Measuring composite indicators using a multiplicative and multilayer DEA model: a case study on the Digital Economy and Society Index (DESI)

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Abstract

A Composite indicator based on data envelopment analysis (DEA) has gained significant consideration in recent years. However, the large number of indicators as well as the ratio measures makes the use of DEA models less effective in the composite indicators. In the literature, many articles have proposed approaches based on DEA models to address the problem of ratio measures. Others researches to address the problem of discrimination related to the data dimension. In this paper, we have developed an indicator composite based on a multiplicative and multilayer DEA model (DEA-MM). The multiplicative DEA model is based on the concept of the geometric mean of invariant units which is highly desirable when all measures are in ratios. In addition, the use of the hierarchical multilayer structure of the indicators allows the DEA-MM model to produce a high degree of discrimination between the scores of the decision units. A numerical application on the Digital Economy and Society Index (DESI) is also illustrated in this paper.

Key word: Composite Indicator, Data Envelopment Analysis, e-Readiness, ratio measures, DEA multilayer.

1. Introduction

Composite indicators are popular multi-criteria assessment tools for benchmarking performance across countries, regions, or companies. Several composite indicators have been developed with different methodologies depending on the nature and size of the data. Indeed, the variety of methodologies used in the construction of composite indicators depends on data standardization method, weighting approach, and the aggregation technique. Generally, two approaches are well distinguished in the construction of composite indicators: (i) Multi-criteria decision analysis (MCDA), a mathematical combination of the individual standardized indicators. The final score is obtained by the arithmetic or geometric aggregation of the weighted indicators. (ii) Data Envelope Analysis (DEA), a non-parametric method based on linear programming to evaluate the effectiveness of decision-making units. Depending on the objective of the evaluation, the efficiency score can be calculated in DEA model by two different approaches: (1) the set of indicators is divided into two parts, input indicators that represent the cost variables to be minimized and output indicators that represent the benefit variables to be maximized. The efficiency score is obtained by a ratio between a virtual output variable and a virtual input variable. (2) the set of indicators is composed of output (or input) variables only, this approach is known as benefit of doubt (BoD), the score obtained evaluates the performance of the decision-making units (DMU) (El Gibari et al., 2019).

However, each of these two approaches has some drawbacks that remain the subject of several studies of composite indicators. Indeed, for the MCDA approach, the assignment of weights to individual indicators can be objective or subjective. In both cases, the weighting remains sensitive and little variation in the weights can change the scores, especially the ranking of the decision-making units [2].

On the other hand, weighting is not a problem in the DEA approach, because the latter uses a linear programming model where each DMU obtains its own weighting coefficients, the best possible ones that maximize its efficiency score (Shen et al., 2013). In addition, the MCDA approach supports a large dimension of indicators, i.e., whatever the number of indicators used, the MCDA can aggregate them into a single index.

While the DEA approach places some constraints between the number of indicators and the number of DMU. Indeed, a set of n DMU evaluated by a m indicators where n≤2m can reduce the discriminating power of the DEA model between the DMU scores (Cook et al., 2014). Generally, the greater the number of indicators, the greater the number of DMU with an efficiency score equal 1 (less discrimination). to go beyond this limitation, Yongjun Shen et al (Shen et al., 2011) proposed a new DEA model based on a multi-layered hierarchical structure to increase the discrimination power of the model. In addition, its hierarchical structure allows each indicator to be treated within its category as they belong to different categories.

Another problem arises in DEA models when some or all indicators are measured in ratio. In fact, the presence of ratio measures in DEA models leads to errors in the efficiency computation results, because the convexity properties of ratio measures can be a problem in defining the set of production possibilities (Emrouznejad and Cabanda, 2010). In addition to convexity, ratio measures do not respect the axiom of proportionality in DEA models, because if the numerator and denominator increase or decrease by the same proportion, the ratio remains the same (Emrouznejad and Cabanda, 2010). To overcome this problem, Emrouznejad et al (Emrouznejad and Cabanda, 2010) propose to use the multiplicative DEA model based on the concept of the geometric mean of the measures with invariance of the units which is very desirable for indicators in ratio measure and which also allows to keep the proportionality.

The objective of this paper is to propose a composite indicator based on the multiplicative and multilayer DEA-MM model to make possible the use of ratio measures and increase the discrimination power using a hierarchical multilayer structure of the indicators.

The remainder of this article is organized as follows: the multiplicative DEA model and the multilayer DEA model are introduced respectively in sections 2 and 3. Section 4 presents the composite indicator based on DEA-MM. Next, a numerical application of the DESI index is illustrated in Section 5. The results are discussed in section 6 and the conclusion is presented in section 7.

2. Composite indicators based on the multiplicative DEA model

Explaining On the GNCP (General Non-parametric Corporate Performance) model proposed by Fernandez-Castro and Smith (1994) shown in Model 1 of figure 1, whose concept is to reformulate the DEA model to combine a set of ratio variables into a single score (Fernandez-Castro and Smith, 1994), Emrouznejad et al proposed the MNCP (Multiplicative Non-parametric Corporate Performance) model based on the concept of the geometric mean of the non-dimensional measures (invariant units) shown in Model 2 of figure 1 (Emrouznejad and Cabanda, 2010).

For n decision-making units evaluated by m ratio variables $\{r_{ij}\,; i = 1..m and j = 1..n\}$, the efficiency of the DMU j_0 is given by the inverse of the optima value of ρ.

Model 1: GNCP	Model 2: MNCP
$Max\rho$ $s.t$ $\sum_{j=1}^{n} \lambda_j r_{ij} \geq \rho r_{ij_0} \quad ; i = 1 \ldots m \quad (1)$ $\sum_{j=1}^{n} \lambda_j = 1$ $\lambda_j \geq 0 \quad ; j = 1 \ldots. n$	$Max\rho$ $s.t$ $\prod_{j=1}^{n} {r_{ij}}^{\lambda_j} \geq \rho r_{ij_0} \quad ; i = 1 \ldots m \quad (2)$ $\sum_{j=1}^{n} \lambda_j = 1$ $\lambda_j \geq 0 \quad ; j = 1 \ldots. n$

Figure1: GNCP and MNCP model

Both models use only output (or input) variables. The decision variable is ρ where all ratio variables r_{ij} weighted by $\lambda_j, j = 1..n$ can increase to form the frontier of efficiency of the DMU j_0.

The MNCP model is linearized by the following transformation:

$$\rho r_{ij_0} = e^{-S_i} \prod_{j=1}^{n} {r_{ij}}^{\lambda_j} \qquad (3)$$

The objective function of model 2 is replaced by $\rho exp(\varepsilon \sum_{i=1}^{m} S_i)$where $S_i \geq 0$ represents slacks and ε is the infinitesimal non-archimedian.

With $h = log(\rho)$ and $g_{ij} = log(r_{ij})$ so the logarithm of equation (3) of the model 2 becomes:

Model 3:

$$h + \varepsilon \sum_{i=1}^{m} S_i$$

$s.t$

$$\sum_{j=1}^{n} \lambda_j g_{ij} - S_i = h + g_{ij_0} \qquad (4)$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \ \ et S_i \geq 0 \qquad \forall i = 1 \dots m ; j = 1 \dots . n$$

Figure 2: Logarithmic linearization of the GNCP multiplicative model

The efficiency of DMU j_0 is given by $\frac{1}{e^h}$.

The MNCP model has proven its effectiveness for ratio measurements. However, its discrimination between the DMU scores is very weak when the number of indicators is large against to the number of DMU.

3. Development of composite indicators based on multiplicative and multilayer dea models (DEA-MM)

Several composite indicators are composed of a large dimension of indicators that generally belong to different categories and structured in a hierarchy of multilayer. For example, Network Readiness Index (NRI) (Soumitra and Bruno, 2019), Internet Inclusion Index (III) (III, 2019), ICT Development Index (IDI) (UIT, 2017), etc. This hierarchical structure is ignored by the standard DEA models since they treat indicators as if they belong to the same category and in a single layer. Moreover, the discriminating power of standard DEA models between the DMU to be evaluated is increasingly weakened if the number of indicators is large compared to the number of DMU (Shen et al., 2011). To overcome this limitation, *Yongjun Shen et al.* proposed to introduce the properties of the hierarchical structure of indicators and their categories into DEA models to increase their discriminative power. Consider n DMU evaluated by s output indicators having a hierarchical structure of k layers as shown in Figure 3.

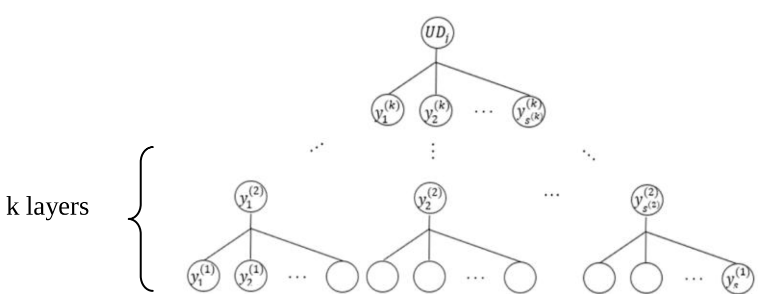

Figure 3: Hierarchical Structure of Indicators

$s^{(i)}$the number of categories in the $i^{ième}$layer $(i = 1 \dots k)$ with $s^{(1)} = s$. The property of indicators in a hierarchical structure assumes that each category (i) is the weighted sum of its output indicators. If $A_{c_i}^{(i)}$ represents the set of output indicators of the $c^{ième}$ category located in the layer (i), so:

$$y_{c_k}^{k} = \sum_{c_{k-1} \in A_{c_k}^{(k)}} w_{c_{k-1}}^{(k-1)} \left(.. \sum_{c_i \in A_{c_{i+1}}^{(i+1)}} w_{c_i}^{(i)} \left(.. \sum_{c_2 \in A_{c_3}^{(3)}} w_{c_2}^{(2)} \left(\sum_{c_1 \in A_{c_2}^{(2)}} w_{c_1}^{(1)} y_{c_1}^{(1)} \right) \right) \right) \qquad (5)$$

$$\sum_{c_i \in A_{c_{i+1}}^{(i+1)}} w_{c_i}^{(i)} = 1, \qquad w_{c_i}^{(i)} \geq \xi, \qquad c_i = 1 \ldots s^{(i)} \qquad (6)$$

With $w_{c_i}^{(i)}$ indicates the internal weight associated with the variable of the $c^{ième}$ category of $i^{ième}$ layer and ξ is a small value.

The replacement of the variable $y_{c_k}^{k}$ in the standard DEA model of Figure 4 leads to the multi-layer DEA model shown in Figure 5.

$$E_0 = \sum_{r=1}^{s} u_r y_{r0}$$
$$s.t. \sum_{r=1}^{s} u_r y_{rj} \leq 1, j = 1,..,n \qquad (7)$$
$$u_r \geq 0, r = 1,..,s$$

Figure 4: Standard DEA Model (CCR)

The objective function of the multilayer AED becomes:

$$CI_0 = \max \sum_{c_k=1}^{s^{(k)}} u_{c_k} \left(\sum_{c_{k-1} \in A_{c_k}^{(k)}} w_{c_{k-1}}^{(k-1)} \left(.. \sum_{c_i \in A_{c_{i+1}}^{(i+1)}} w_{c_i}^{(i)} \left(.. \sum_{c_2 \in A_{c_3}^{(3)}} w_{c_2}^{(2)} \left(\sum_{c_1 \in A_{c_2}^{(2)}} w_{c_1}^{(1)} y_{c_1}^{(1)} \right) \right) \right) \right) \quad (8)$$

The linearization of the model was performed by the transformation below:

$$\hat{u}_{c_1} = \prod_{i=1}^{k-1} w^{(i)}{}_{\substack{c_i \\ c_i \in A_{c_{i+1}}^{i+1}}} . u_{c_k} \qquad (9)$$

The weight of the category c_i located in the (i) is given by the following relationship:

$$w^{(i)}{}_{\substack{c_i \\ c_i \in A_{c_{i+1}}^{i+1}}} = \frac{\sum_{c_1 \in A_{c_i}^{(i)}} \hat{u}_{c_1}}{\sum_{c_1 \in A_{c_{i+1}}^{(i+1)}} \hat{u}_{c_1}}, \quad c_i = 1 \ldots s^{(i)}, \quad i = 1,..,k-1 \qquad (10)$$

Restriction rules on the weight indicated by the set Φ are added to the multi-layer DEA model to ensure consistency between layer-specific weights and prior knowledge of composite indicators. Figure 5 shows the DEA model with a hierarchical multilayer structure.

$$CI_0 = \max \sum_{c_1}^{s} \hat{u}_{c_1} y_{c_1 0}$$
$$s.t. \quad \sum_{c_1}^{s} \hat{u}_{c_1} y_{c_1 j} \leq 1, \quad j = 1,..,n \qquad (11)$$
$$\sum_{c_1 \in A_{c_i}^{(i)}} \hat{u}_{c_1} \Big/ \sum_{c_1 \in A_{c_{i+1}}^{(i+1)}} \hat{u}_{c_1} = w^{(i)}{}_{\substack{c_i \\ c_i \in A_{c_{i+1}}^{i+1}}} \in \Phi, c_i = 1 \ldots s^{(i)}, \quad i = 1,..,k-1$$
$$\hat{u}_{c_1} \geq 0, \qquad c_i = 1,..,s$$

Figure 4: The DEA model of a multi-layered hierarchical structure

4. Composite Indicators Based On The Multiplicative Dea Model

Aspreviously mentioned, standard DEA models do not support ratio measurements. This issue also exists in the multi-layer DEA model presented in Section 3. Our contribution in this paper is to introduce the concept of multilayer hierarchical structure in the multiplicative DEA model MNCP which is desirable for ratio measures in order to build a composite indicator model that supports ratio measures and with a very high discriminative power. The use of the hierarchical multilayer structure presented in Figure 3 in the multiplicative model based on the concept of the geometric mean, formulas (5) and (6) become:

$$y_{c_k,j}^{(k)} = \prod_{c_{k-1}\in A_{c_k}^{(k)}} \left(..\prod_{c_i\in A_{c_{i+1}}^{(i+1)}} \left(..\prod_{c_2\in A_{c_3}^{(3)}} \left(\prod_{c_1\in A_{c_2}^{(2)}} \left(y_{c_1,j}^{1}\right)^{w_{c_1}^{(1)}} \right)^{w_{c_2}^{(2)}} \right)^{w_{c_i}^{(i)}} \right)^{w_{c_{k-1}}^{(k-1)}} \quad (12)$$

With logarithmic transformation

$$\log y_{c_k,j}^{(k)} = \sum_{c_{k-1}\in A_{c_k}^{(k)}} w_{c_{k-1}}^{(k-1)} \left(..\sum_{c_i\in A_{c_{i+1}}^{(i+1)}} w_{c_i}^{(i)} \left(..\sum_{c_2\in A_{c_3}^{(3)}} w_{c_2}^{(2)} \left(\sum_{c_1\in A_{c_2}^{(2)}} w_{c_1}^{(1)} \log y_{c_1,j}^{1} \right)\right)\right) \quad (13)$$

$$\prod_{c_i\in A_{c_{i+1}}^{(i+1)}} w_{c_i}^{(i)} = 1, \qquad w_{c_i}^{(i)} \geq \xi, \qquad c_i = 1 \ldots s^{(i)} \quad (14)$$

By replacing (13) in model 3 of Figure 2, we obtain:

$$\max h + \varepsilon \sum_{c_k=1}^{s} S_{c_k}$$

$s.t$

$$\sum_{j=1}^{n} \lambda_j \left(\sum_{c_{k-1}\in A_{c_k}^{(k)}} w_{c_{k-1}}^{(k-1)} \left(..\sum_{c_i\in A_{c_{i+1}}^{(i+1)}} w_{c_i}^{(i)} \left(..\sum_{c_2\in A_{c_3}^{(3)}} w_{c_2}^{(2)} \left(\sum_{c_1\in A_{c_2}^{(2)}} w_{c_1}^{(1)} \log y_{c_1,j}^{1} \right)\right)\right)\right) - S_{c_k}$$

$$= h + \sum_{c_{k-1}\in A_{c_k}^{(k)}} w_{c_{k-1}}^{(k-1)} \left(..\sum_{c_i\in A_{c_{i+1}}^{(i+1)}} w_{c_i}^{(i)} \left(..\sum_{c_2\in A_{c_3}^{(3)}} w_{c_2}^{(2)} \left(\sum_{c_1\in A_{c_2}^{(2)}} w_{c_1}^{(1)} \log y_{c_1,j_0}^{1} \right)\right)\right)$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad (15)$$

$$\lambda_j \geq 0 \;\; et S_{c_k} \geq 0, \qquad c_k = 1,..,s\,; j = 1,..,n$$

It is assumed that:

$$\hat{\varphi}_{c_1} = \prod_{i=1}^{k-1} {w^{(i)}}_{\substack{c_i \\ c_i\in A_{c_{i+1}}^{i+1}}} \quad (16)$$

$$\hat{\mu}_{c_1,j} = \lambda_j . \hat{\varphi}_{c_1} \quad (17)$$

We have

$$\sum_{c_1\in A_{c_k}^{k}} \hat{\varphi}_{c_1} = 1 \quad (18)$$

$$\sum_{c_1\in A_{c_k}^{k}} \hat{\mu}_{c_1,j} = \lambda_j \quad (19)$$

The weighting coefficients can be deduced from the following formula:

$${w^{(i)}}_{\substack{c_i \\ c_i\in A_{c_{i+1}}^{i+1}}} = \frac{\sum_{c_1\in A_{c_i}^{(i)}} \hat{\varphi}_{c_1}}{\sum_{c_1\in A_{c_{i+1}}^{(i+1)}} \hat{\varphi}_{c_1}}, \quad c_i = 1 \ldots s^{(i)}, \quad i = 1,..,k-1 \quad (20)$$

So, the model (15) becomes:

$$\max h + \varepsilon \sum_{c_k=1}^{s} S_{c_k}$$

$s.t$

$$\sum_{j=1}^{n}\left(\sum_{c_{1=1}}^{s}\hat{\mu}_{c_1,j}.\log y_{c_1,j}^{1}\right)-S_{c_k}=h+\sum_{c_{1=1}}^{s}\hat{\varphi}_{c_1}.\log y_{c_1,j_0}^{1}\ ,\quad c_1=1,..,s$$

$\sum_{c_1\in A_{c_k}^{k}}\hat{\varphi}_{c_1}=1,\qquad c_1=1,..,s$ (21)

$\sum_{c_1\in A_{c_k}^{k}}\hat{\mu}_{c_1,j}=\lambda_j\,,\,c_1=1,..,setj=1,..,n$

$$\sum_{c_1\in A_{c_i}^{(i)}}\hat{\varphi}_{c_1}\Big/\sum_{c_1\in A_{c_{i+1}}^{(i+1)}}\hat{\varphi}_{c_1}=w^{(i)}{}_{\substack{c_i\\ c_i\in A_{c_{i+1}}^{i+1}}}\in\Phi,\quad c_i=1\ldots s^{(i)},\ \ i=1,..,k-1$$

$\sum_{j=1}^{n}\lambda_j=1\,,\ \ j=1,..,n$

$\lambda_j\geq 0\ \ etS_{c_k}\geq 0,\qquad c_k=1,..,s^{(k)}\,,\ \ j=1,..,n$

$\hat{\varphi}_{c_1},\hat{\mu}_{c_1,j}\geq\varepsilon,\quad c_1=1,..,setj=1,..,n$

Figure 5: Multiplicative and Multilayer DEA Model (DEA-MM)

ε is a small non-zero value imposed to ensure the contribution of all indicators and categories of all layers.

5. Case Study: Digital Economy And Society Index (Desi)

To demonstrate the effectiveness of the DEA-MM model, we have constructed a composite indicator based on DEA-MM for the Digital Economy and Society Index (DESI) for the year 2020.

DESI is a composite index developed by the European Commission to monitor the overall digital performance of all European Union member countries in order to measure their progress in digital competitiveness (DESI, 2020).

The DESI Index is composed of 37 indicators grouped into 5 categories and 12 sub-categories as shown in Figure 6. The description of the indicators and the categories of the different layers are shown in Table 1 of the Appendix. The indicator measures are retrieved from the database published by the DESI 2020 Index. The indicators are normalized by the maxmin transformation in a scale of [0-1] where 1 denotes the highest score and 0 denotes the lowest score. As all values are less than or equal to 1, as well as some values are null, the DEA-MM model cannot be used directly since the logarithmic transformation of null values is not defined and the logarithmic transformation of values less than 1 is negative. To avoid this problem, we proceeded as follow: (1) the null values are replaced by small positive values. (2)

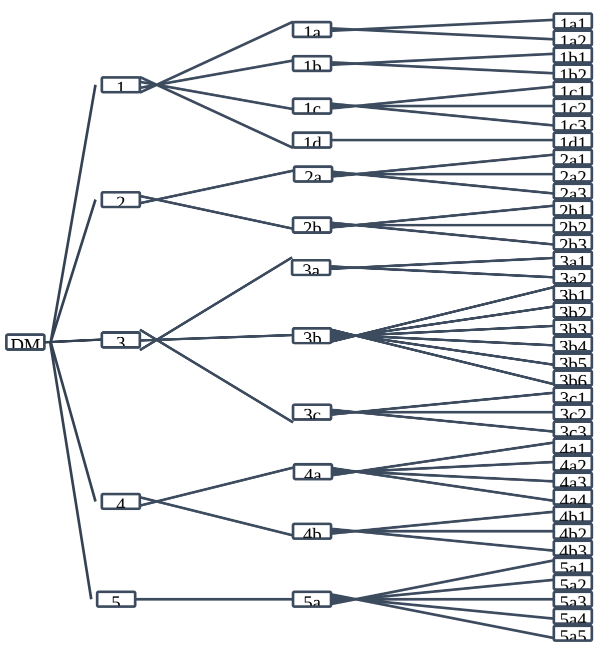

Figure 6: Hierarchical structure of 3 layers of DESI indicators

All the values are divided by a constant $R = min\{y_{c_1,j}, c_1 = 1,..,s and j = 1,..,n\} - \varepsilon$. Where ε is a positive value sufficiently lower than the $min\{y_{c_1,j}, c_1 = 1,..,s and j = 1,..,n\}$(Emrouznejad and Cabanda, 2010).

According to the basic rule of thumb that links the number n of DMU and the number of indicators m In a standard DEA model, the number of DESI indicators (37) is greater than the number of countries covered by the index (28). None of these rules $\{n \geq 2m or n \geq 3m\}$ is valid, because $37 \geq 28$(Cook et al., 2014). In this case, the use of the multiplicative DEA model is not possible because all countries get a score of 1. However, using the DEA-MM model proves a great discrimination and the countries' scores are different.

5.1. Restriction conditions on weighting coefficients

The weighting system used in the DESI index is subjective. Indeed, all indicators in layer 1 that belong to the same category have the same importance (equal weight). Categories of layer 3 and sub-categories of layer 2 have different weights depending on the digital priority of the EU (European Commission, 2020). However, in the DEA-MM model, each country obtains their optimal weights. No restriction rule on the weights has been imposed in the DEA-MM model, but we oblige the participation of all indicators and categories of all layers by the following condition: $\hat{\varphi}_{c_1}, \hat{\mu}_{c_1,j} \geq \varepsilon,\ \ c_1 = 1,..,s and j = 1,..,n$. we assume $\varepsilon = 0.003246$ the smallest value that guarantees the feasibility of calculating the efficiency and weighting coefficients of all DESI countries by the DEA-MM model.

5.2. Results

The DEA-MM model evaluates the performance score of 28 countries using the 37 indicators of the DESI index for the year 2020. Table 2 presents the result of the DEA-MM model, with scores ranging from 0 to 100%. To analyze the difference in country rankings between the two approaches, a comparison with the scores of DESI 2020 is made in the same table.

Table 2. Performance scores of the DESI 2020 index by the DEA-MM model

Country	DESI		DEA-MM		Difference ranks
	score	**rank**	**score**	**rank**	
ES	57,5	11	98,62	1	-10
NL	67,7	4	95,16	2	-2
FI	72,3	1	93,09	3	2
LV	50,7	18	92,13	4	**-14**
IE	61,8	6	87,78	5	-1
DK	69,1	3	84,77	6	3
EE	61,1	7	83,09	7	0
AT	54,3	13	81,09	8	-5
SI	51,2	16	72,09	9	-7
LU	57,9	10	71,16	10	0
BE	58,7	9	68,8,0	11	2
SE	69,7	2	68,00	12	10
PL	45,0	23	66,79	13	-10
LT	53,9	14	65,44	14	0
CY	44,0	24	62,79	15	-9
CZ	50,8	17	62,59	16	-1
PT	49,6	19	51,75	17	-2
MT	62,7	5	51,53	18	13
DE	56,1	12	50,82	19	7
FR	52,2	15	49,26	20	5
HR	47,6	20	44,02	21	1
BG	36,4	28	42,28	22	-6
SK	45,2	22	40,97	23	1
HU	47,5	21	40,06	24	3
IT	43,6	25	40,04	25	0
UK	60,4	8	39,29	26	**18**

EL	37,7	27	30,41	27	0
RO	40,0	26	24,04	28	2

The DEA-MM model shows a large discrimination between country scores. The best score is 98.62% obtained by Spain (ES) and the lowest score is 24.04% obtained by Romania (RO). The difference between the two approaches ranking scores has an absolute average of 4,78. The ranking of the United Kingdom (UK) has experienced a degradation from 8 (DESI) to 26 in the DEA-MM, while the ranking of Latvia (LV) got an improvement from 18 (DESI) to 4 in the DEA-MM. This difference in ranking is due not only to the difference in methodology between DESI and DEA-MM, but also to the difference in DESI weight assigned to each indicator and category. Indeed, the DEA-MM model calculates for each country the best weights that maximize its efficiency. On the other hand, the DESI composite index proposes for each indicator and category a fixed subjective weighting for all countries.

The DEA-MM model is flexible and allows to add subjective restriction rules on indicator weights according to the user's priority policy. This can vary the results of the scores and the ranking of countries. Indeed, DEA-MM allows to combine the objective weighting calculated by the model for each country and the subjective weighting proposed by the user in the form of weights restriction rules. To compare fairly the composite indicator approach used by the DESI index and the DEA-MM model, we recalculated the DESI index using an equal weighting for all indicators of the same category in all layers. For the DEA-MM model, we added a restriction rules on weights that require an equal weighting of all indicators in each category and each layer. Table 3 presents the scores of the two approaches with their rankings. Country scores and rankings have changed for both models. For the DEA-MM model, the first score is 99.03% obtained by Estonia (EE) and the last score is 48.31% obtained by Romania RO. The absolute average of the ranking difference between the two approaches the is 3.71. There is a decrease in the absolute average of the difference between the rankings of the two approaches.

Table 3: Comparison between DESI and DEA-MM using equal weights

Country	DESI		DEA-MM		Difference ranks
	score	**rank**	**score**	**rank**	
EE	46,86	6	99,03	1	-5
ES	46,87	5	98,49	2	-3
FI	51,35	3	97,94	3	0
DK	52,7	1	96,18	4	3
LV	43,46	13	95,94	5	**-8**
NL	51,53	2	93,03	6	4
IE	46,13	8	89,77	7	-1
AT	43,21	14	89,56	8	-6
LT	43,63	12	89,41	9	-3
SE	50,91	4	88,16	10	6
FR	41,67	16	82,34	11	-5
LU	44,96	11	81,35	12	1
MT	46,14	7	80,98	13	6
PT	41,34	17	80,39	14	-3
SI	40,69	18	80,35	15	-3
BE	45,29	10	79,92	16	6
CY	38,11	21	78,21	17	-4
PL	38,38	20	76,88	18	-2
CZ	40,31	19	71,31	19	0
DE	42,56	15	71,02	20	5
IT	36,2	25	69,23	21	-4
UK	45,32	9	68,37	22	**13**
BG	30,6	28	67,43	23	-5
HR	37,42	22	65,25	24	2
HU	37,32	23	60,99	25	2
SK	36,62	24	58,89	26	2
EL	32,31	26	55,22	27	1
RO	30,93	27	48,31	28	1

6. Conclusion

This paper proposes a method for constructing composite indicators using the Multiplicative and Multilayer Data Envelope Analysis model (MDA-MM). This approach is essentially built on two concepts: (a) The multiplicative DEA model based on the geometric mean of indicators with invariant units which allows to use measures in ratios for efficiency computation. (b) The use of multi-layered hierarchical structure of the indicators makes it possible to overcome the limitation related to the approximation rules between the number of DMU and the number of variables used in a DEA model. Thus, the DEA-MM model has shown a great capacity for discrimination between the DMU scores even if the number of indicators used is greater than the number of DMU to be evaluated.

The use of the DEA-MM model without weights restriction on indicator produces an objective result that depends only on the indicator measurements. However, as the DEA-MM model is flexible, it allows the user to introduce weights restriction rules on indicator and category on all layers that reflect his priority policy.

The paper also presents a case study for the DESI 2020 Index which covers 28 countries defined by a data hierarchical structure of 3 layers (layer 1: 37 indicators, layer 2: 12 sub-categories and layer 3: 5 categories). Two applications of the DEA-MM model have been carried out: (i) DEA-MM without restriction on the weights compared to the original DESI index result. The mean absolute of the difference between the rankings of the two models is 4,78. (ii) Both DEA-MM and DESI based on equal weighting gives an absolute mean difference between the scores ranking of 3.71. The DEA-MM model shows a great capacity for discrimination between scores despite the large number of indicators (37) compared to the number of countries (28). However, the application of a simple DEA model as a multiplicative DEA or DEA CCR on the DESI data is not feasible.

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Appendex

Table 1: Description of indicators and categories of the DESI 2020 index

Category	Sub-category	Indicator
1) Connectivity	1a) Adoption of fixed broadband	1a1) Overall adoption of fixed broadband
		1a2) Adoption of fixed broadband of at least 100 Mbps
	1b) Fixed broadband coverage	1b1) High-speed broadband coverage (NGA)
		1b2) Very High-Capacity Fixed Network (VHCN) Coverage
	1c) mobile broadband	1c1) 4G coverage

		1c2) Mobile broadband adoption
		1c3) Preparation for 5G
	1d) Broadband price index	1d1) Broadband Price Index
2) Human Capital	2a) Skills of Internet users	2a1) At least basic digital skills
		2a2) Above basic digital skills
		2a3) At least basic software skills
	2b) Advanced skills and development	2b1) ICT Specialists
		2b2) Women ICT specialists
		2b3) ICT graduates
3) Use of Internet services	3a) Use of the Internet	3a1) People who have never used the Internet
		3a2) Internet users
	3b) Online activities	3b1) News
		3b2) Music, videos and games
		3b3) Video on Demand
		3b4) Video calls
		3b5) Social Networks
		3b6) Taking an online course
	3c) Transactions	3c1) Bank
		3c2) Shopping
		3c3) Online sales
4) Integration of digital technology	4a) Digitization of companies	4a1) Electronic information sharing
		4a2) Social media
		4a3) Big data
		4a4) Cloud
	4b) E-Commerce	4b1) SME selling online
		4b2) e-Commerce revenues
		4b3) Cross-border online sales
5) Digital Utilities	5a) e-Government	5a1) Users of e-Government services
		5a2) Pre-filled forms
		5a3) Completion of online service
		5a4) Digital utilities for businesses
		5a5) Open data