OPTIMAL PRODUCTION PLANNING FOR ICI PAKISTAN USING LINEAR PROGRAMMING AND SENSITIVITY ANALYSIS

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1. Abstract

This paper estimates an optimal production levels for the different products manufactured at ICI, a multinational company in Pakistan. The revised simplex method is used to maximize the profit generated in 2010 subjected to cost resource constraints. The production of Polyester, Soda Ash, Paints and Chemicals are taken into consideration. The production of the Soda Ash is most productive contributing more to the objective function. In the year 2010, the company was earning R.s 3, 273,756,000 from the production of these products. This amount raises by R.s 189,708, R.s 989,238, R.s 15,594,377 and R.s 45,408,040 by changing production patterns within the first, second, third and fourth digits respectively. The company can earn significant profit by operating on the proposed production forecasts. The top management and decision makers can maximize the profit of the company within the name plate production capacity, setting up the future goals and outlook of the company.

KEYWORDS: Production Planning, Linear Programming, Sensitivity, Simplex Method, Supply Chain, Management Science.

2. Introduction

Linear programming is a powerful tool for the optimal allocation of scares resources with the objective of maximization of profit. Simplex method first devised by Dantzig in 1947 is used to solve LP's. He then extended the method for planning/scheduling dynamically. As such the development of a mathematical model is necessary in order to make best choice among several alternatives using its numerical values (Dantzig, 1963), (Adams, 1969), (Hiller *et al.*, 1995). The noble laureate Leonid Kantorovich (USSR) and Tjalling Koopmas (USA) were awarded for their work on the optimal allocation of resources using the technique of linear programming. Bierman and Bonini (1973) pointed its usefulness in decision making process of making the best choice with several different alternatives. Linear programming is about making rules and relations with limited funds and technological restrictions (Andrade, 1990). David (1982) and Nearing and Tucker (1993) emphasized the application of the tool in tactical and strategic management. The simplex method is regarded as the most important and credible method devised of the mid 20th century. Now a day it is a benchmark optimizing tool saving thousands and millions of dollars in many organizations. Linear programming can be effectively applied to diverse fields including transportation, telecommunication, energy, blending and production, airline crew scheduling, network flows (Winston and Albright, 2000), (Anderson *et al.*, 2002).

Linear programming has been used in operational management such as aggregate production planning, service productivity, product planning, product routing, process control, inventory control and distribution scheduling, plant location and material handling (Manley and Threadgill, 1991), (Zappe *et al.*, 1993), (Jacob *et al.*, 1996). Linear programming works for maximizing the company's profits with the minimal consumption of resources (Chopra and Meindl, 2001), (Thomas, 2002), (Stadtler, 2000), (Taghrid and Hassan, 2009), (Fagoyinbo *et al.*, 2011). This research takes into consideration the sale/production of the four main products of ICI Pakistan Limited. The profit and loss data has been obtained from the keenly prepared annual book of the year 2010. The study points out the product that is contributing more to the objective function (profit). The simplex method is used to get the best possible consumption of the resources (cost) of the problem for ICI Pakistan. As a matter of nature some bottleneck may occur, e.g. the demand for one product may be greater than other. This research considers such bottlenecks in the formulation and modeling of the linear programming problem.

3. MATERIALS AND METHODS

A linear programming problem with "n" decision variables and "m" constraints can be mathematically modeled as (Taha, 1975), (Zeleny, 1982), (Winston, 1995), (Higle and Wallence, 2003).

$$Maximize..z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

$$s.t$$

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2$$

$$.$$

$$.$$

$$.a_{n1}x_1 + a_{22}x_2 + \ldots + a_{mn}x_n \le b_m$$

$$x_{1,}x_2,\ldots x_n \ge 0$$

This can be written as,

$$\max z = c^t x$$

$$s.t,$$

$$Ax \le b,$$

$$x \ge 0$$

ICI Pakistan is a limited company enlisted in Karachi, Lahore and Islamabad Stock Exchange. The company is engaged in manufacturing of Polyester, Soda Ash, Paints and Chemicals. The profit and loss data of the manufactured products of the company is obtained from the annual book of the year 2010. In this regard, the commission and discounts paid by the company are presented in Table 1.

Table 1 Tax, Commission, Duty, Discounts to Distributors and Customers.

Table 2 contains the cost of goods sold of the four mentioned products. The selling and distribution expenses and the administration and general expenses incurred on the products in 2010 are presented in Table 2 and Table 3 respectively.

Table 2 Cost of Goods Sold.

Table 3 Selling and Distribution Expenses.

Table 4 Administration and General Expenses.

The per metric tons/per kilo liters profit and expenses of the company are summarized in the Table 5.

Table 5 Summarized Profit and Loss of the Company.

Let x1, x2, x3, and x4 represent the quantity sold/produced of the Polyester, Soda Ash, Paints and Chemicals respectively, then the initial linear programming model can be formulated as,

Maximize z=15586.93x1+3066.59x2+3885.23x3+14404.69x4

Subject to

408.02x1+967.14x2+28533.76x3+9711.33x4<=1472112000

120411.1x1+17385.47x2+96443.35x3+138703.3x4<=26179724000

559.18x1+696.13x2+22133.74x3+9085.12x4<=1181683000
2044.18x1+996.05x2+12985.97x3+9413.92x4<=1150763000
x1>=129730
x2>=291860
x3>=34566
x4>=15508

The next section discusses the solution of the initial model using the Excel Solver.

4. RESULTS AND DISCUSSIONS

The solution of the initially proposed model is obtained by utilizing the Microsoft Excel® solver. The solution of the model is presented in Figure 1.

Figure 1 Answer Report.

Figure 2 Limits Report.

The answer report of the initial linear programming model shows that the company can generate a profit of R.s 3273786300, an amount R.s 30,300 greater than the presently operating profit. The cost of goods sold, quantity sold of polyester, paints and chemicals are binding constraints and they are consumed fully whereas all the other constraints are non binding and are available for the future production runs. The limits report in Figure 2 shows the lower and upper limits of the variables in which the solution is optimal.

4.1 Sensitivity Analysis and Different Production Runs

The sensitivity analysis of the model gives important information about the manufacturing process (Kinc, 2008).The sensitivity report of the model is given in the Figure 3.

Figure 3 Sensitivity Report.

The reduced cost shows that quantity produced of the soda ash and then that of polyester takes less amount of cost in the production run whereas that of quantity of paints and chemicals consume much of the cost. The shadow price shows that an R.s 1 spent as the cost of goods sold is contributing at the rate of R.s 0.176 to the maximization of the profit for future production runs based on the initial linear programming model. Several production runs of the optimized model on various production spaces within the nameplate production capacity are summarized in the Table 6.

Table 6 Various Production Runs in The Name Plate Capacity.

5. CONCLUSIONS AND RECOMMENDATIONS

The techniques of linear programming and sensitivity analysis were used to maximize the profit generated from the production patterns of the ICI Pakistan. Four different products manufactured at the company were taken into consideration. The analysis predicted that the production of Soda Ash is contributing more than other products to the objective function. The company is already a prime supplier of the Soda Ash in the region. The sensitivity analysis reveals the fact that a cost of R.s 1 spent as the cost the goods sold returns at the rate of R.s 0.176. The company can save profits of R.s 189,708, R.s 989,238, R.s 15,594,377 and R.s 45,408,040 by changing its production space within the first, second, third and fourth digits respectively. The research reveals that among the other products Soda Ash is more profitable to the company and the company should give more attention to its production to maximize its profit. The research is significant in the sense that it will assist the top management of the company in making corrective decisions well in time using the methods of linear programming. This will determine the future production patterns and outlook resulting in the establishment of new production units, while planning for maximizing profits of the company.

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7. TABLES

Table 1 Tax, Commission, Duty, Discounts to Distributors and Customers.

	Polyester (000) R.s	Soda Ash (000) R.s	Paints (000) R.s	Chemicals (000) R.s
Sales Tax	0	1,263,579	900,599	290,709
Excise Duty	0	60,160	54,115	7,546
Commission and Discount to Distributors and Customers	52,933	282,270	986,298	150,611
Total	52933	1606009	1941012	448866

Table 2 Cost of Goods Sold.

	Polyester (000) R.s	Soda Ash (000) R.s	Paints (000) R.s	Chemicals (000) R.s
Raw Material Consumed	13,419,368	1,748,082	2,931,861	945,199
Salaries, Wages and Benefits	336,069	540,016	96,312	39,211
Stores and Spares Consumed	118,723	119,593	3,704	3,510
Conversion Fee Paid to Contract Manufacturers	0	0	0	4,341
Oil, Gas and Electricity	1,394,075	1,962,927	28,851	7,523
Rent, Rates and Taxes	1,234	1,379	15,893	8,612
Insurance	17,722	16,236	27,849	973
Repairs and Maintenance	1,670	695	15,585	3,242
Depreciation and Amortization	319,963	455,542	75,205	15,087
Technical Fees	0	0	23,270	5,750
Royalty	0	0	0	24,862
General Expenses	106,805	83,627	67,637	10,965
Opening Stock of Work in Progress	54,163	0	15,600	1,655
Closing Stock of Work in Progress	-24,388	0	-10,976	-725
Cost of Goods Manufactured (Total)	15,745,404	4,928,097	3,290,791	1,070,205
Opening Stock of Finished Goods	509,236	207,554	246,586	141,658
Finished Goods Purchased	91,316	0	64,800	1,193,235
Closing Stock of Finished Goods	-725,027	-58,912	-246,547	-233,838
Provision for Obsolete Stocks	0	-2,615	-21,969	-20,250
Total	15620929	5,074,124	3,333,661	2,151,010

Table 3 Selling and Distribution Expenses.

	Polyester (000) R.s	Soda Ash (000) R.s	Paints (000) R.s	Chemicals (000) R.s
Salaries and Benefits	46,473	21,759	220,879	61,696
Repair and Maintenance	14	1,079	3,933	1,381
Advertising and Publicity	1,163	10,948	253,121	817
Rent, Rates and Taxes	0	1,282	19,323	739
Insurance	0	1,028	0	3,530
Lighting, Heating and Cooling	15	1,088	6,683	2,164
Depreciation and Amortization	0	277	0	2,151
Outward Freight and Handling	9,626	149,095	179,945	31,538
Traveling Expenses	7,022	2,703	30,082	12,171
Postage, Telegram, Telephone and Telex	529	1,325	7,046	3688
General Expenses	7,701	12,589	44,063	21,017
Total	72,543	203,173	765,075	140,892

Table 4 Administration and General Expenses.

	Polyester (000) R.s	Soda Ash (000) R.s	Paints (000) R.s	Chemicals (000) R.s
Salaries and Benefits	129,271	191,647	168,579	81,934
Repair and Maintenance	2,986	3,930	9,147	1,003
Advertising and Publicity	1,837	3,681	1,372	866
Rent, Rates and Taxes	2,675	2,954	5,060	680
Insurance	735	1,858	692	425
Lightening, Heating and Cooling	3,637	6,200	4,655	1,416
Depreciation and Amortization	15,070	19,781	15,439	9,651
Provision for Doubtful Debts-Trade	0	0	138,262	401
Others	0	381	2,500	0
Provision for Obsolete Stock	0	2,615	21,969	20,250
Provision for Obsolete Spare	59,100	0	5,000	0
Traveling Expenses	10,138	9,652	12,732	6,791
Postage, Telegram, Telephone and Telex	2,329	3,948	5,814	1,678
General Expenses	37,143	44,061	57,652	20,896
Total	265,191	290,708	448,873	145,991

Table 5 Summarized Profit and Loss of the Company.

	Polyester Per Metric Tones (R.s)	Soda Ash Per Metric Tones(R.s)	Paints Per Kilo Liters (R.s)	Chemicals Per Metric Tones (R.s)	Total Available
Profit	15586.93	3066.59	3855.23	14406.69	
Commission and Discount	408.02	967.14	28533.76	9711.33	1472112000
Cost of Goods Sold	120411.1	17385.47	96443.35	138703.3	26179724000
Selling and Distribution Expenses	559.18	696.13	22133.74	9085.12	1181683000
Administration and General Expenses	2044.18	996.05	12985.97	9413.92	1150763000

Table 6 Various Production Runs in The Name Plate Capacity.

Production Runs		First digit Production Dynamics	Second Digit Production Dynamics	Third digit Production Dynamics	Forth Digit Production Dynamics
1	Allowable Range (Production Space) Metric tones, x3 in (000) Liters	X1 ↓ 6 X2 ↓4 X3 ↓6 X4 ↓8	X1 ↓ 30 X2 ↓60 X3 ↓60 X4 ↑00	X1 ↓ 700 X2 ↓800 X3 ↓500 X4 ↓500	X1 ↓ 6000 X2 ↓6000 X3 ↓1000 X4 ↓2000
	Optimal Solution	Z=3273945708 x1 = 129730 x2=291957.13 x3 = 34560 x4 = 15500	Z=3274745238 x1 = 129700 x2=292400.64 x3 = 34506 x4 = 15508	Z=3289350377x 1 = 129030 x2=303470.93 x3 = 34066 x4 = 15008	Z=3319164040 x1 =127588.57 x2 =328195.06 x3 = 33566 x4 = 13508
	Benefits (R.S)	189,708	989,238	15,594,377	45,408,040
2	Allowable Range (Production Space) Metric tones, x3 in (000) Liters	X1 ↓ 5 X2 ↓3 X3 ↓5 X4 ↓7	X1 ↓ 20 X2 ↓50 X3 ↓50 X4 ↑10	X1 ↓ 600 X2 ↓700 X3 ↓400 X4 ↓400	X1 ↓ 5000 X2 ↓5000 X3 ↓0000 X4 ↓1000
	Optimal Solution	Z=3273916841 x1 = 129731 x2 =291936.68 x3 = 34561 x4 = 15501	Z=3274456565 x1 = 129710 x2 =292196.13 x3 = 34516 x4 = 15518	Z=3286463647 x1 = 129130 x2 =301425.79 x3 = 34166 x4 = 15108	Z=3285553934x 1 =129427.68 x2=301931.95 x3 = 34566 x4 = 14508
	Benefits (R.S)	160,841	700,565	12,707,647	11,797,934
3	Allowable Range (Production Space) Metric tones, x3 in (000) Liters	X1 ↓ 4 X2 ↓2 X3 ↓4 X4 ↓6	X1 ↓ 10 X2 ↓40 X3 ↓40 X4 ↑20	X1 ↓ 500 X2 ↓600 X3 ↓300 X4 ↓300	No Optimal Solution
	Optimal Solution	Z=3273887974 x1 = 129732 x2 =291916.23 x3 = 34562 x4 = 15502	Z=3274167892 x1 = 129720 x2 =291991.61 x3 = 34526 x4 = 15528	Z=3283576918 x1 = 129230 x2 =299380.64 x3 = 34266 x4 = 15208	
	Benefits (R.S)	131,974	411,892	9,820,918	
4	Allowable Range (Production Space) Metric tones, x3 in (000) Kilo Liters	X1 ↓ 3 X2 ↓1 X3 ↓3 X4 ↓5	No Optimal Solution	X1 ↓ 400 X2 ↓500 X3 ↓200 X4 ↓200	No Optimal Solution
	Optimal Solution	Z=3273859107 x1 = 129733 x2 =291895.78 x3 = 34563 x4 = 15503		Z=3280508598 x1=129362.12 x2 = 297112.98 x3 = 34366 x4 = 15308	
	Benefits (R.S)	103,107		6,752,598	
5	Allowable Range (Production Space) Metric tones, x3 in (000)Liters	X1 ↓ 2 X2 ↓0 X3 ↓2 X4 ↓4	No Optimal Solution	X1 ↓ 300 X2 ↓400 X3 ↓100 X4 ↓100	No Optimal Solution
	Optimal Solution	Z=3273830239 x1 = 129734 x2 =291875.33 x3 = 34564 x4 = 15504		Z=3277147587x 1 = 129546.03 x2 = 294486.67 x3 = 34466 x4 = 15408	
	Benefits (R.S)	74,239		3,391,587	

↓ = Quantiy Decreased By
↑= Quantity Increased By

8. FIGURES

Figure 1 Answer Report.

Name	Original Value	Final Value
PROFIT	3273786300	3273786300

Name	Original Value	Final Value
QUANTITY SOLD POLYSTER	129730	129730
QUANTITY SOLD SODA ASH	291860.0276	291860.0276
QUANTITY SOLD PAINTS	34566	34566
QUANTITY SOLD CHEMICALS	15508	15508

Name	Cell Value	Status	Slack
COMISSION AND DISTCOUNT	1472103196	Not Binding	8804.48129
COST OF GOODS SOLD	26179724000	Binding	0
SELLING AND DISTRIBUTION COST	1181681840	Not Binding	1159.768352
ADMIN AND GENERAL COST	1150762762	Not Binding	237.7026069
QUANTITY SOLD POLYSTER	129730	Not Binding	129730
QUANTITY SOLD SODA ASH	291860.0276	Not Binding	291860.0276
QUANTITY SOLD PAINTS	34566	Not Binding	34566
QUANTITY SOLD CHEMICALS	15508	Not Binding	15508
QUANTITY SOLD PAINTS	34566	Binding	0
QUANTITY SOLD POLYSTER	129730	Binding	0
QUANTITY SOLD SODA ASH	291860.0276	Not Binding	0.027626518
QUANTITY SOLD CHEMICALS	15508	Binding	0

Figure 2 Limits Report.

Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
QUANTITY SOLD POLYSTER	129730	129730	3273786300	129730	3273786300
QUANTITY SOLD SODA ASH	291860.0276	291860	3273786215	291860.0276	3273786300
QUANTITY SOLD PAINTS	34566	34566	3273786300	34566	3273786300
QUANTITY SOLD CHEMICALS	15508	15508	3273786300	15508	3273786300

Figure 3 Sensitivity Report.

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
QUANTITY SOLD POLYSTER	129730	-5652.151475	15586.93	5652.151475	1E+30
QUANTITY SOLD SODA ASH	291860.0276	0	3066.59	1E+30	816.0819578
QUANTITY SOLD PAINTS	34566	-13156.22915	3855.23	13156.22915	1E+30
QUANTITY SOLD CHEMICALS	15508	-10058.91256	14406.69	10058.91256	1E+30

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
COMISSION AND DISTCOUNT	1472103196	0	1472112000	1E+30	8804.48129
COST OF GOODS SOLD	26179724000	0.176388099	26179724000	4148.959933	480.2999976
SELLING AND DISTRIBUTION COST	1181681840	0	1181683000	1E+30	1159.768352
ADMIN AND GENERAL COST	1150762762	0	1150763000	1E+30	237.7026069