Selecting an Innovation Project under Discount Rate Uncertainty

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The graphical method presented herein allows solving the problem of selecting an innovation project under discount rate uncertainty. The principle of the method consists in plotting NPV graphs of two projects under review on the interval of discount rates, where NPV of at least one project is positive, and calculating the two obtained areas between the intersecting graphs. The presented method enables to expressly find out which of the projects is most attractive, i.e. to solve the known problem of inconsistent assessments of the traditional project selection criteria, such as NPV, PI, PP, IRR and MIRR. As of today, there are plenty of quantitative criteria for selecting an innovation project out of the existing alternatives. These criteria, for instance, include a net present value, a productivity index, a payback period, an internal rate of return and a modified internal rate of return of a project. However, as a matter of practice, their calculation may lead to contradicting results. In addition, the problem of selecting a project becomes more complicated by the fact that it is often very difficult to predict a discount rate. To solve these problems, a graphical method described below may be used.Let us consider the following example. A certain company has drawn up a business plan for packing cereal products into consumer packages with renting out warehouse and office premises and letting out vacant spaces for a parking place. The company considers a strategic (long-term) plan for 11 years.

The first alternative of the long-term plan represents a roadmap specifying 1) the total cost of construction of a new factory and 2) additional investments in launching a new automated line (using a flour bulk storage and the V2-SR-500 drying plant) indicating annual production volumes, the cost of production and selling price of macaroni products. The first alternative also provides for modification of the following metrics by index year: volume of production, unit price, fixed costs and out-of-pocket unit costs. The second alternative of the long-term plan provides for construction of a new factory without additional investments in a new automated line. Also, the second alternative specifics include the fact that the indices of the above mentioned by-year metrics will not change, which is conditioned by the production and technical capabilities of the old automated line. For the sake of convenience for further calculations, the first alternative of the long-term plan will be hereinafter referred to as Project 1, and the second alternative – as Project 2. The cash flows of Projects 1 and 2 are shown in Table 1. The data is presented throughout each year. It is required to select the best project. As is well known, the net present value (NPV) of a project, using discrete discount rate k, may be calculated according to the formula:

$$NPV = \sum_{t=0}^{n} \frac{CIF_t - COF_t}{(1+k)^t},$$

where t – year number, n – total project period (number of years), CIF_t – cash inflow in year t, COF_t – cash outflow in year t, and k – annual price of project capital (in %).

Table 1. Cash Flows of Projects 1 and 2 (million rubles)

Year	1	2	3	4	5	6
Project 1	-25.601	11.99	12.01	12.86	13.73	14.63
Project 2	-20.3	11.99	11.99	11.99	11.99	11.99

Year	7	8	9	10	11
Project 1	15.55	16.5	17.47	18.47	19.48
Project 2	11.99	11.99	11.99	11.99	11.99

Referring to CF_t as any cash flow in year t, i.e. both cash inflow and outflow, the NPV of a project may be calculated according to the formula:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+k)^t}.$$

Hereinafter referring to ΔCF_t as incremental cash flow between two different projects in year t, i.e. CF_t of one project minus CF_t of the other project, the difference between NPV of the two projects at the set value of discount rate k, i.e. ΔNPV, may be calculated as:

$$\Delta NPV = \sum_{t=0}^{n} \frac{\Delta CF_t}{(1+k)^t}.$$

In passing from discrete discount rate k to continuous rate δ and in view of the known ratio between them for one year $1+k=e^{\delta}$ [1], we have:

$$\Delta NPV = \sum_{t=0}^{n} \frac{\Delta CF_t}{e^{\delta t}}.$$

The principle of a further graphical method of comparing innovation projects consists in plotting the NPV graphs of either project under review on the interval of discount rates at which the NPV of at least one project is positive and calculating the two obtained areas between the intersecting graphs (Figure 1). The largest area represents the maximum economic benefit of the project whose graph is larger in terms of such area.

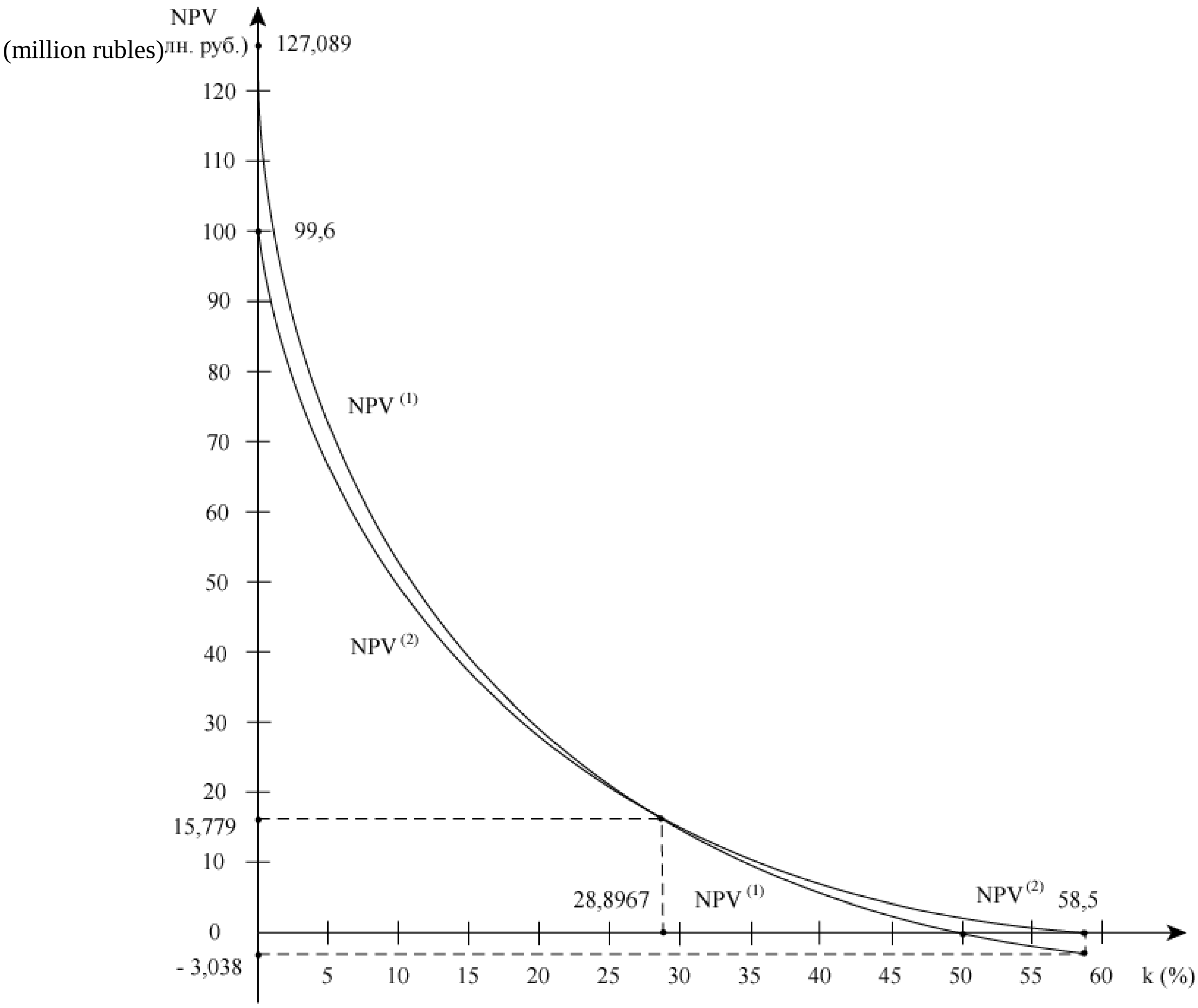

Figure 1. NPV Graphs of Projects 1 and 2

In order to calculate an area between the NPV graphs of two projects on the interval of continuous discount rates from δ_0 to δ_1, it is required to integrate from function ΔNPV from δ_0 to δ_1.

Then

$$\Delta NPV = \int_{\delta_0}^{\delta_1} \sum_{t=0}^{n} \frac{\Delta CF_t}{e^{\delta t}} d\delta = \sum_{t=0}^{n} \Delta CF_t \int_{\delta_0}^{\delta_1} e^{-\delta t} d\delta = \sum_{t=0}^{n} \Delta CF_t \frac{e^{-\delta t}}{-t} \Bigg|_{\delta_0}^{\delta_1} =$$

$$= -\sum_{t=0}^{n} \frac{\Delta CF_t}{t} e^{-\delta t} \Bigg|_{\delta_0}^{\delta_1} = -\sum_{t=0}^{n} \frac{\Delta CF_t}{t} \left(e^{-\delta_1 t} - e^{-\delta_0 t}\right) = \sum_{t=0}^{n} \frac{\Delta CF_t}{t} \left(e^{-\delta_0 t} - e^{-\delta_1 t}\right) =$$

$$= \sum_{t=0}^{n} \frac{\Delta CF_t}{t} \left(\frac{1}{e^{\delta_0 t}} - \frac{1}{e^{\delta_1 t}} \right).$$

In passing back from continuous discount rates to discrete ones, we obtain the following ratio:

$$\Delta NPV = \sum_{t=0}^{n} \frac{\Delta CF_t}{t} \left(\frac{1}{(1+k_0)^t} - \frac{1}{(1+k_1)^t} \right).$$

It is required to individually take into consideration a peculiarity that if t = 0, the latter ratio makes no sense, that is why it is more rational to start summing t from 1. From economic point of view, this means that all cash flows with regard to the both projects, for the purposes of their comparison, should be brought to an earlier period than the year of the first cash flow by time. Consequently, in general, the area between the NPV graphs of two projects on the interval of discount rates from k_0 to k_1 may be defined as:

$$\Delta NPV = \sum_{t=1}^{n} \frac{\Delta CF_t}{t} \left(\frac{1}{(1+k_0)^t} - \frac{1}{(1+k_1)^t} \right).$$

Subsequently, considering Projects 1 and 2 under review, it is required to take into account the fact that all cash flows according to them are uniformly distributed within each year. In this case, for the preset discrete discount rate [1]

$$\Delta NPV = \sum_{t=0}^{n} \frac{\Delta CF_t}{(1+k)^t} (1+k)^{\frac{1}{2}}$$

or in passing from the continuous discount rate,

$$\Delta NPV = \sum_{t=0}^{n} \frac{\Delta CF_t}{e^{\delta t}} e^{\frac{\delta}{2}} = \sum_{t=0}^{n} \frac{\Delta CF_t}{e^{\delta\left(t-\frac{1}{2}\right)}}.$$

Calculating the area between the NPV graphs as an integral from the function ΔNPV from δ_0 to δ_1, we obtain as follows:

$$\Delta NPV = \int_{\delta_0}^{\delta_1} \sum_{t=0}^{n} \frac{\Delta CF_t}{e^{\delta\left(t-\frac{1}{2}\right)}} d\delta = \sum_{t=0}^{n} \Delta CF_t \int_{\delta_0}^{\delta_1} e^{-\delta\left(t-\frac{1}{2}\right)} d\delta = \sum_{t=0}^{n} \Delta CF_t \int_{\delta_0}^{\delta_1} e^{\delta\left(\frac{1}{2}-t\right)} d\delta =$$

$$= \sum_{t=0}^{n} \Delta CF_t \frac{e^{\delta\left(\frac{1}{2}-t\right)}}{\frac{1}{2}-t} \Bigg|_{\delta_0}^{\delta_1} = \sum_{t=0}^{n} \frac{\Delta CF_t}{\frac{1}{2}-t} \left(e^{\delta_1\left(\frac{1}{2}-t\right)} - e^{\delta_0\left(\frac{1}{2}-t\right)}\right) = \sum_{t=0}^{n} \frac{\Delta CF_t}{t-\frac{1}{2}} \left(e^{\delta_0\left(\frac{1}{2}-t\right)} - e^{\delta_1\left(\frac{1}{2}-t\right)}\right) =$$

$$= \sum_{t=0}^{n} \frac{\Delta CF_t}{t-\frac{1}{2}} \left(\frac{1}{e^{\delta_0\left(t-\frac{1}{2}\right)}} - \frac{1}{e^{\delta_1\left(t-\frac{1}{2}\right)}} \right).$$

In passing from continuous discount rates to discrete ones and considering that, as in the general case, all cash flows are brought to an earlier period than the period of the first cash flows by time, we have the final formula for calculating the area between NPV of the projects the cash flows under which are uniformly distributed within each year:

$$\Delta NPV = \sum_{t=1}^{n} \frac{\Delta CF_t}{t-0{,}5} \left(\frac{1}{(1+k_0)^{t-0{,}5}} - \frac{1}{(1+k_1)^{t-0{,}5}} \right). \qquad (1)$$

Then let us plot NPV graphs of Projects 1 and 2, i.e. $NPV^{(1)}$ and $NPV^{(2)}$.

For this purpose, let us first calculate their values at the discount rate of 0%.

$$NPV_{0\%}^{(1)} = -25.601 + 11.99 + 12.01 + 12.86 + 13.73 + 14.63 + 15.55 + 16.5 + 17.47 + $$

$$+18.47 + 19.48 = 127.089$$ (million rubles).

$$NPV_{0\%}^{(2)} = -20.3 + 11.99 \cdot 10 = 99.6$$ (million rubles).

Pursuant to Figure 1, the point of intersection of each NPV graph with the horizontal axis may be defined from the condition NPV = 0. In this case, the discount rate will represent a project internal rate of return (IRR) which may be defined, for example, by using the method of proportional parts [1]. The IRR rate of Project 1 turns out to be equal to the value $IRR^{(1)}$=50%, and of Project 2 – to the value $IRR^{(2)}$=58.5%.

It is further necessary to calculate points of intersection of the two NPV graphs. To do this, let us first form the incremental cash flow $\Delta CF_t = CF_t^{(1)} - CF_t^{(2)}$ (Table 2).

Table 2. Incremental Cash Flow ΔCF_t (million rubles)

Year	1	2	3	4	5	6
ΔCF_t	-5.301	0	0.02	0.87	1.74	2.64

Year	7	8	9	10	11
ΔCF_t	3.56	4.51	5.48	6.48	7.49

The IRR rate of such cash flow may be also found by using the method of proportional parts. So we have approximately IRR=28.8967%.

Let us assess the NPV of either project at the discount rate found.

The NPV of a project, as it was described above, provided that all cash flows thereunder are uniformly distributed within each year, may be found according to the formula:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+k)^t}(1+k)^{0,5} = \sum_{t=0}^{n} \frac{CF_t}{(1+k)^{t-0,5}}.$$

Then the NPV of Project 1 will be equal to:

$$NPV_{28.8967\%}^{(1)} = -\frac{25.601}{1.288967^{0.5}} + \frac{11.99}{1.288967^{1.5}} + \frac{12.01}{1.288967^{2.5}} + \frac{12.86}{1.288967^{3.5}} +$$

$$+\frac{13.73}{1.288967^{4.5}} + \frac{14.63}{1.288967^{5.5}} + \frac{15.55}{1.288967^{6.5}} + \frac{16.5}{1.288967^{7.5}} + \frac{17.47}{1.288967^{8.5}} +$$

$$+\frac{18.47}{1.288967^{9.5}} + \frac{19.48}{1.288967^{10.5}} = 15.77913$$ (million rubles).

The NPV of Project 2 may be calculated more easily taking into consideration the fact that receipts of net income thereunder represent a yearly annuity [1]. Then the NPV of Project 2 may be calculated according to the formula:

$$NPV = \left(-\frac{K}{1+k} + NI \cdot \frac{1-(1+k)^{-n}}{k} \cdot \frac{1}{1+k}\right)(1+k)^{0,5} =$$

$$= \left(-K + NI \cdot \frac{1-(1+k)^{-n}}{k}\right)\frac{1}{(1+k)^{0,5}},$$

where K – capital investment, and NI – net income. Then the NPV of Project 2 will be

$$NPV_{28.8967\%}^{(2)} = \left(-20.3 + 11.99\frac{1-1.288967^{-10}}{0.288967}\right)\frac{1}{1.288967^{0.5}} =$$

$$= 15.779641$$ (million rubles).

Let us plot the graphs $NPV^{(1)}$ and $NPV^{(2)}$ in Figure 1.

Let us additionally calculate the NPV of Project 1 at the discount rate of 58.5%, i.e. at the IRR rate of Project 2. In this case, the NPV of Project 1 will constitute the negative value of -3.038184 million rubles.
Then let us consider the entire interval of possible positive NPVs according to Figure 1, i.e. where k changes from 0% to 58.5%.

Subject to the method described above, using the formula (1), let us first calculate the area where $NPV^{(1)} > NPV^{(2)}$, i.e. where Project 1 is more profitable than Project 2.

$$\Delta NPV_{(1\succ 2)} = -\frac{5.301}{0.5}\left(1-\frac{1}{1.288967^{0.5}}\right)+\frac{0.02}{2.5}\left(1-\frac{1}{1.288967^{2.5}}\right)+$$
$$+\frac{0.87}{3.5}\left(1-\frac{1}{1.288967^{3.5}}\right)+\frac{1.74}{4.5}\left(1-\frac{1}{1.288967^{4.5}}\right)+\frac{2.64}{5.5}\left(1-\frac{1}{1.288967^{5.5}}\right)+$$
$$+\frac{3.56}{6.5}\left(1-\frac{1}{1.288967^{6.5}}\right)+\frac{4.51}{7.5}\left(1-\frac{1}{1.288967^{7.5}}\right)+\frac{5.48}{8.5}\left(1-\frac{1}{1.288967^{8.5}}\right)+$$
$$+\frac{6.48}{9.5}\left(1-\frac{1}{1.288967^{9.5}}\right)+\frac{7.49}{10.5}\left(1-\frac{1}{1.288967^{10.5}}\right)=2.319889\ .$$

Then let us assess the area where $NPV^{(2)} > NPV^{(1)}$, i.e. where Project 2 is more profitable than Project 1.

$$\Delta NPV_{(2\succ 1)} = \frac{5.301}{0.5}\left(\frac{1}{1.288967^{0.5}}-\frac{1}{1.585^{0.5}}\right)-\frac{0.02}{2.5}\left(\frac{1}{1.288967^{2.5}}-\frac{1}{1.585^{2.5}}\right)-$$
$$-\frac{0.87}{3.5}\left(\frac{1}{1.288967^{3.5}}-\frac{1}{1.585^{3.5}}\right)-\frac{1.74}{4.5}\left(\frac{1}{1.288967^{4.5}}-\frac{1}{1.585^{4.5}}\right)-$$
$$-\frac{2.64}{5.5}\left(\frac{1}{1.288967^{5.5}}-\frac{1}{1.585^{5.5}}\right)-\frac{3.56}{6.5}\left(\frac{1}{1.288967^{6.5}}-\frac{1}{1.585^{6.5}}\right)-$$
$$-\frac{4.51}{7.5}\left(\frac{1}{1.288967^{7.5}}-\frac{1}{1.585^{7.5}}\right)-\frac{5.48}{8.5}\left(\frac{1}{1.288967^{8.5}}-\frac{1}{1.585^{8.5}}\right)-$$
$$-\frac{6.48}{9.5}\left(\frac{1}{1.288967^{9.5}}-\frac{1}{1.585^{9.5}}\right)-\frac{7.49}{10.5}\left(\frac{1}{1.288967^{10.5}}-\frac{1}{1.585^{10.5}}\right)=0.400731\ .$$

The above calculations show that the first area is larger; consequently, Project 1 is more profitable than Project 2 since the larger area, the more economic benefit.

Thus, let us formulate the following final conclusions:

1. The graphical method under review allows solving the problem of selecting an innovation project under discount rate uncertainty. Specifically, if it is rather difficult to predict it under the conditions of a high risk, then taking as a basis the whole interval of rates from zero to the value at which the NPV of one of the projects under review is equal to zero, and the NPV of the other project is negative, it is possible to calculate the areas between the NPVs of the both projects on the preset interval of rates. The largest out of the areas obtained denotes the maximum economic benefit of that project whose NPV graph goes higher.

2. The graphical method enables to expressly find out which of the projects is most attractive, i.e. to solve the known problem of inconsistent assessments of the traditional project selection criteria, such as net present value, productivity index, payback period, internal rate of return and modified internal rate of return.

References

1. E.M. Chetyrkin. Financial Mathematics: Textbook. – 2nd edition revised – M.: Delo, 2002.