

## Modeling the Structural Shifts in Real Exchange Rate with Cubic Spline Regression (CSR). Turkey 1987-2008

**Dr. Bahar BERBEROĞLU**

Anadolu University, Open Education Faculty.  
Eskisehir, 26470. Turkey

E-mail: bdirem@anadolu.edu.tr, Phone: 90 222 3350580/2553

**Prof. Dr. C. Necat BERBEROĞLU** (Corresponding Author)

Faculty of Economic and Administrative Sciences  
Anadolu University

Eskisehir, 26470. Turkey

E-mail: nberbero@anadolu.edu.tr, Phone: 90 222 3350585/3241

### Abstract

*Economic life is continuous but time series presenting economic data contain some break points because of sharp changes and shifts because of important shocks such as economic crises. In such cases many important structural shifts occur in some parameters of economic models. These shifts on economic parameters generally may differ and occur with different time lags. In this study it is aimed to expose and analyze the occurrence of structural shifts and structural changes in Real Exchange Rate in Turkey during the period of 1987-2008. We used cubic spline regression in modeling the structural changes in real exchange rate in Turkey. With this technique, several cubic spline regression models of real exchange rate are constructed and discussed, and the most significant one is chosen. Then in this study, we also tried to put forward how prediction sum of squares statistic residuals can improve the analysis in cubic spline models.*

**Keywords: Cubic Spline Regression, Structural Change, Real Exchange Rate, PRESS statistics.**

**JEL Codes: C01, C10, C22, C51**

### 1. INTRODUCTION

Economic time series are usually affected by numerous internal or external shocks and variables. Although the economic life is continuous, some events such as wars, economic or political crises generally cause abrupt and sharp changes in the data collected in their periods. But mostly leaps or jumps do not occur in these time series, so they keep their continuity with some structural shifts or structural changes. When a statistician and of course any other researcher such as an econometrician sees some break points, knots or structural shifts in a time series in which he attempts to use in forecasting, or simply he has a priori reasons to suspect that important structural changes have occurred in that time series, spline functions offer a way to use these prior information. Structural shifts are unexpected changes in the data pattern, but these changes do not always imply discontinuity in the data. Spline functions preserve the continuity of a time series without creating jumps and by this way these functions improve forecasts when applied to data sets containing all kinds of structural changes.

Spline functions offer a relatively simple technique to fit a function through a series of points. A spline function occurs when two or more polynomial functions are connected end to end. The biggest advantage of spline functions is their flexibility in approximation. Each portion of the function may be structurally similar or different. The spline function is less restrictive than imposing the same structure on all subsets of the sample. With the use of regression analysis, trends are estimated to predict the future. For example, when important structural changes have occurred in almost all kinds of economic time series or data sets such as national income values, economic growth rates, exports, imports, price indexes and interest rates, spline functions provide a very useful additional tool for time series forecasting. Spline functions have been used primarily in statistics then in business and econometrics. They were previously used by statisticians for testing structural shifts in various data sets; afterwards other researchers used splines in testing time series of profits, interest rates, economic growth rates, exports and imports which usually contain structural shifts (Marcus, 1987/1988). Whenever there is the suspicion of a structural change in a time series, a spline function allows that a priori belief to be included in the model. In the language of Bayesian statistics, spline functions can be said to offer a 'prior' that structural change has occurred.

A spline function joins a time series of observations at points called join points, knots, or break points occurring in time (Marcus, 1987/1988). Marcus (1987/1988) explained a quadratic spline with a single join point and estimated quadratic spline with using dummy variables. Poirier (1973) showed how cubic splines can be used to test the structural changes. In Poirier's study, spline theory and piecewise regression theory were integrated to provide a framework in which structural change is viewed as occurring smoothly. Specifically, structural change occurs at given points through jump discontinuities in the third derivative of a continuous piecewise cubic estimating function. In the study testing procedures were developed for detecting structural change. Poirier's study used cubic spline because of its actual piecewise nature which is justified from a theoretical standpoint. Attention is focused on those piecewise regression models when; (1) the piecewise nature reflects the occurrence of structural change, (2) the pieces are defined over adjacent intervals of the independent variable, (3) the overall model is continuous with structural change revealing itself with jump discontinuities in the third derivative, and (4) the points of structural change are known (Poirier, 1973).

Poirier (1973) proposed an excellent discussion of the basic theory of cubic regression splines. Arguing from an economic point of view, he developed the idea that structural change occurs in a smooth fashion so that splines form a natural tool for analyzing structural changes. He argued that knots should occur at a point in time near the point of structural change. Poirier illustrated his point with an example based on Indianapolis 500 data. Buse and Lim (1977) followed up Poirier's paper, arguing that cubic spline regression is a special case of restricted least squares and that the latter approach offered a richer menu of procedures (Wegman and Wright, 1983). Gallant and Fuller (1973) treated the join points (knots) as unknown parameters and developed procedures for estimating their location. In the study of Buse A., and Lim L., (1977) the standard cubic spline regression method was used and this method was shown as to be a special case of the restricted least squares (RLS) estimator. In that study the equivalence of the two procedures under a common set of restrictions was proved. The greater flexibility of the restricted least squares estimator in terms of the number of restrictions and test of the hypotheses were also obviously shown in the study.

Wold (1974) reflected a lot of experience with fitting regression splines. Based on his practical experience, Wold (1974) made some useful recommendations for knot points selection. These recommendations were based on the assumption of fitting a cubic spline, which is the most popular case, and may need some modification for degree of the spline function,  $m > 3$ .

1. Knot points should be located at data points.
2. A minimum of four or five observations should fall between knot points.
3. No more than one extremum and one inflection point should fall between knots (because a cubic could not fit more).
4. Extrema should be centered in intervals and inflection points should be located near knot points.

A useful statistical approach for comparing the forecasting performance of different models is the (Predicted Residuals Sum of Squares) PRESS statistics proposed by Allen (1974). PRESS statistics is based on the leave-one-out or jackknife technique. The idea is to fit the model without the  $i$ th observation  $x_i$  and use this fitted model to predict the response  $\hat{y}_i$  at  $x_i$ . There is a well-known formula for computing prediction sum of squares (PRESS) residuals in a regression problem without having to refit the curve for each observation. Tarpey (2000a) showed that the same basic results hold for fitting a regression function when the regression coefficients are subject to linear restrictions.

## **2. TURKISH REAL EXCHANGE RATE IN THE PERIOD OF 1987-2008**

The rapid economic development in Turkey which started in early 1960s with the implementation of planned industrialization and modernization model based upon import substitution strategy, has turned into a severe crisis as a result of the adjustment problems and the oil crisis of 1970s. In the beginning of crisis period the growth rate slowed down and then turned to negative, inflation began to increase, finding funds for imports became harder and scarcity of goods emerged in the internal markets, and Turkey had almost come to an end economically in 1980. The Stability Measures of the 24th of January, 1980 aimed maintaining short-term economic stability, structural adjustment in the economy, and changing long-term accumulation regime. Even though IMF supported stability programs had been implemented before 1980, with the military intervention of the 12th of September, 1980 the relations with IMF altered and, the 24th of January program and its results changed in content and influences. The structural adjustment policies of the IMF and World Bank became the major determinants of Turkey's economic policies in the early 1980s.

International trade played a minor role in the Turkish economy before 1980, but grew rapidly after the 1980 reforms. Especially exports and imports started to increase rapidly after 1983 with the “Government Change”. New government emphasized foreign trade liberalization. After 1983 economic reforms promoted foreign trade by removing price controls, decreasing subsidies, reducing tariffs. In addition to rapid growth in both exports and imports, the reforms brought a change in the structure of foreign trade, and the predominant role of agricultural products came to an end with the emergence of a greater emphasis on industrial products. Turkey signed a free trade agreement with the European Free Trade Association (EFTA) in 1991. In 1992, Turkey and 10 other nations in the Black Sea region formed the Black Sea Economic Cooperation Organization. Turkey became a member of the World Trade Organization (WTO) in 1995, and a year later signed a Customs Union Agreement with the EU. Turkey also became a member of the Organization for Economic Cooperation and the Organization of the Islamic Conference. Separately, Turkey has entered into free trade agreements with Israel, and with several Central and Eastern European countries.

During these developments in Turkey’s international relations, Turkish Economy experienced several economic crises’ and implemented several stabilization programs. Some of these economic crises’ emerged from domestic political problems and government changes and some others arise from wars and international crises and consequently all of these developments had great impacts on Turkey’s exports and imports. The most important stabilization programs in 1987-2008 period were, 5th of April 1994 Stabilization Program which had impacts in the same year, and 21st of January 2001 Stabilization Program whose impacts were seen in 2002. The major effects of all these events were seen of course on the values of Real Exchange Rate. The changes in Turkey’s Real Exchange Rate in this period can easily be seen in the Figure 1.

Figure 1. Based on 1 USD+1.5 EUR basket, in the relative price calculations and consumer prices for Turkey are used. (1987. JAN. = 100)

In the years of economic crises and sharp depreciations of TL almost all of macroeconomic parameters of Turkish Economy were affected negatively at different degrees and with different time lags. When the value of Real Exchange Rate is considered we assumed that, structural changes occurred in data without any time lag. So our hypothesis is that, the sharp depreciation of TL in the year of 1994 and the Economic Crisis in 2001 were the most important events of 1987-2008 period in Turkish Economy and structural changes in the value of Real Exchange Rate occurred in these years. For this reason, we applied cubic spline regression on these three parameters and formed the knots  $t_1=8$  for 1994 and  $t_2=15$  for 2001. When Figure 1 is observed it could be understood that the model fits four (4) conditions of Wold (1974) which was mentioned before, and also it could be understood that cubic spline regression provides good modeling in such time series. Since in this technique knots should be located at data points and five observations should fall between knot points, and the inflection points should fall between knots and should be located near knot points.

**3. MODEL (CUBIC SPLINE REGRESSION (CSR) AND PRESS STATISTICS)**

Spline regressions are splines which are computed according to a regression model. The model is generally as follows,

$$y_i = S_{\Delta}(x_i) + \varepsilon_i, \quad i = 1,2, \dots, n \tag{3.1}$$

Where  $S_{\Delta}(x)$  is a piecewise polynomial spline with mesh  $\Delta$  and  $\varepsilon_i$  are a white noise sequence. The problem then is to compute  $S_{\Delta}(x)$  based on a least squares approach. Given the mesh of knots,  $\Delta=\{t_1<t_2<\dots<t_N\}$ , Smith (1979) shows that a piecewise or segmented polynomial may be represented in the Heaviside function notation; that is, if we let  $u_+ = u$  if  $u>0$  and  $u_+ = 0$  if  $u \leq 0$ , then the general form of a segmented polynomial regression model, (3.1), with  $N$  knots in the mesh  $\Delta$  having  $N+1$  polynomial pieces each of degree  $m$  may be rewritten as follows (Wegman and Wright, 1983):

$$y_i = \sum_{j=0}^m \beta_{0j} x_t^j + \sum_{k=1}^N \sum_{j=0}^m \beta_{kj} (x_i - t_k)_+^j + \varepsilon_i \tag{3.2}$$

In general, with  $N$  knots and the  $N+ 1$  polynomial pieces each of degree  $m$  may be written a spline model with no continuity restrictions as follows:

$$S_{\Delta}(x) = \sum_{j=0}^m \beta_{0j} x_t^j + \sum_{k=1}^N \sum_{j=0}^m \beta_{kj} (x_i - t_k)_+^j. \tag{3.3}$$

As in the special case just discussed, the presence of a  $\beta_{kj}(x_i - t_k)_+^j$  term allows a discontinuity at  $t_k$  in the  $j$ th derivative of  $S_\Delta$ ,  $S_\Delta^{(j)}$  ( $S_\Delta^{(0)}=S_\Delta$ ), and its absence forces the continuity of  $S_\Delta^{(j)}$  at  $t_k$ . Thus,  $S_\Delta$  can be made continuous at knot  $t_k$  by omitting from (3.3) the constant term  $\beta_{k0}(x_i - t_k)_+^0$ , and  $S_\Delta^{(j)}$  can be made continuous at  $t_k$  by omitting the term  $\beta_{kj}(x_i - t_k)_+^j$ . If we wanted continuity in the  $j$ th derivative of  $S_\Delta$  at a particular knot  $t_k$ , we would probably also want continuity in all lower-order derivatives at  $t_k$ . This continuity is accomplished by omitting the terms  $\beta_{kn}(x_i - t_k)_+^n$ ,  $n=0,1,\dots,j$ . Different continuity restrictions can be imposed at different knots simply by deleting the appropriate terms. The more terms we delete, the worse the fit will be. Because the deletion of a ‘+’ term is equivalent to adding a continuity restriction, however, the curve will be smoother. In order to determine which continuity restrictions do not result in a significantly worse fit, we test in (3.3) whether the appropriate coefficients are zero.

The smoothest possible spline with polynomial pieces of degree  $m$  with  $N$  knots is given by (Smith, 1979),

$$S_\Delta(x) = \sum_{j=0}^m \beta_{0j} x_t^j + \sum_{k=1}^N \beta_{km} (x - t_k)_+^m \tag{3.4}$$

The degree of the spline function from (3.2),  $m$ , depends on what is a realistic assessment of the number of derivatives available in the regression function. Obviously, this knowledge is frequently not available. Often the choice is simply  $m=3$ , which yields a cubic spline and which is the smallest  $m$  yielding visual smoothness. The choice  $m=2$  or  $m=1$  will yield, respectively, piecewise quadratics or piecewise lines (i.e., quadratic and linear splines) (Wegman and Wright, 1983). The cubic spline function is

$$y_i = \sum_{j=0}^3 \beta_{0j} x_t^j + \sum_{k=1}^N \sum_{j=0}^3 \beta_{kj} (x_i - t_k)_+^j + \varepsilon_i \tag{3.5}$$

Formula (3.5) may be rewritten as follows:

$$E(y_i) = \begin{cases} \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{03}x^3 & , 0 < x \leq t_1 \\ \beta_{10} + \beta_{11}(x - t_1) + \beta_{12}(x - t_1)^2 + \beta_{13}(x - t_1)^3 & , t_1 < x \leq t_2 \\ \beta_{20} + \beta_{21}(x - t_2) + \beta_{22}(x - t_2)^2 + \beta_{23}(x - t_2)^3 & , t_2 < x \end{cases} \tag{3.6}$$

Then as given by (Smith, 1979) smoothest cubic spline with polynomial pieces of degree  $m=3$  with  $N$  knots will be,

$$S_\Delta(x) = \sum_{j=0}^3 \beta_{0j} x_t^j + \sum_{k=1}^N \beta_{k3} (x - t_k)_+^3 \tag{3.7}$$

Formula (3.7) may be rewritten as follows:

$$E(y_i) = \begin{cases} \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{03}x^3 & , 0 < x \leq t_1 \\ \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{03}x^3 + \beta_{13}(x - t_1)^3 & , t_1 < x \leq t_2 \\ \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{03}x^3 + \beta_{13}(x - t_1)^3 + \beta_{23}(x - t_2)^3 & , t_2 < x \end{cases} \tag{3.8}$$

Thus;

$$E(y_i) = \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{03}x^3 + \beta_{13}(x - t_1)_+^3 + \beta_{23}(x - t_2)_+^3 \tag{3.9}$$

The ordinary least squares fit, the  $i$ th PRESS residual is defined as  $e_i = y_i - \hat{y}_i$ , where  $\hat{y}_i$ , is the predicted  $i$ th response from a model fitted with all the data except the  $i$ th observation. A simple way the compute the PRESS residuals without having to refit the model for each of the observations is to note that;  $e_{(i)} = y_i - \hat{y}_i / 1 - h_{ii}$ , where  $h_{ii}$  is the  $i$ th diagonal element of the hat matrix  $H = X(X'X)^{-1}X'$  (Allen 1974). For the restricted linear regression, there also exists a simple formula for computing PRESS residuals. It can be expressed as the vector of predicted responses under the restricted model by

$$\tilde{y} = (H - J)y,$$

where

$$J = X(X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}R(X'X)^{-1}X'$$

The PRESS residual for the leave-one-out analysis using the restricted least squares fit is

$$\tilde{e}_i = y_i - \tilde{y}_i.$$

It can be expressed  $\tilde{e}_i$  as

$$\tilde{e}_{(i)} = y_i - \tilde{y}_i/1 - h_{ii} + j_{ii},$$

Where  $j_{ii}$  denote the  $i$ th diagonal element of the matrix  $J$  (Tarpey 2000b). In another work (Tarpey 2000a), Tarpey argued that PRESS statistic performs better job for Restricted Least Squares. In both works (Tarpey, 2000a and 2000b) he supported this argument by comparing PRESS statistics of restricted and unrestricted models.

**4. ESTIMATION RESULTS: REAL EXCHANGE RATE CSR MODELS**

In our work, we produced four different models from the same data set by implementing cubic spline regression from piecewise polynomial structure to the smoothest structure and showed the produced models in Table 1. Model III was not statistically significant but the others were. So we tested the significance of the coefficients of Models I, II and IV.

Table 1: Coefficients of constructed CSR models for Real Exchange Rate

We will present all of the models which we produced and summarized in Table 1 in graphics one by one. In these graphics one can see, how the curve transforms to the smoothest shape from piecewise form. As it can be seen, Model I is in piecewise form and Model IV is so smooth. While forming cubic spline regression models, knots appeared in the years of structural changes. For this reason when we estimate the equation (3.6), our data set knots appeared at  $t_1=8$  and  $t_2=15$ . So the equation could be written as below,

$$E(y_i) = \begin{cases} \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{03}x^3 & , 0 < x \leq 8 \\ \beta_{10} + \beta_{11}(x - 8) + \beta_{12}(x - 8)^2 + \beta_{13}(x - 8)^3 & , 8 < x \leq 15 \\ \beta_{20} + \beta_{21}(x - 15) + \beta_{22}(x - 15)^2 + \beta_{23}(x - 15)^3 & , 15 < x \end{cases}$$

Figure 2. CSR model (Model I) for Turkey’s Real Exchange Rate in 1987-2008

In the Figure 2 which presents Model I, it is shown that cubic spline curve is indeed a discontinuous function at the knots as in Smith’s (1979) work. Also since Model I is a discrete function, constants of  $\beta_{00}$ ,  $\beta_{10}$  and  $\beta_{20}$  are shown separately.

If we wish to fit polynomial pieces of different degrees, model (3.3) can be modified and multiple regression techniques applied as usual. In some cases, the modification may necessarily be substantial. The simplest type of modification occurs when only polynomial pieces of degrees  $n$  and  $n-1$  are used in the model. We must then impose the condition in model (3.3) that some parameters are zero and certain pairs of parameters sum to zero. If polynomial pieces of degrees other than  $n$  and  $n-1$  are desired, the modifications to (3.3) must be involved more (Smith, 1979).

We begin by using (3.3) to write the model for a three-piece cubic (largest-degree piece) continuous spline:

$$E(y_i) = \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{03}x^3 + \beta_{11}(x - 8)_+ + \beta_{12}(x - 8)_+^2 + \beta_{13}(x - 8)_+^3 + \beta_{21}(x - 15)_+ + \beta_{22}(x - 15)_+^2 + \beta_{23}(x - 15)_+^3$$

Figure 3. CSR model (Model II) for Turkey’s Real Exchange Rate in 1987-2008

The absence of the  $\beta_{10}$  and  $\beta_{20}$  terms guarantees that the function is continuous, and the presence of higher-order terms allow for possible discontinuities in the derivatives.

Our third continuous spline is;

$$E(y_i) = \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{03}x^3 + \beta_{12}(x - 8)_+^2 + \beta_{13}(x - 8)_+^3 + \beta_{22}(x - 15)_+^2 + \beta_{23}(x - 15)_+^3$$

Figure 4. CSR model (Model III) for Turkey’s Real Exchange Rate in 1987-2008

In this model in addition to the terms  $\beta_{10}$  and  $\beta_{20}$ , two other terms  $\beta_{11}$  and  $\beta_{21}$  do not exist because of the 0 restriction on them. Testing the significance of the model also means testing the significance of the restrictions at the same time. Although Model III was not significant, in order to show how the curve is getting smoother we presented its graphics in Figure 4.

Our fourth continuous spline is;

$$E(y_i) = \beta_{00} + \beta_{01}x + \beta_{02}x^2 + \beta_{03}x^3 + \beta_{13}(x - 8)_+^3 + \beta_{23}(x - 15)_+^3$$

Figure 5. CSR model (Model IV) for Turkey’s Real Exchange Rate in 1987-2008

In Model IV, besides the terms  $\beta_{10}$ ,  $\beta_{20}$ ,  $\beta_{11}$ ,  $\beta_{21}$  we used the 0 restriction for  $\beta_{12}$  and  $\beta_{22}$  terms. Because of the significance of the model which can be seen easily in Table 1, all of these restrictions are also significant. Furthermore this model is the smoothest one among all cubic splines and which is produced according to the formula (3.3), so without any hesitation we choose Model IV as the best one. If the fourth continuous spline is observed carefully it can be seen that, two extreme points exist in this spline, the year 1991 is the maximum and 1995 is the minimum. As Wold (1974) mentioned, two extremes must exist in a cubic function and only one be placed between knots. Furthermore, in our function the year 1993 is the inflection point, and as the year 1994 is the first knot, the inflection point is placed near the knot.

Table 2: Comparison of CSR model (Model IV) and OLS models with residuals sum of squares

In order to test the significance of the CSR model constructed in Table 2 and to obtain a positive F test statistical value, the RSS value of CSR model should be greater than the RSS of OLS model. Model IV is significant at 0.05 levels. But according to Tarpey's approach the PRESS statistics of CSR model should be smaller than OLS model in order the restrictions which are determined according to the knots should be significant. Here it can be seen that the PRESS statistic of our CSR model Model IV is smaller than the PRESS statistic of OLS model.

### 5. CONCLUSIONS

The economic issues and problems experienced in a country cause big changes particularly in economic time series. They are usually affected by various internal or external events. Such cases as wars, economic or politic crises generally cause abrupt and sharp changes in the data collected in their periods but these time series usually keep their continuity with some structural shifts or structural changes. In this study our attempt is to shed light on the question that whether Real Exchange Rate which are in interaction, change in the years of economic crises. In the years of economic crises and sharp depreciation of TL almost all of the economic parameters of Turkish Economy were affected negatively but the effects on many parameters seen usually at different degrees and time lags. When we consider the values of Real Exchange Rate, our assumption was that the effects were similar and the structural changes occurred in data without any time lag. So we proposed that, sharp depreciation of TL in 1994 and the 2001 Economic Crises were the most important events of 1987-2008 period in Real Exchange Rate. For this reason we applied cubic spline regression on these three parameters and formed the knots  $t_1=8$  for 1994 and  $t_2=15$  for 2001.

In our study we used cubic spline regression model in order to expose how the economic policies which were implemented to promote Turkish foreign trade caused structural changes in related data. With this effort we realized that the devaluation in 1994 and the big economic crisis experienced in 2001 caused structural breaks in Real Exchange Rates of Turkey. These structural breaks can only be examined with real data (without any transforming or smoothing) by employing Cubic Spline Regression. According to spline regression theory, spline regression is a preferred method in interpolation. With this method, one can easily reach minimum residual sum of squares and achieve this result by using the real economic data without changing their nature. If an economist cares and takes this advantage into account, he/she will model the breaks in time series with splines and observe the deep impacts of crises better.

In our interpolation process, we are quite enlightening the question "What should be in economic sense". In 1987-2008 period 2001 crisis started because of a political problem without any expectations in governmental and economic areas. But 1994 crisis was foreseen, so government implemented 5 th April Stabilization Program before it. It is so interesting that, in our statistically significant model the inflection point is the year 1993 and by the turn of diminishing to an increase the change in the concavity of our spline occurs in this year. When observed carefully, it can be seen that the increase continues both before and after the year 2001 and also the structural break in 2001 crisis seen briefly. We don't have theoretical proofs to offer but, when we remember that 2001 crisis was unforeseen and evaluate our model in this sense, we can easily suggest that our model fits the structure of such economic case and data.

When we examine the Real Exchange Rates of Turkey, it can be seen that the rate was 133.8 in 1993 and declined to 101.3 in 1994. This decline might be accepted as a requested result of devaluation in the same year. But also in 2001 Crisis which started suddenly in February, although an economic stability program based on exchange rate was implemented in 1999, similar changes occurred and Real Exchange Rate declined to 123.1 in 2001 although it was 152 in 2000. In conclusion it can be said that, our argument on spline regression is supported with the results of this work.

From the perspective of spline theory, cubic spline regression provides a good model in interpolation when there are significant structural breaks in time series. Although 2008 Crisis had caused another structural break in the values of real exchange rate values in 2009, we could not include that year, because in this method in order to include and analyze this new structural break we should need to model a time series which also covers the year 2012. As mentioned above there must be a period of at least 4 or 5 years between the knot values. Furthermore, again in the perspective of regression analysis the estimation of the values of 2009 and 2010 with extrapolation method will be useless. Because as known, 2008 Crisis created a structural break in the time series and the value of real exchange declined and became 183.24 in 2009.

In the study we demonstrated how Cubic Spline Regression (CSR) becomes a very useful tool in the existence of structural shifts or changes in time series, we also showed that that prediction sum of squares residuals (PRESS) statistics improve the analysis in cubic spline models. Buse and Lim (1977) anyway showed the equality of Restricted Least Squares and Cubic Spline Regression. The Real Exchange Rate is a well known subject in economics and Real Exchange Rate is analyzed with various models, but if great structural shifts occur in Real Exchange Rate parameters in the cases of big economic crises and devaluations cubic spline models which describe the break points as knots can do better job in estimations. The models in this study will be more useful in understanding the real effects of economic crisis in a country and they will be helpful in realizing the size and length of crisis experienced in reality.

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Figures:

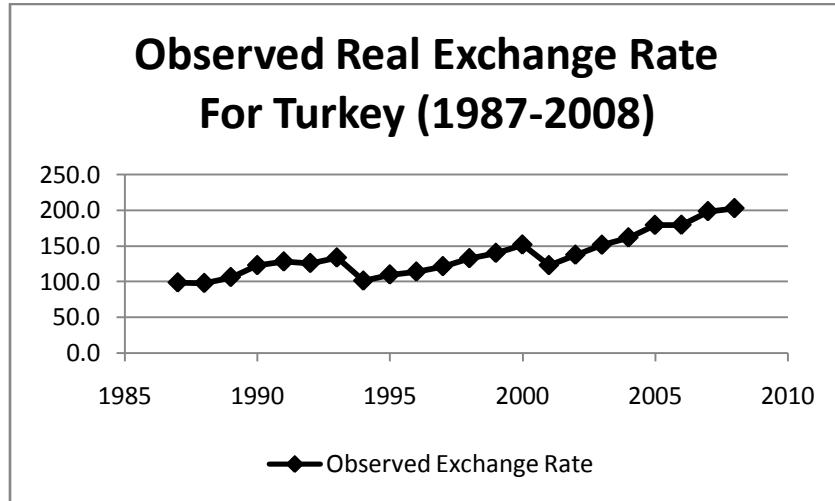


Figure 1. Based on 1 USD+1.5 EUR basket, in the relative price calculations and consumer prices for Turkey are used. (1987. JAN. = 100)

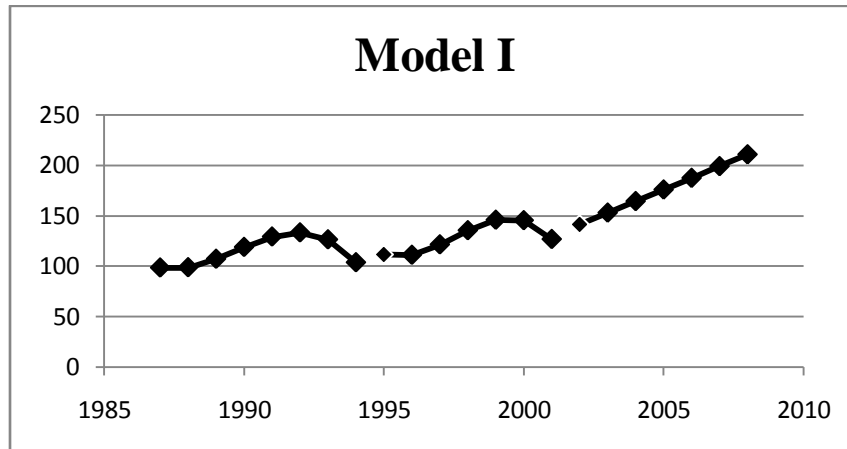


Figure 2. CSR model (Model I) for Turkey's Real Exchange Rate in 1987-2008

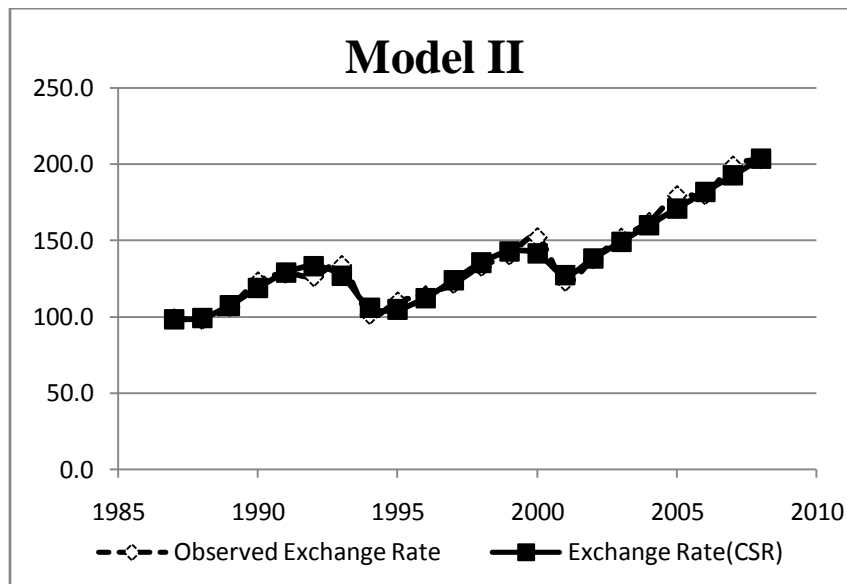


Figure 3. CSR model (Model II) for Turkey's Real Exchange Rate in 1987-2008



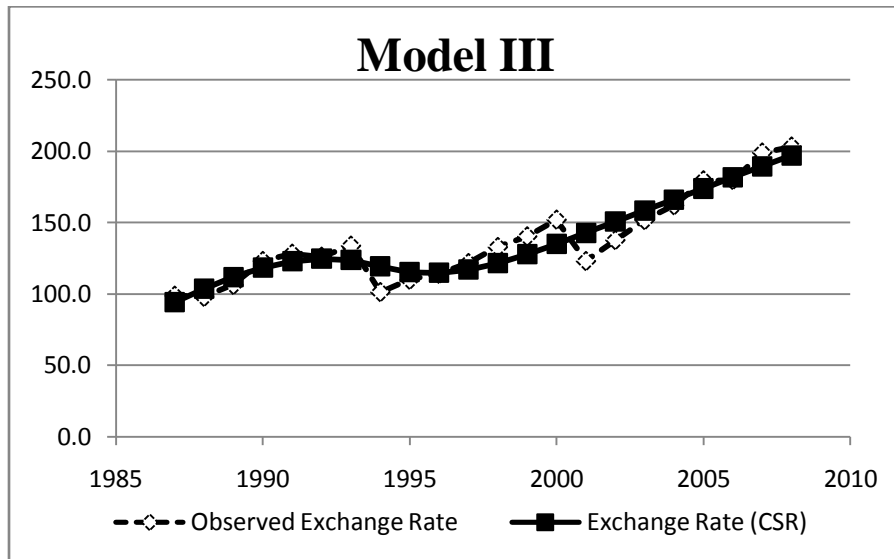


Figure 4. CSR model (Model III) for Turkey's Real Exchange Rate in 1987-2008

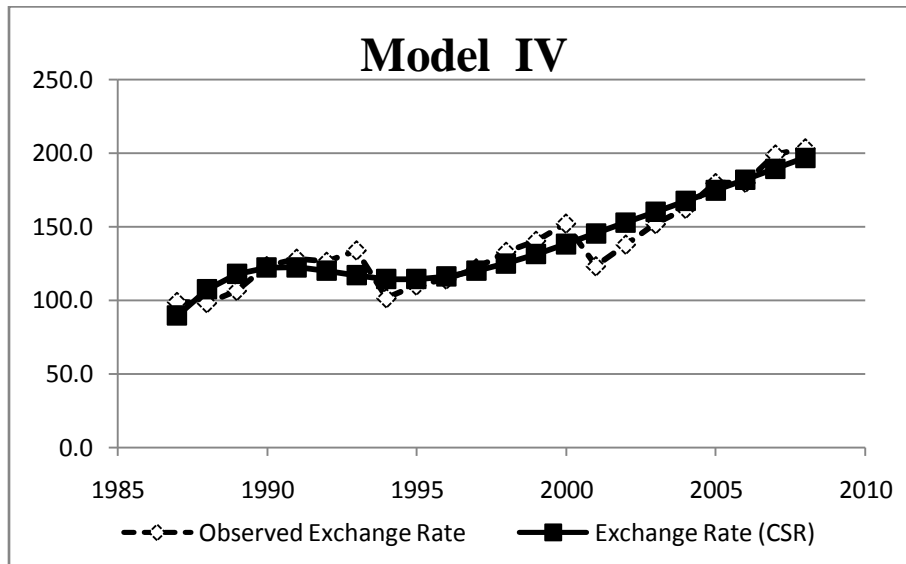


Figure 5. CSR model (Model IV) for Turkey's Real Exchange Rate in 1987-2008

Tables:

**Table 1: Coefficients of constructed CSR models for Real Exchange Rate**

Coefficients		Model I		Model II		Model III		Model IV	
		Estimated Coefficients (and s.e.)	Partial t	Estimated Coefficients (and s.e.)	Partial t	Estimated Coefficients (and s.e.)	Partial t	Estimated Coefficients (and s.e.)	Partial t
0<x≤8	$\beta_{00}$	111.166* (13.764)	8.076	109.763* (14.063)	7.805	83.932	-	62.40006* (17.133)	3.642
	$\beta_{11}$	-20.683 (12.445)	-1.657	-18.9 (12.7)	-1.488	10.77	-	32.333* (10.317)	3.134
	$\beta_{12}$	8.824* (3.124)	2.824	8.271* (3.177)	2.603	-0.27	-	-5.46703* (1.677)	-3.260
	$\beta_{13}$	-0.795* (0.229)	-3.469	-0.746* (0.232)	-3.209	-0.065	-	0.279824* (0.08)	3.493
8<x≤15	$\beta_{10}$	26.828 (17.056)	1.573	-	-	-	-	-	-
	$\beta_{11}$	1.746 (17.813)	0.098	23.007** (12.07)	1.906	-	-	-	-
	$\beta_{12}$	22.990* (5.211)	4.412	16.021* (2.901)	5.522	3.878	-	-	-
	$\beta_{13}$	-0.416 (0.444)	-0.938	0.034 (0.349)	0.097	-0.036	-	-0.33929* (0.092)	-3.681
15<x	$\beta_{20}$	3.497 (7.159)	0.488	-	-	-	-	-	-
	$\beta_{21}$	41.351* (8.567)	4.827	32.988* (6.514)	5.064	-	-	-	-
	$\beta_{22}$	12.719* (3.475)	3.66	8.552* (2.27)	3.767	0.065	-	-	-
	$\beta_{23}$	1.212* (0.38)	3.188	0.712* (0.218)	3.259	0.101	-	0.059464* (0.013)	4.497
F		1.135***		0.9***		5.9		1.427***	

Note: \* Coefficients are significant at 95% levels,

\*\* Coefficients are significant at 90% levels,

\*\*\* CSR Models are significant at 95% levels.

Model 3 was not significant in F test, so we didn't need to compute t values.

Table 2: Comparison of CSR model (Model IV) and OLS models with residuals sum of squares

Residuals sum of squares of Constructed CSR Models and OLS Models		RSS $\sum e_{(i)}$	PRESS $\sum \tilde{e}_{(i)}$
Real Exchange Rate	CSR Model (Model IV)	2185,936	4230,006
	OLS Model	1854,991	4384,508