A Note on the Scale Efficiency Test of Simar and Wilson

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Abstract

Simar and Wilson (2002) procedure to test for returns to scale has a drawback as it may generates inconsistent values. We use a counter example to show that their returns to scale test procedure generates non admissible pseudo-values. Consequently, the test results are not consistent and cannot be used to test for returns to scale. We propose a remedy that consists in a correction of the simulation algorithm to ensure the consistency of the estimated test statistic.

Keywords: Returns to scale, Data envelopment analysis, Bootstrap, Test of hypothesis.

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1. Introduction

Simar and Wilson (2002) bootstrap method offers a hypothesis testing procedure for returns to scale that is interesting and seems very satisfactory as it does not suffer from the convergence problem of the others[1]. The procedure is based on the efficiency scale ratio developed by Färe and Grosskopf (1985) and this opens the door to some a potential problem that we address here. Färe and Grosskopf scale efficiency measure is defined as the ratio of two efficiency scores, those being measured under two different assumptions on the return to scale. By construction, these measures are always bounded by one. This must hold because the data are generated under only one Data Generating Process (the true one) that belongs to either one or the other assumption. In the bootstrap simulations proposed by Simar and Wilson the efficiency scores are generated from two different Data Generating Processes (DGP). This is not consistent with the theory and might ultimately lead to values of the score that are not admissible: in a simple example we show that the scale efficiency pseudo-scores may easily end up taking values above one and this is clearly not allowed, by definition. The problem is traced back to the construction of the DGP for the simulations. To correct this problem, we develop a smooth bootstrap method that guaranties that the pseudo-scores of the efficiency scale have all admissible values. This feature allows us to anchor the test procedure to a consistent DGP of the scale efficiencies.

[1] Recently, Kneip *et al.* (2008, 2009) developed a consistent bootstrap method for inference about the efficiency of a single, fixed point. However, this method requires complicated coding and is not appropriate for approximating the sampling distribution of a test statistic (Simar and Wilson, 2011).

2. Production technology and returns to scale

Consider a production activity that uses the sets of inputs, $x=\{x_i, i=1,\ldots,m\}$ to produce the output vector, $y=\{y_r, r=1,\ldots,s\}$. The production possibility set of this activity is defined as:[2]

$$\Psi=\left\{(x,y)\in \mathbb{R}_{+}^{m+s} \middle| (x,y) \text{ is feasible}\right\}. \quad (1)$$

The returns to scale of the firms are characterized by the way one can either expand the scale in the production set or shrink it or both. That is, the technology exhibits non increasing (*nirs*), non decreasing (*ndrs*) or constant (*crs*) returns to scale depending on the value assumed by the positive scalar α in the following set:

$$\Psi^k=\left\{(x,y)\in\Psi \middle| (\alpha x,\alpha y)\in\Psi \text{ for all } \alpha\in \mathrm{K}^k\right\}, \text{ for } k=nirs, ndrs \text{ and } crs \quad (2)$$

where $\mathrm{K}^{nirs}=[0,1)$, $\mathrm{K}^{ndrs}=[1,\infty)$ and $\mathrm{K}^{crs}=[0,\infty)$. A technology that exhibits *ndrs*, *nirs* or *crs* in different regions of the production frontier is said to be characterized by variable returns to scale (*vrs*). This production possibility set is denoted Ψ^{vrs}.

It is possible to define an input oriented technical efficiency measure in the sense of Farrell (1957) with respect to the various assumptions concerning the returns to scale. That is:

$$\theta^k(x,y)=\min\left\{\theta \middle| (\theta x,y)\in\Psi^k\right\}, \text{ where } k = nirs, ndrs, crs \text{ and } vrs \quad (3)$$

From Färe and Grosskopf (1985), the (technical) efficiency measures defined in (3) can be used to construct scale efficiency measures for each organization. A scale efficiency measure is the ratio of the efficiency measure under *crs* technology and a *vrs* type technology. That is:

$$S_1(x,y)=\theta^{crs}(x,y)/\theta^{vrs}(x,y)\le 1. \quad (4)$$

We say that the production technology is of the *crs* type if $S_1(x,y)=1$. To determine the returns to scale of the technology when $S_1(x,y)<1$ we compute a second ratio that is less restrictive than the initial ratio. That is:

$$S_2(x,y)=\theta^{nirs}(x,y)/\theta^{vrs}(x,y)\le 1. \quad (5)$$

When $S_2(x,y)=1$ the technology is said to exhibit decreasing returns to scale and for values strictly less than one, i.e. $S_2(x,y)<1$, it is said to exhibit increasing returns to scale.

3. Efficiency estimation and statistical model

Because they are unknown, the efficiency measures must be estimated. Given a sample of *n* observations, $\Psi_0=\left\{(x_j,y_j)\right\}_{j=1}^{n}$, a standard Data Envelopment Analysis (DEA) estimator is constructed to assess the Decision Making Units (DMU) efficiency. The smallest convex envelop of the sample gives the DEA estimator of Ψ in the *vrs* case. That is:

$$\hat{\Psi}^{vrs}=\left\{(x,y)\in \mathbb{R}^{m+s} \middle| x\ge\sum_{j=1}^{j=n}\lambda_j x_j,\ y\le\sum_{j=1}^{j=n}\lambda_j y_j, \sum_{j=1}^{j=n}\lambda_j=1\ \forall\ j=1,\ldots,n\right\} \quad (6)$$

To obtain the estimators for the other types of returns to scale, $\hat{\Psi}^{nirs}$ and $\hat{\Psi}^{crs}$ it is sufficient to slightly alter the constraint on the sum of the λ_j. That is, in the *nirs* case we have $\sum_{j=1}^{j=n}\lambda_j\le 1$ and to obtain a *crs* envelop, one simply removes the summation constraints on the λ_j, but obviously keeps the non negativity of the λs.

Farrell's efficiency measures are obtained by substituting $\hat{\Psi}^k$ for Ψ^k in equations (3). This gives:

$$\hat{\theta}^k(x,y)=\min\left\{\theta \middle| (\theta x,y)\in\hat{\Psi}^k\right\} \quad k = vrs, nirs \text{ and } crs. \quad (7)$$

[2] We suppose that the production set Ψ is closed, satisfies free disposal of inputs, is bounded for finite inputs and positive outputs require positive inputs. These are standard assumptions on the technology and are discussed in Färe (1988), among others.

The resulting scale ratio estimators are $\hat{S}_1 = \hat{\theta}^{crs}(x,y)/\hat{\theta}^{vrs}(x,y)$ and $\hat{S}_2 = \hat{\theta}^{nirs}(x,y)/\hat{\theta}^{vrs}(x,y)$.

To ensure the consistency of the estimator, it is necessary to specify a statistical model that allows us to fully characterize the DGP. This is the content of the following assumptions.

Assumption A1: The set of observations $\{(x_j, y_j)\}_{j=1}^{n}$ are identically and independently distributed (*i.i.d.*) random variables with probability density function $f(x,y)$ defined on Ψ.

Assumption A2: The probability density function $f(x,y)$ is continuous on the interior of Ψ, and $f(x^{\partial}(y), y) > 0$ where $x^{\partial}(y) = \theta(x,y)x$ is any point on the frontier of Ψ.

Assumption A3: The efficiency measure $\theta(x,y)$ is differentiable in *x* and *y*.

Assumption A1, A2 and A3 together define the statistical model that allows us to characterize the DGP, denoted $\Im$. In fact, the DGP is entirely characterized by the production possibility set Ψ and the density *f*. That is, $\Im = (\Psi, f)$.

4. Bootstrapping Test Statistics for Returns to Scale

The test procedure is in two steps. The first step consists in testing the null that a given combinations (x,y) is scale efficient, i.e. its technology is of the *crs* type. The alternative hypothesis has to be less restrictive. One natural hypothesis is that the combination (x,y) is characterized by a *vrs*-technology. Then we have:

$$Test\#1: H_0 : S_1(x,y) = 1$$
$$H_A : S_1(x,y) < 1$$

If the hypothesis H_0 is rejected, we still have to identify whether the returns to scale are increasing or decreasing. This will work if we can find a "new" null hypothesis that is less restrictive than the one in the first test. One way of doing this is to suppose that under the null hypothesis the combination (x,y) is subject to decreasing returns to scale (*drs*). Then, the alternative hypothesis would be that the combination (x,y) is subject to an increasing return to scale (*irs*) technology, since constant returns to scale have already been rejected in the first test. Thus, the second test is:

$$Test\#2: H_0^{'} : S_2(x,y) = 1$$
$$H_A^{'} : S_2(x,y) < 1$$

The test statistics for the first and second test are $\hat{S}_1 = \hat{\theta}^{crs}(x,y)/\hat{\theta}^{vrs}(x,y)$ and $\hat{S}_2 = \hat{\theta}^{nirs}(x,y)/\hat{\theta}^{vrs}(x,y)$, respectively. To apply these tests we need to find an approximation of the sampling distribution of the estimators of both scale efficiency ratios, $\hat{S}_1$ and $\hat{S}_2$. This approximation rests on the bootstrap method that consists in identically replicating the empirical DGP many times and study the behavior this set of bootstrapped estimates. To implement the procedure, we first generate, from the original sample Ψ_0, *B* pseudo-samples: Ψ_b^*, $b = 1, \ldots, B$. Then, the original estimation method (DEA in our case) is applied to each pseudo-samples to obtain the bootstrap estimator of the test statistic $\hat{S}_1^*$ for $\hat{S}_1$ (and $\hat{S}_2^*$ for $\hat{S}_2$) which are functions of $\hat{\theta}^{*crs}$, $\hat{\theta}^{*vrs}$ and $\hat{\theta}^{*nirs}$. To generate the pseudo-efficiencies $\hat{\theta}^{*k}, k = crs, vrs, nirs$, we use a homogenous bootstrap methodology developed by Simar and Wilson (1998). This procedure rests on the assumption that the efficiency structure is homogenous.[3] That is, the efficiency score θ is independent of (η, y): $f(\theta|\eta, y) = f(\theta)$.

[3] Because Farrell's measure is radial, we are allowed to write the input vector x in polar coordinates. That is, the modulus of x is $\omega(x) = \|x\| = \sqrt{x^T x}$ and the angle is $\eta = \eta(x) \in [0, \pi/2]^{m-1}$. This allows us to write the density as $f(x,y) = f(\omega, \eta, y)$.

A consistent estimator of *f*, obtained using a kernel estimator and corrected by Schuster's (1985) and Silverman (1985) is defined as follows:

$$\hat{f}^c(t)=\begin{cases}2\hat{g}(t) & \text{if } t\le 1\\ 0 & \text{otherwise}\end{cases}, \text{ where } \hat{g}(t)=\frac{1}{2nh}\sum_{j=1}^{j=n}\left[\phi\left(\frac{t-\hat{\theta}_j}{h}\right)+\phi\left(\frac{t-2+\hat{\theta}_j}{h}\right)\right]. \quad (8)$$

We use a normal Gaussian kernel, denoted ϕ, and the bandwidth, *h*, is set following the normal reference rule (Silverman (1986)). In the procedure proposed by Simar et Wilson (2002), the pseudo-scores $\hat{\theta}^{*k}, k=crs,vrs,nirs$ are generated from $\hat{f}^c$, independently from the null hypothesis. The five step procedure to recover the simulated efficiencies is:

Step 1: Given *k*, compute $\hat{\theta}_j^k=\hat{\theta}_j^k(x_j,y_j)\ \forall j=1,\ldots,n$ using equation (7).

Step 2: Generate smoothed resampled pseudo-efficiencies as follows. First generate $\{\rho_j^*, j=1,\ldots,n\}$ by resampling with replacement a sample of size *n*, from the empirical distribution $\{\hat{\theta}_j^k, j=1,\ldots,n\}$. Then generate the sequence $\{\tilde{\rho}_j^*, j=1,\ldots,n\}$ as follows:

$$\tilde{\rho}_j^*=\begin{cases}\rho_j^*+h\varepsilon_j^* & \text{if } (\rho_j^*+h\varepsilon_j^*)\le 1\\ 2-(\rho_j^*+h\varepsilon_j^*) & \text{otherwise}\end{cases}, \text{ where } \varepsilon_j^* \square N(0,1).$$

Then, generate the pseudo-efficiencies γ_j^* using $\gamma_j^*=\bar{\rho}^*+(\tilde{\rho}_j^*-\bar{\rho}^*)/\sqrt{1+h^2/\hat{\sigma}^2_{\hat{\theta}^k}}$ where $\bar{\rho}^*=(1/n)\sum_{j=1}^n\rho_j^*$.

Step 3: Compute the pseudo variable inputs, $x_j^*=(1/\gamma_j^*)\hat{\theta}_j^k x_j$.

Step 4: Compute the bootstrapped efficiency measures $\hat{\theta}_j^{*k}$ using the pseudo variable inputs based on the following program:

$$\hat{\theta}^{*k}(x,y)=\min\left\{\theta|\theta x\ge\sum_{j=1}^{j=n}\lambda_j x_j^*,\ y\le\sum_{j=1}^{j=n}\lambda_j y_j,\sum_{j=1}^{j=n}\lambda_j=1,\ \lambda_j\ge 0\right\} \text{ if } k = vrs.$$

When *k* = *nirs* or *k* = *crs*, it is sufficient to slightly alter the constraint on the sum of the λ_j. That is, in the *nirs* case we have $\sum_{j=1}^{j=n}\lambda_j\le 1$, and to obtain a *crs* envelop, no constraints on the λ_j are necessary, other than the non negativity ones.

Step 5: Repeat steps 2-5 *B* times to obtain *B* efficiency measures $\{\hat{\theta}_{bj}^{*k}, j=1,\ldots,n, b=1,\ldots,B\}$.

This algorithm is repeated for each type of returns to scale, *k=vrs*, *nirs* and *crs*, and the simulation results are used to compute the pseudo-scores $\hat{S}_1^*=\hat{\theta}^{*crs}/\hat{\theta}^{*vrs}$ and $\hat{S}_2^*=\hat{\theta}^{*nirs}/\hat{\theta}^{*vrs}$. These simulated values are used to estimate the empirical distribution of $(\hat{S}_1^*-\hat{S}_1)$ (and of course $(\hat{S}_2^*-\hat{S}_2)$), the one used to approximate the unknown distribution of the statistic $(\hat{S}_1-S_1)$.

Simar de Wilson (2002) procedure does not take internalize the null hypothesis however when generating the pseudo-scores $\{\hat{\theta}_{bj}^{*k}, j=1,\ldots,n, b=1,\ldots,B, k=vrs,nirs,crs\}$. This is a problem because this procedure may well returns non admissible values of the scale efficiency estimates. That is, as we show in our example below, it may well be possible to obtain test statistic above one, $\hat{S}_1^*>1$ or $\hat{S}_2^*>1$.

To solve this problem, we propose to take into account the null hypothesis in the pseudo-scores generation procedure. The algorithm of Simar and Wilson (2002) is adjusted in this case as follows:

Step 1: Compute $\hat{\theta}_j^{crs}=\hat{\theta}_j^{crs}(x_j,y_j)\ \forall j=1,\ldots,n$ using equation (7).

Steps 2 and 3: As above.

Step 4: Compute the bootstrapped efficiency measures $\hat{\theta}_j^{*crs}$ using the pseudo variable inputs based on the following program:

$\hat{\theta}^{*crs}(x,y) = \min\left\{\theta|\theta x \geq \sum_{j=1}^{j=n}\lambda_j x_j^*,\ y \leq \sum_{j=1}^{j=n}\lambda_j y_j, \lambda_j \geq 0\right\}$;

and

$\hat{\theta}^{*vrs}(x,y) = \min\left\{\theta|\theta x \geq \sum_{j=1}^{j=n}\lambda_j x_j^*,\ y \leq \sum_{j=1}^{j=n}\lambda_j y_j, \sum_{j=1}^{j=n}\lambda_j = 1,\ \lambda_j \geq 0\right\}$.

Step 5: Repeat steps 2-5 *B* times to obtain *B* efficiency measures $\left\{\hat{\theta}_{bj}^{*k}, j = 1,\ldots,n, b = 1,\ldots,B, k = vrs, crs\right\}$.

If the test concludes that we reject the null hypothesis, then one conduct the second test. To generate the data in this second case, we use the following:

Step 1: Compute $\hat{\theta}_j^{nirs} = \hat{\theta}_j^{nirs}(x_j, y_j)\ \forall\ j = 1,\ldots,n$ using equation (7).

Steps 2 and 3: As above.

Step 4: Compute the bootstrapped efficiency measures $\hat{\theta}_j^{*nirs}$ and $\hat{\theta}_j^{*vrs}$ using the pseudo variable inputs based on the following program:

$\hat{\theta}^{*nirs}(x,y) = \min\left\{\theta|\theta x \geq \sum_{j=1}^{j=n}\lambda_j x_j^*,\ y \leq \sum_{j=1}^{j=n}\lambda_j y_j, \sum_{j=1}^{j=n}\lambda_j \leq 1,\ \lambda_j \geq 0\right\}$;

and

$\hat{\theta}^{*vrs}(x,y) = \min\left\{\theta|\theta x \geq \sum_{j=1}^{j=n}\lambda_j x_j^*,\ y \leq \sum_{j=1}^{j=n}\lambda_j y_j, \sum_{j=1}^{j=n}\lambda_j = 1,\ \lambda_j \geq 0\right\}$.

Step 5: Repeat steps 2-5 *B* times to obtain *B* efficiency measures $\left\{\hat{\theta}_{bj}^{*k}, j = 1,\ldots,n, b = 1,\ldots,B, k = vrs, nirs\right\}$.

The correction we have made to the algorithm of Simar et Wilson (2002) ensures that the values assumed by $\hat{S}_1^* = \hat{\theta}^{*crs}/\hat{\theta}^{*vrs}$ and $\hat{S}_2^* = \hat{\theta}^{*nirs}/\hat{\theta}^{*vrs}$ are all admissible. That is, $\hat{S}_2^* \leq 1$ and $\hat{S}_1^* \leq 1$.

5. An Example

To illustrate the difference between the two procedures, we use a simple example based on six DMU producing one output with two inputs.

Table 1 : The Data

DMU	Output	Input1	Input2
A	54	86	16
B	24	29	10
C	36	43	11
D	14	24	6
E	7	11	2
F	12	11	4

The tables below present the results for ten bootstrap simulations, B=10. (We have decided to keep the size of the problem manageable and not complicate it through unnecessary computations.) The first two tables presents the results using the approach proposed by Simar and Wilson (2002) and then the next two tables present the results with our proposed method. These simulations are conducted using a SAS program developed by the authors.

Tables 2 and 3 presents the simulation results for $\hat{S}_1^*$ and $\hat{S}_2^*$ based on Simar and Wilson (2002). In both tables, the first column gives the initial value of the score, that is either $\hat{S}_1$ or $\hat{S}_2$. The rest of the tables consist of the pseudo-values. In the Table 1, there are seven pseudo values that are larger than one, that is $\hat{S}_1^*$ > 1. These pseudo scale efficiency measures are deemed not admissible. Table 2 contains one pseudo-value that exceeds one, $\hat{S}_2^*$ > 1.

Table 2: Pseudo-scores for $\hat{S}_1^*$ (Simar et Wilson approach)

Initial value	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9	COL10
0,969231	0,951194	0,967418	1	0,999755	0,977953	1	0,951203	0,879412	0,669312	1
0,849268	**1,016939**	**1,01232**	0,974726	1	**1,148769**	0,878386	0,718342	**1,002204**	**1,032732**	0,870379
1	0,875658	1	**1,058365**	0,960059	1	0,781173	0,78774	1	0,996856	0,923959
0,991553	**1,070533**	0,944172	0,904234	0,936011	0,7959	0,944589	0,918447	0,993264	1	0,990099
1	1	1	0,929621	1	0,692547	0,799471	0,888334	1	0,72845	0,886349
1	1	0,989176	1	0,828294	0,879016	1	1	1	1	1

Table 3: Pseudo-scores for $\hat{S}_2^*$ (Simar et Wilson approach)

Initial value	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9	COL10
1	1	1	1	1	1	1	1	1	1	1
1	0,991706	0,999977	1	1	0,96343	1	0,99811	0,99093	0,999942	1
1	0,991178	1	0,9919	1	1	0,990446	1	1	1	0,991651
0,991553	0,999814	0,992814	1	0,981533	0,923768	1	**1,001051**	0,996495	1	1
1	1	1	1	1	0,716604	0,757568	1	1	0,722879	0,988685
1	1	1	0,984433	1	0,888273	1	1	1	1	1

Table 4 and 5 present the simulation results after we corrected Simar and Wilson's algorithm. These results present no anomalies, all pseudo values are less than or equal to one, as expected.

Table 4: Corrected pseudo-scores for $\hat{S}_1^*$

Initial value	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9	COL10
0,969231	0,951194	0,967418	1	0,999755	0,977953	1	0,951203	0,879412	0,669312	1
0,849268	0,923675	1	0,974726	1	0,966736	0,878386	0,657699	0,888374	1	0,870379
1	0,905552	1	0,983375	0,960059	1	0,878834	0,78774	1	0,996856	0,866408
0,991553	0,982198	0,979792	0,983211	0,999964	0,7959	0,975296	0,990665	0,993264	1	0,990099
1	1	1	0,929621	1	0,692547	0,799471	0,888334	1	0,72845	0,886349
1	1	0,989176	1	0,828294	0,879016	1	1	1	1	1

Table 5: Corrected pseudo-scores for $\hat{S}_2^*$

Initial value	COL1	COL2	COL3	COL4	COL5	COL6	COL7	COL8	COL9	COL10
1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	0,966736	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
0,991553	1	0,979792	0,983211	1	0,7959	0,975296	1	0,993264	1	0,990099
1	1	1	0,929621	1	0,692547	0,799471	0,888334	1	0,72845	0,886349
1	1	0,989176	1	0,828294	0,879016	1	1	1	1	1

6. Conclusion

In this note we have developed a procedure to test non parametric statistical hypothesis concerning the scale efficiency of organization. Because there is no asymptotic distribution for the test statistic under the null hypothesis, we used a smooth bootstrap methodology to approximate it. This approach allows us to estimate the *p*-values of the test and to determine a decision rule to accept or reject the null hypothesis.

The main characteristic of our test is its consistency under the null hypothesis. In the procedure developed by Simar and Wilson (2002), the data generating process to generate the test statistic may return values that are above one, which is not possible theoretically. Our procedure corrects this flaw and allows us to test the returns to scale with a consistent test.

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