{(Equation 21)}$$

Since *m* is the starting point of the value range of the item, any personal value of the player's *v* must always be greater or equal than *m*, that is $v\geq m$; so $v-m\geq 0$. Here, if we examine the $v=m$ case, we may realize that it yields zero earning and, in such a situation, it is illogical to enter the auction, for this reason, we may admit that *v-m* is always positive; therefore,

$$E''(b^*_1)>0 \qquad \textit{(Equation 22)}$$
$$E''(b^*_2)<0 \qquad \textit{(Equation 23)}$$

From elementary calculus rules we come to the conclusion that b^*_2 is the bid amount which maximizes $E(b)$, because of the always negative sign of $E''(b^*_2)$. For simplicity, we may recognize b^*_2 *as* b^*, that is to say $b^*= b^*_2$. To summarize, in First Prize Private Value Auction with three players with the value range *[m,M]*, the optimal bid of each player is

$$b^*=(m+2v)/3= m/3 + 2v/3 \qquad \textit{(Equation 24)}$$

where it is the sum of one third of minimum value *m* and two thirds of his personal value *v* and independent of M.

2.1. First Prize Private Value Auction with N Players

In the last step, we extend our findings in Private Value Auction with two and three players to one with *N* players. So this model is the general case, which can be applied to any number of players, where *N* symbolizes any positive integer. To find an optimal bid amount in this auction with *N* players, that is equal to determine the maximizing bid b^* dependent on *v, m* and *N*, we use the same approach as the one with three players.

In this auction model, $v_1 , v_2 , ... , v_N$ are personal values of the *N* players while $b_1 , b_2 , ... ,b_N$ are their related bids. Here $v_1 , v_2 , ... , v_N$ belong to the usual interval *I=[m, M]* and $b_1 , b_2 , ... ,b_N$ have a uniform distribution on this interval. Similar to the previous model, because of the uniform distribution, all bids are distributed on the same length intervals $I_1 , I_2 , ... , I_N$ where $I= I_1 \cup I_2 \cup ... \cup I_N$ and $|I_1|=|I_2|= ... =|I_N|$. According to the same assumption, the bid which lies on the last interval I_N is always the highest, so in this model the range of the probability function is $I_1 \cup I_2 \cup ... \cup I_{N-1}$ which is equal to *[m, M-(M-m)/N]***.** That means the bid amount of the first player *b* ($b=b_1$ for simplicity) is at any point on this interval, so the probability function is

$$p(b)=(b-m)/[M-(M-m)/N-m]=[N(b-m)]/[(N-1)(M-m)] \; ; m\leq b\leq M-(M-m)/N \qquad \textit{(Equation 25)}$$

The function in *(24)* gives the probability that the first player has a higher bid than the second one. But there are *N-1* players in all, apart from the first one. And to win the auction this player's bid must exceed all the other players' bids, since these *N-1* events are independent and their probabilities are equal to each other for the symmetry, the probability function in which the first player wins the auction is

$$[p(b)]^{N-1} = p^{N-1}(b) = [N(b-m)]^{N-1}/[(N-1)(M-n)]^{N-1} \; ; m\leq b\leq M-(M-m)/N \qquad \textit{(Equation 26)}$$

The earning of the first player is *v-b* as usual, therefore the Expected Function is

$$E(b)=(v-b)[N(b-m)]^{N-1}/[(N-1)(M-n)]^{N-1} \; ; m\leq b\leq M-(M-m)/N \qquad \textit{(Equation 27)}$$

Similarly, we maximize this by equaling its derivative to zero

$$E'(b)=\{N/[(N-1)(M-n)]\}^{N-1} [(v-b)(N-1)(b-m)^{N-2}-(b-m)^{N-1}]$$
$$= \{N/[(N-1)(M-n)]\}^{N-1} (b-m)^{N-2} [(v-b)(N-1)-(b-m)]=0 \qquad \textit{(Equation 28)}$$
$$b_1=m \; , b_2=m/N+ (1-1/N)v \qquad \textit{(Equation 29)}$$

In light of the three-player model, it is obvious that b_1 minimizes, while b_2 maximizes the Expected Function. Thus , in First Prize Private Value Auction with *N* players the optimal bid b^* with a value range *[m, M]* is

$$b^* = m/N+ (1-1/N)v \qquad \textit{(Equation 30)}$$

To compare this result with the previous ones, we may set *N=2* and *N=3* into the *(31)*

$$N=2 \rightarrow b^* = m/2 + v/2 \quad \textit{(Equation 32)}$$
$$N=3 \rightarrow b^*= m/3 + 2v/3 \quad \textit{(Equation 33)}$$

As we can immediately realize, we have the same findings as in the two and three player cases.
In a further study we may examine the behavior of *(29)* for a lot of players. This situation can be handled in a mathematical approach as *N* approaches to infinity, in math terms, we should take the limit of *(29)* as $N\rightarrow\infty$. So

$$\lim_{N\rightarrow\infty}[\frac{m}{N}+(1-\frac{1}{N})] = v \qquad \textit{(Equation 34)}$$

To interpret this consequence, in the First Prize Private Value Auction with a lot of players, the optimal bid of a player approaches to his personal value.

3. Experimental Approach

In this section, we conduct an experiment on First Prize Private Value Auction. Without any comment, we explain the way of these and exhibit their results. This experiment is a three-player auction with 30 auctioned items.
Firstly, we conduct the experiment with three players who are graduate students. In this experiment, there are 30 items which are independent from each other and the prize of each is between 25 and 75 unit money. That means, the minimum value is $m=25$ while the maximum one is $M=75$. As it can be immediately realized, we do not change the value interval of the items to provide an equal condition in each round.

There are three players who enter the auction for each of these 30 items; the first player is "Player 1" who always applies the related optimal bid formula for each auctioned item independent of his own opinion or feeling. The others are "Player 2" and "Player 3". These both have absolutely no knowledge about the optimal bid amount formula, so they decide to their bid amount regarding their initiative which is related to their personal risk-averse, risk-neural or risk-alike character.

Because of the characteristic of the First Prize Private Value Auction, each player only knows his own personal value and value range of the item. In light of this limited information, each one makes a bid to win a maximum earning from each auctioned item. Because of the sealed envelope style of the auction, after auctioning each item, the players learn neither the other's personal values nor their bids. They are told only the winner player with the highest bid.

"The Personal Values of the Players" columns are determined by using the help of Excel function "RAND()" which generates uniformly distributed random real numbers between 0 and 1. If we modify this function by *(75-25)* RAND()+25*, we may obtain a random real number between 25 and 75. So, we generate 30 random values between 25 and 75, and then distribute them to three players in a mixed order as entered in columns of three players. In spite of mixed order, these three columns consist of the same 30 values in fact and that brings the result that the average personal value for each player is same and that is equal to 53.72, showed in the last row of the "The Personal Values of Players" columns.

"The Bids of Players" columns consist of bids of each player for each item. As we explained above, the first player's bids, that is to say, Player 1's bids are constituted by using *equation 24* which determines the optimum bid amount in a First Prize Private Value Auction with three players. For example, by the auction of the first item, Player 1's personal value is 44,94 $(v=44{,}94)$ and the minimum value is 25 for each player $(m=25)$; so the bid, which is expected to be the optimal, is $25/3+2*44{,}94/3=38{,}30$. Beside that, the second and third player's bids are between 25 and 75 according to their personal value for each item dependent on their personal choice. These personal values and bids of players are shown below:

Table 1. Personal values and bids of players

	The Personal Values of Players			The Bids of Players		
Item No:	Player 1 (v1)	Player 2 (v2)	Player 3 (v3)	Player 1 (b1=b*)	Player 2 (b2)	Player 3 (b3)
1	44,94	40,82	35,32	38,30	37,00	29,00
2	73,15	48,49	46,24	57,10	43,00	36,00
3	57,19	48,14	33,85	46,46	43,50	27,00
4	41,51	60,92	62,97	36,01	55,92	50,00
5	44,47	29,64	62,34	37,98	27,44	56,50
6	61,55	58,20	69,63	49,37	54,52	60,30
7	73,61	74,76	58,96	57,41	70,00	51,75
8	65,06	56,03	31,58	51,70	53,50	26,00
9	73,12	60,69	42,85	57,08	58,69	36,00
10	44,05	47,86	63,59	37,70	45,80	55,00
11	40,82	35,32	44,94	35,55	32,85	38,85
12	48,49	46,24	73,15	40,66	40,50	67,50
13	48,14	33,85	57,19	40,42	29,50	56,00
14	60,92	62,97	41,51	48,94	57,00	39,00
15	29,64	62,34	44,47	28,09	57,00	42,00
16	58,20	69,63	61,55	47,13	63,20	58,00
17	74,76	58,96	73,61	58,18	55,26	69,50
18	56,03	31,58	65,06	45,69	31,00	60,60
19	60,69	42,85	73,12	48,79	40,00	63,00
20	47,86	63,59	44,05	40,24	55,00	43,00
21	35,32	44,94	40,82	31,88	44,00	39,75
22	46,24	73,15	48,49	39,16	60,00	45,40
23	33,85	57,19	48,14	30,90	54,00	48,00
24	62,97	41,51	60,92	50,31	41,00	53,00
25	62,34	44,47	29,64	49,89	44,00	28,50
26	69,63	61,55	58,20	54,76	56,00	54,00
27	58,96	73,61	74,76	47,64	62,00	69,00
28	31,58	65,06	56,03	29,39	56,00	55,00
29	42,85	73,12	60,69	36,90	66,00	59,00
30	63,59	44,05	47,86	50,72	44,00	47,00
	53,72	**53,72**	**53,72**			

Average of Personal Values for each player

For each auctioned item, name of the winner player with the highest bid is stated in the column with the title "The Winner". Furthermore, "the Earning of Players" columns show earning of the winner player for each item that is calculated by the difference of the bid from the personal value *(v-b)*. It is clear that the other two players who do not have the highest bid have zero earning. Next to it, "the Cumulative Earning of Players" columns show the cumulative sum of earnings of each player after each round. The 30th row of this column also indicates the final result showing the totals earning of players after auctioning of the 30 items.

Table 2. Earnigs and cumulative earnings of players

Item No:	The Winner	The Earning of Players			Cumulative Earning of Players		
		Player 1	Player 2	Player 3	Player 1	Player 2	Player 3
1	Player 1	6,6	0,0	0,0	6,65	0,00	0,00
2	Player 1	16,0	0,0	0,0	22,70	0,00	0,00
3	Player 1	10,7	0,0	0,0	33,43	0,00	0,00
4	Player 2	0,0	5,0	0,0	33,43	5,00	0,00
5	Player 3	0,0	0,0	5,8	33,43	5,00	5,84
6	Player 3	0,0	0,0	9,3	33,43	5,00	15,17
7	Player 2	0,0	4,8	0,0	33,43	9,76	15,17
8	Player 2	0,0	2,5	0,0	33,43	12,29	15,17
9	Player 2	0,0	2,0	0,0	33,43	14,29	15,17
10	Player 3	0,0	0,0	8,6	33,43	14,29	23,76
11	Player 3	0,0	0,0	6,1	33,43	14,29	29,85
12	Player 3	0,0	0,0	5,6	33,43	14,29	35,50
13	Player 3	0,0	0,0	1,2	33,43	14,29	36,69
14	Player 2	0,0	6,0	0,0	33,43	20,26	36,69
15	Player 2	0,0	5,3	0,0	33,43	25,60	36,69
16	Player 2	0,0	6,4	0,0	33,43	32,03	36,69
17	Player 3	0,0	0,0	4,1	33,43	32,03	40,80
18	Player 3	0,0	0,0	4,5	33,43	32,03	45,25
19	Player 3	0,0	0,0	10,1	33,43	32,03	55,37
20	Player 2	0,0	8,6	0,0	33,43	40,62	55,37
21	Player 2	0,0	0,9	0,0	33,43	41,56	55,37
22	Player 2	0,0	13,1	0,0	33,43	54,71	55,37
23	Player 2	0,0	3,2	0,0	33,43	57,90	55,37
24	Player 3	0,0	0,0	7,9	33,43	57,90	63,29
25	Player 1	12,4	0,0	0,0	45,87	57,90	63,29
26	Player 2	0,0	5,6	0,0	45,87	63,45	63,29
27	Player 3	0,0	0,0	5,8	45,87	63,45	69,05
28	Player 2	0,0	9,1	0,0	45,87	72,51	69,05
29	Player 2	0,0	7,1	0,0	45,87	79,62	69,05
30	Player 1	12,9	0,0	0,0	58,74	79,62	69,05

In the next column, the number of won items is stated again in a cumulative manner with the title of "Number of Won Items". For example, by the auction of the 22nd item, this number is 9 for Player 2, that means, Player 2 won 9 of the 22 auctioned items. So, with help of this "Number of Won Items" column, we formed the "Average Earning per Item" column which is constituted by the division of the cumulative earning by the number of won items. For example, by the auction of the 18th item, Player 3's cumulative earning is 45,25 and he won 8 of the 18 items, thus, his average earning is the division of 45,25 by 8 and it equals to 5,66. In the special situation where there is a no won item, zero is entered in the average Earning cell. Lastly, we have the "Risk coefficient" column which gives an idea about the risk character of each player. To constitute this column, we used the definition of risk coefficient β which is calculated by the formula (Menezes and Monterio, 2005)

$$\beta=(b-m)/(v-m) \quad (Equation\ 35)$$

In this definition, it is clear that the risk coefficient is between 0 and 1 since $m \leq b \leq v$ and β of a risk averse player approaches to 1 while a risk alike one's to 0. It can be realized that Player 1 has a constant risk coefficient β_1 in this first experiment with three players for each auctioned item (2/3=0,67), to obtain this value, we put Equation *24* in Equation *35*, we have

$$\beta_1 = (m/3 + 2v/3 - m)/(v-m) = 2/3 \qquad \text{(Equation 36)}$$

Table 3. Average earnig per item and risk coefficient of players

Item No:	**Number of Won Items**			**Average Earning per Item**			**Risk Coefficient: (b-m)/(v-m)**		
	Player 1	**Player 2**	**Player 3**						
	0	**0**	0	**Player 1**	**Player 2**	**Player 3**	**Player 1**	**Player 2**	**Player 3**
1	1	0	0	6,65	0,00	0,00	0,67	0,76	0,39
2	2	0	0	11,35	0,00	0,00	0,67	0,77	0,52
3	3	0	0	11,14	0,00	0,00	0,67	0,80	0,23
4	3	1	0	11,14	5,00	0,00	0,67	0,86	0,66
5	3	1	1	11,14	5,00	5,84	0,67	0,53	0,84
6	3	1	2	11,14	5,00	7,59	0,67	0,89	0,79
7	3	2	2	11,14	4,88	7,59	0,67	0,90	0,79
8	3	3	2	11,14	4,10	7,59	0,67	0,92	0,15
9	3	4	2	11,14	3,57	7,59	0,67	0,94	0,62
10	3	4	3	11,14	3,57	7,92	0,67	0,91	0,78
11	3	4	4	11,14	3,57	7,46	0,67	0,76	0,69
12	3	4	5	11,14	3,57	7,10	0,67	0,73	0,88
13	3	4	6	11,14	3,57	6,11	0,67	0,51	0,96
14	3	5	6	11,14	4,05	6,11	0,67	0,84	0,85
15	3	6	6	11,14	4,27	6,11	0,67	0,86	0,87
16	3	7	6	11,14	4,58	6,11	0,67	0,86	0,90
17	3	7	7	11,14	4,58	5,83	0,67	0,89	0,92
18	3	7	8	11,14	4,58	5,66	0,67	0,91	0,89
19	3	7	9	11,14	4,58	6,15	0,67	0,84	0,79
20	3	8	9	11,14	5,08	6,15	0,67	0,78	0,94
21	3	9	9	11,14	4,62	6,15	0,67	0,95	0,93
22	3	10	9	11,14	5,47	6,15	0,67	0,73	0,87
23	3	11	9	11,14	5,26	6,15	0,67	0,90	0,99
24	3	11	10	11,14	5,26	6,33	0,67	0,97	0,78
25	4	11	10	11,47	5,26	6,33	0,67	0,98	0,75
26	4	12	10	11,47	5,29	6,33	0,67	0,85	0,87
27	4	12	11	11,47	5,29	6,28	0,67	0,76	0,88
28	4	13	11	11,47	5,58	6,28	0,67	0,77	0,97
29	4	14	11	11,47	5,69	6,28	0,67	0,85	0,95
30	5	14	11	11,75	5,69	6,28	0,67	1,00	0,96
							0,67	**0,83**	**0,78**
							Average of Risk Coefficient for each player		

4. Interpretation of Experiment Results

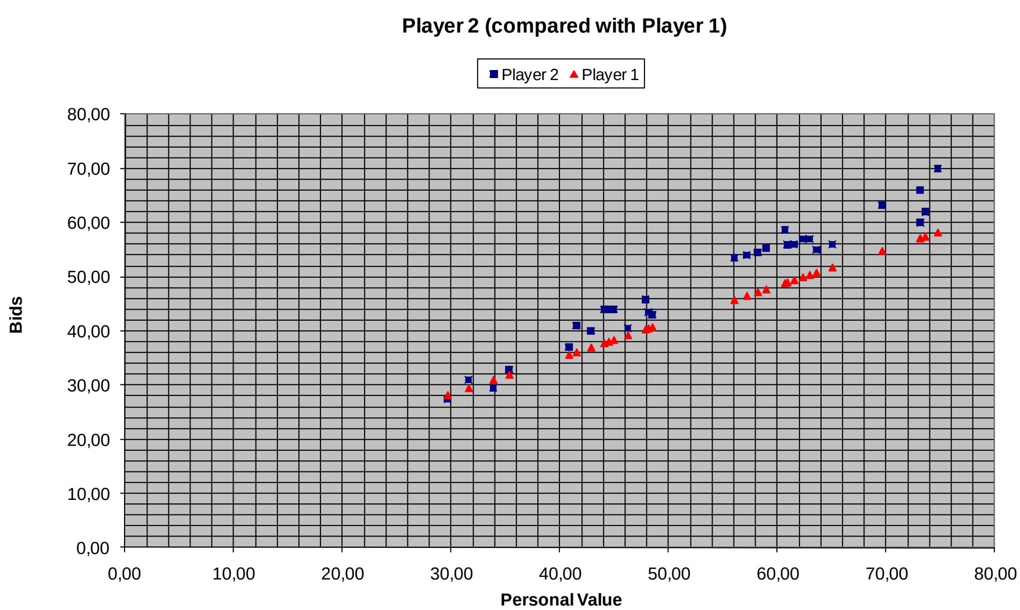
In this part we will examine the results of this experiment and then explain them by using our theoretical findings. There are some main outputs of the experiment; these are "the cumulative earning of each player", "the average earning of each player" and "the average risk coefficient of each player". In this experiment, since Player 1's results reflect the theoretical side, we will compare them with the results which belong to other players.

The cumulative earning of both Player 3 and Player 2 is higher than Player 1. *(79,62>58,74* and *69,05>58,74)* But these two players together won 25 of 30 auctioned items while Player 1 won only 5. That brings the fact that "the average earning per item" value of Player 1 is too much higher than Player 3 and Player 2. *(11,75>5,69* and *11,75>6,28)*

If we observe the average risk coefficients of three players we may see that the ones of Player 3 and Player 2 are higher than the Player 1's which may be admitted as the risk neutral limit. In the light of the above explanations about risk coefficient we can say that both Player 3 and Player 2 are risk adverse players. Furthermore, Player 2 had the highest average risk coefficient (0,83) and that yielded him the highest cumulative earning because he earned the most of auctioned items (14 of 30) by giving relatively high bids. On the other side, because of this risk averse behavior he could not make good earnings in his won items and so he had the lowest average earning per item.

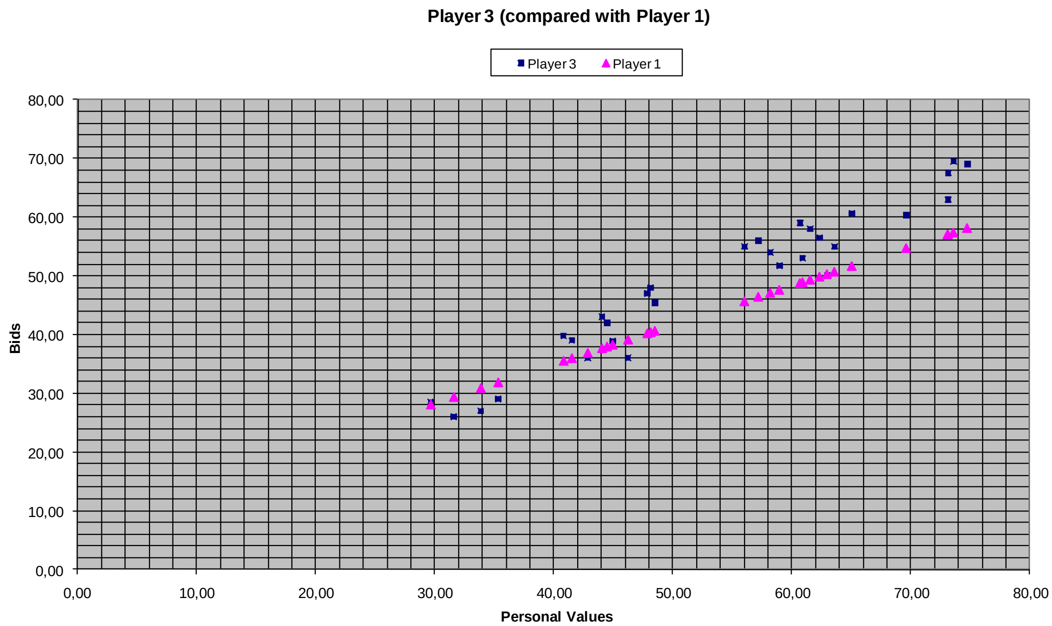
To observe the relationship between personal values and their related bids that is equal to the risk coefficients of players we can firstly draw the graph the "Personal Value-Bid" Graph for Player 2. In this graph, the square-shaped points show the original bids of Player 2 which were offered according to his personal values during the conducted experiment. On the other hand, triangle-shaped points demonstrate imaginary bids of Player 1 which would be offered by using the optimal bid formula if he had the same personal values of Player 2.

Figure 2.



As we similarly draw the graph for Player 3,

Figure 2.



As we discussed above, Player 1 has a constant risk coefficient of 0,67; that means the line equation of Player 1's points is *b=0,67*v*, because of that the related personal value-bid points lie on this line. And since both Player 3 and Player 2 have higher average risk coefficients than 0,67; their points usually lie above this risk neutral line which belongs to Player 1.

5. Conclusion

In this paper, using the principle of maximum expected payoff we introduced a mathematical function to find the optimal bid for a first prize private value auction for N players, displayed in Equation 30. By generating this function we assumed that each player has restricted information about the others' personal value of the auctioned item (only minimum and maximum values). After deriving this formula, we applied it in a three player experiment where first player bids according to the formula.

In this experiment the other players behaved with a risk averse manner compared to Player 1's risk neutral optimum bid strategy. And with this risk averse manner, they won most of the auctioned items and because of that, their cumulative earning became higher than Player 1's one. But since their little earnings per item, their averages earning per item are less than the Player 1's one.

According to the results of this experiment, it can be stated that the optimal bid strategy maximized the average earning per item and created the smallest risk coefficient. It is obvious that this optimal strategy does not give the highest returns in each category and the concequences are so much dependent on the decisions of other players apart from Player 1 who bids by using the formula.

For a further research, the number of experiments can be increased where players' risk characteristics, number of auctioned items and the information about the items differ.

6. References

Engelbrecht-Wiggans (1980), "Auctions and bidding models: A survey", *Management Science,* Vol. 26(2), pp. 119–142.

Holt, C., (1980) "Competitive bidding for contracts under alternative auction procedures", *Journal of Political Economy,* Vol. 88 (3), pp 433–445.

Klemperer, P. (1999), "Auction theory: A guide to the literature", *Journal of*

Economic Surveys, Vol. 13(3), pp. 227–286.

Klemperer, P. (2004), "Auctions: Theory and Practice", *Princeton University Press*

Leufkens, K., Peeters, R., (2007), "Synergies are a reason to prefer first-price auctions**!",** *Economics Letters,* Vol. 97, pp. 64-69.

Lunander, A. (1999), "Procurement Bidding in First-Price and Second-Price Sealed Bid Common Value Auctions", *Working Paper Series,* Vol. 17

Maskin, E. S., and Riley, J. G. (1985), "Auction theory with private values", *American Economic Review,* Vol. 75, pp. 150–155.

McAfee, R. P. and McMillan, J. (1987), "Auctions and bidding", *Journal of Economic Literature*, Vol. 25(2), pp. 699–738.

McAfee, R. P., McMillan, J. (1996), "Analyzing the Airwaves Auction", *Journal of Economic Perspectives,* Vol. 10, pp. 159-175.

Menezes, F. M., Monterio, P. K. (2005), "An introduction to auction theory", *Oxford University Press*

Milgrom, P. R. (1985), "The economics of competitive bidding: A selective Survey", *In L. Hurwicz, D. Schmeidler, and H. Sonnenschein (eds.), Social Goals and Social Organization: Essays in Memory of Elisha Pazner, Cambridge: Cambridge University Press.*

Milgrom, P. R. (1987), "Auction theory", *in Truman F. Bewley (ed.), Advances in Economic Theory: Fifth World Congress, Cambridge University Press.*

Milgrom, P. R. (1989), "Auctions and bidding: A primer", *Journal of EconomicPerspectives,* Vol. 3, pp. 3–22.

Milgrom, P. R. (2004), "Putting auction theory to work", Cambridge University Press

Riley, J. G. (1989), "Expected revenue from open and sealed bid auctions", *Journal of Economic Perspectives,* Vol: 3(3), pp. 41–50.

Stegeman, M., (1996) "Participation cost and efficient auctions*", Journal of Economic Theory,* Vol. 71, pp. 228-259.

Tan, G., (1992) "Entry and R and D in Procurement Contracting", *Journal of Economic Theory,* Vol. 58, pp. 41—60.

Wilson, R. (1992), "Strategic analysis of auctions.' *Elsevier Science,* Vol. 1, pp. 227–280.

Wolfstetter (1996), "Auctions: An introduction", *Journal of Economic Surveys*, Vol: 10, pp. 367–421.